GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Mark Schemes for the Units

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Oxford Cambridge and RSA Examinations

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MARK SCHEME FOR THE UNITS

4751 (C1) Introduction to Advanced Mathematics

Section A

1	(i) 0.125 or 1/8 (ii) 1	1 1	as final answer	2
2	$y = 5x - 4$ www	3	M2 for $\frac{y-11}{-9-11}=\frac{x-3}{-1-3}$ o.e. or M1 for grad = $\frac{11-(-9)}{3-(-1)}$ or 5 eg in $y = 5x + k$ and M1 for $y - 11$ = their $m(x - 3)$ o.e. or subst (3, 11) or (−1, −9) in y = their $mx + c$ or M1 for $y = kx - 4$ (eg may be found by drawing)	3
3	$x > 9/6$ o.e. or $9/6 < x$ o.e. www isw	3	M2 for $9 < 6x$ or M1 for $-6x < -9$ or $k < 6x$ or $9 < kx$ or $7 + 2 < 5x + x$ [condone $\le$ for Ms]; if 0, allow SC1 for 9/6 o.e found	3
4	$a = -5$ www	3	M1 for f(2) = 0 used and M1 for $10 + 2a = 0$ or better long division used: M1 for reaching $(8 + a)x - 6$ in working and M1 for $8 + a = 3$ equating coeffts method: M2 for obtaining $x^3 + 2x^2 + 4x + 3$ as other factor	3
5	(i) 4[x^3] (ii) 84[x^2] www	2 3	ignore any other terms in expansion M1 for $-3[x^3]$ and $7[x^3]$ soi; M1 for $\frac{7\times6}{2}$ or 21 or for Pascal's triangle seen with 1 7 21 … row and M1 for 2^2 or 4 or $\{2x\}^2$	5

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6	1/5 or 0.2 o.e. www	3	M1 for $3x + 1 = 2x \times 4$ and M1 for $5x = 1$ o.e. or M1 for $1.5+\frac{1}{2x}=4$ and M1 for $\frac{1}{2x}=2.5$ o.e.	3
7	(i) $5^{3.5}$ or $k = 3.5$ or 7/2 o.e. (ii) $16a^6b^{10}$	2 2	M1 for $125 = 5^3$ or $\sqrt{5}=5^{\frac{1}{2}}$ SC1 for $5^{\frac{3}{2}}$ o.e. as answer without working M1 for two 'terms' correct and multiplied; mark final answer only	4
8	$b^2 - 4ac$ soi $k^2 - 4 \times 2 \times 18 < 0$ o.e. $-12 < k < 12$	M1 M1 A2	allow in quadratic formula or clearly looking for perfect square condone $\le$; or M1 for 12 identified as boundary may be two separate inequalities; A1 for $\le$ used or for one 'end' correct if two separate correct inequalities seen, isw for then wrongly combining them into one statement; condone b instead of k; if no working, SC2 for $k < 12$ and SC2 for $k > -12$ (ie SC2 for each 'end' correct)	4
9	$y + 5 = xy + 2x$ $y - xy = 2x - 5$ oe or ft $y(1 - x) = 2x - 5$ oe or ft $[y =]\frac{2x-5}{1-x}$ oe or ft as final answer	M1 M1 M1 M1	for expansion for collecting terms for taking out y factor; dep on xy term for division and no wrong work after ft earlier errors for equivalent steps if error does not simplify problem	4
10	(i) $9\sqrt{3}$ (ii) $6 + 2\sqrt{2}$ www	2 3	M1 for $5\sqrt{3}$ or $4\sqrt{3}$ seen M1 for attempt to multiply num. and denom. by $3 + \sqrt{2}$ and M1 for denom. 7 or $9 - 2$ soi from denom. mult by $3 + \sqrt{2}$	5

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Section B

11	**i**	C, mid pt of AB = $\left(\frac{11+(-1)}{2}, \frac{4}{2}\right)$ = (5, 2)	B1	evidence of method required – may be on diagram, showing equal steps, or start at A or B and go half the difference towards the other		
		[AB^2 =] $12^2 + 4^2$ [= 160] oe or [CB^2 =] $6^2 + 2^2$ [=40] oe with AC	B1	or square root of these; accept unsimplified		
		quote of $(x - a)^2 + (y - b)^2 = r^2$ o.e with different letters	B1	or (5, 2) clearly identified as centre and $\sqrt{40}$ as r (or 40 as r^2) www or quote of *gfc* formula and finding c = −11		
		completion (ans given)	B1	dependent on centre (or midpt) and radius (or radius^2) found independently and correctly	4	
	ii	correct subst of $x = 0$ in circle eqn soi	M1			
		$(y - 2)^2 = 15$ or $y^2 - 4y - 11$ [= 0]	M1	condone one error		
		$y - 2 = \pm\sqrt{15}$ or ft	M1	or use of quad formula (condone one error in formula); ft only for 3 term quadratic in y		
		[y =]$2 \pm \sqrt{15}$ cao	A1	if $y = 0$ subst, allow SC1 for (11, 0) found alt method: M1 for y values are $2 \pm a$ M1 for $a^2 + 5^2 = 40$ soi M1 for $a^2 = 40 - 5^2$ soi A1 for [y =]$2 \pm \sqrt{15}$ cao	4	
	iii	grad AB = $\frac{4}{11-(-1)}$ or 1/3 o.e.	M1	or grad AC (or BC)		
		so grad tgt = – 3	M1	or ft −1/their gradient of AB		
		eqn of tgt is $y - 4 = -3(x - 11)$	M1	or subst (11, 4) in $y = -3x + c$ or ft (no ft for their grad AB used)		
		$y = -3x + 37$ or $3x + y = 37$	A1	accept other simplified versions		
		(0, 37) and (37/3, 0) o.e. ft isw	B2	B1 each, ft their tgt for grad ≠ 1 or 1/3; accept $x = 0$, $y = 37$ etc NB alt method: intercepts may be found first by proportion then used to find eqn	6	14

12	**i**	$3x^2 + 6x + 10 = 2 - 4x$	M1	for subst for x or y or subtraction attempted		
		$3x^2 + 10x + 8$ [= 0]	M1	or $3y^2 - 52y + 220$ [=0]; for rearranging to zero (condone one error)		
		$(3x + 4)(x + 2)$ [=0]	M1	or $(3y - 22)(y - 10)$; for sensible attempt at factorising or formula or completing square		
		$x = -2$ or $-4/3$ o.e. $y = 10$ or $22/3$ o.e.	A1 A1	or A1 for each of $(-2, 10)$ and $(-4/3, 22/3)$ o.e.	5	
	ii	$3(x + 1)^2 + 7$	4	1 for $a = 3$, 1 for $b = 1$, 2 for $c = 7$ or M1 for $10 - 3 \times$ their b^2 soi or for $7/3$ or for $10/3 -$ their b^2 soi	4	
	iii	min at $y = 7$ or ft from (ii) for positive c (ft for (ii) only if in correct form)	B2	may be obtained from (ii) or from good symmetrical graph or identified from table of values showing symmetry condone error in x value in stated min ft from (iii) [getting confused with 3 factor] B1 if say turning pt at $y = 7$ or ft without identifying min or M1 for min at $x = -1$ [e.g. may start again and use calculus to obtain $x = -1$] or min when $(x + 1)^{[2]} = 0$; and A1 for showing y positive at min or M1 for showing discriminant neg. so no real roots and A1 for showing above axis not below eg positive x^2 term or goes though (0, 10) or M1 for stating bracket squared must be positive [or zero] and A1 for saying other term is positive	2	11

13	**i**	any correct y value calculated from quadratic seen or implied by plots	B1	for $x \neq 0$ or 1; may be for neg x or eg min.at (2.5, −1.25)	
		(0, 5)(1, 1)(2, −1)(3, −1)(4,1) and (5,5) plotted	P2	tol 1 mm; P1 for 4 correct [including (2.5, −1.25) if plotted]; plots may be implied by curve within 1 mm of correct position	
		good quality smooth parabola within 1mm of their points	C1	allow for correct points only [accept graph on graph paper, not insert]	4
	ii	$x^2 - 5x + 5 = \frac{1}{x}$	M1		
		$x^3 - 5x^2 + 5x = 1$ and completion to given answer	M1		2
	iii	divn of $x^3 - 5x^2 + 5x - 1$ by $x - 1$ as far as $x^3 - x^2$ used in working	M1	or inspection eg $(x - 1)(x^2 \ldots\ldots + 1)$ or equating coeffts with two correct coeffts found	
		$x^2 - 4x + 1$ obtained	A1		
		use of $b^2 - 4ac$ or formula with quadratic factor	M1	or $(x - 2)^2 = 3$; may be implied by correct roots or $\sqrt{12}$ obtained	
		$\sqrt{12}$ obtained and comment re shows other roots (real and) irrational <u>or</u> for $2 \pm \sqrt{3}$ or $\frac{4 \pm \sqrt{12}}{2}$ obtained isw	A2	[A1 for $\sqrt{12}$ and A1 for comment] NB A2 is available only for correct quadratic factor used; if wrong factor used, allow A1 ft for obtaining two irrational roots or for their discriminant and comment re irrational [no ft if their discriminant is negative]	5

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4752 (C2) Concepts for Advanced Mathematics

Section A

1	$4x^5$ $-12x^{-\frac{1}{2}}$ $+c$	1 2 1	M1 for other $kx^{-\frac{1}{2}}$	4
2	95.25, 95.3 or 95	4	M3 ½×5×(4.3+0+2[4.9+4.6+3.9+2.3+1.2]) M2 with 1 error, M1 with 2 errors. Or M3 for 6 correct trapezia.	4
3	1.45 o.e.	2	M1 for $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$ oe	2
4	105 and 165	3	B1 for one of these or M1 for $2x = 210$ or 330	3
5	(i) graph along $y = 2$ with V at (3,2) (4,1) & (5,2) (ii) graph along $y = 6$ with V at (1,6) (2,3) & (3,6)	2 2	M1 for correct V, or for f(x+2) B1 for (2,k) with all other elements correct	4
6	(i) 54.5 (ii) Correct use of sum of AP formula with n = 50, 20, 19 or 21 with their d and a = 7 eg S_{50} = 3412.5, S_{20} = 615 Their $S_{50} - S_{20}$ dep on use of ap formula 2797.5 c.a.o.	2 M1 M1 A1	B1 for $d = 2.5$ or M2 for correct formula for S_{30} with their d M1 if one slip	5
7	$8x - x^{-2}$ o.e. their $\frac{dy}{dx} = 0$ correct step x = ½ c.a.o.	2 M1 DM1 A1	B1 each term s.o.i. s.o.i.	5
8	(i) 48 geometric, or GP (ii) mention of $\|r\| < 1$ condition o.e. S = 128	1 1 1 2	M1 for $\frac{192}{1 - -\frac{1}{2}}$	5
9	(i) 1 (ii) (A) 3.5 $\log_a x$ (ii) (B) $-\log_a x$	1 2 1	M1 for correct use of 1st or 3rd law	4

Section B

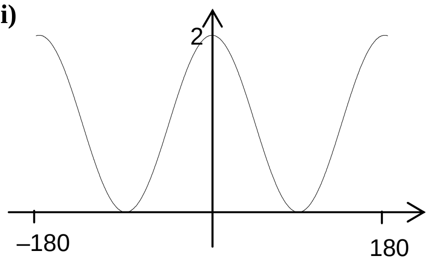
10	**i**	$7 - 2x$	M1		
		$x = 2$, gradient $= 3$	A1	differentiation must be used	
		$x = 2, y = 4$	B1		
		y – their 4 = their grad $(x - 2)$	M1	or use of y = their $mx + c$ and subst (2, their 4), dependent on diffn seen	
		subst $y = 0$ in their linear eqn	M1		
		completion to $x = \frac{2}{3}$ (ans given)	A1		6
	ii	f(1) = 0 or factorising to $(x - 1)(6 - x)$ or $(x - 1)(x - 6)$	1	or using quadratic formula correctly to obtain $x = 1$	
		6 www	1		2
	iii	$\frac{7}{2}x^2 - \frac{1}{3}x^3 - 6x$	M1	for two terms correct; ignore $+c$	
		value at 2 – value at 1	M1	ft attempt at integration only	
		$2\frac{1}{6}$ or 2.16 to 2.17	A1		
		$\frac{1}{2} \times \frac{4}{3} \times 4$ – their integral	M1		
		0.5 o.e.	A1		5
11	**i**(A)	150 (cm) or 1.5 m	2	M1 for 2.5 × 60 or 2.5 × 0.6 or for 1.5 with no units	2
	i(B)	$\frac{1}{2} \times 60^2 \times 2.5$ or 4500	M1	or equivalents in m^2	
		$\frac{1}{2} \times 140^2 \times 2.5$ or 24 500	M1		
		subtraction of these	DM1		
		20 000 (cm^2) isw	A1	or 2 m^2	4
	ii(A)	attempt at use of cosine rule	M1	condone 1 error in substitution	
		$\cos \text{EFP} = \dfrac{3.5^2 + 2.8^2 - 1.6^2}{2 \times 2.8 \times 3.5}$ o.e.	M1		
		26.5 to 26.65 or 27	A1		3
	ii(B)	2.8 sin (their EFP) o.e.	M1		
		1.2 to 1.3 [m]	A1		2

12	**i**	$\log a + \log (b^t)$ www clear use of $\log (b^t) = t \log b$ dep	B1 B1	condone omission of base throughout question	2
	ii	(2.398), 2.477, 2.556 , 2.643, 2.724 points plotted correctly f.t. ruled line of best fit f.t.	T1 P1 1	On correct square	3
	iii	$\log a = 2.31$ to 2.33 $a = 204$ to 214 $\log b = 0.08$ approx $b = 1.195$ to 1.215	M1 A1 M1 A1	ft their intercept ft their gradient	4
	iv	eg £210 million dep	1	their £a million	1
	v	$\frac{\log 1000 - \text{their intercept}}{\text{their gradient}} \approx \frac{3-2.32}{0.08}$ = 8.15 to 8.85	M1 A1	or B2 from trials	2

4753 (C3) Methods for Advanced Mathematics

Section A

1 $\lvert x-1\rvert<3 \Rightarrow -3<x-1<3$ $\Rightarrow \quad -2<x<4$	M1 A1 B1 [3]	or $x - 1 = \pm 3$, or squaring $\Rightarrow$ correct quadratic $\Rightarrow$ $(x + 2)(x - 4)$ (condone factorising errors) or correct sketch showing $y = 3$ to scale $-2 <$ < 4 (penalise $\le$ once only)
2(i) $y = x\cos 2x$ $\Rightarrow \dfrac{dy}{dx} = -2x\sin 2x + \cos 2x$	M1 B1 A1 [3]	product rule d/dx (cos2x) = –2sin2x oe cao
(ii) $\int x\cos 2x dx = \int x\dfrac{d}{dx}(\dfrac{1}{2}\sin 2x)dx$ $=\dfrac{1}{2}x\sin 2x - \int \dfrac{1}{2}\sin 2x dx$ $=\dfrac{1}{2}x\sin 2x + \dfrac{1}{4}\cos 2x + c$	M1 A1 A1ft A1 [4]	parts with $u = x$, $v = ½ \sin 2x$ $+\dfrac{1}{4}\cos 2x$ cao – must have + c
3 *Either* $y=\dfrac{1}{2}\ln(x-1)$ $\quad x\leftrightarrow y$ $\Rightarrow \quad x=\dfrac{1}{2}\ln(y-1)$ $\Rightarrow \quad 2x = \ln(y - 1)$ $\Rightarrow \quad e^{2x} = y - 1$ $\Rightarrow \quad 1 + e^{2x} = y$ $\Rightarrow \quad g(x) = 1 + e^{2x}$	 M1 M1 E1	or $y = e^{(x-1)/2}$ attempt to invert and interchanging x with y o.e. (at any stage) $e^{\ln y-1} = y - 1$ or ln (e^y) = y used www
or $gf(x) = g(½ \ln (x - 1))$ $= 1 + e^{\ln(x-1)}$ $= 1 + x - 1$ $= x$	M1 M1 E1 [3]	or fg(x) = … (correct way round) $e^{\ln(x-1)} = x - 1$ or $\ln(e^{2x}) = 2x$ www
4 $\int_0^2 \sqrt{1+4x}dx \quad$ let $u=1+4x,\ du=4dx$ $=\int_1^9 u^{1/2}.\dfrac{1}{4}du$ $=\left[\dfrac{1}{6}u^{3/2}\right]_1^9$ $=\dfrac{27}{6}-\dfrac{1}{6}=\dfrac{26}{6}=\dfrac{13}{3}$ or $4\dfrac{1}{3}$	M1 A1 B1 M1 A1cao	$u = 1 + 4x$ and du/dx = 4 or du = 4dx $\int u^{1/2}.\dfrac{1}{4}du$ $\int u^{1/2}du=\dfrac{u^{3/2}}{3/2}$ soi substituting correct limits (u or x) dep attempt to integrate
or $\dfrac{d}{dx}(1+4x)^{3/2}=4.\dfrac{3}{2}(1+4x)^{1/2}=6(1+4x)^{1/2}$ $\Rightarrow \int_0^2 (1+4x)^{1/2}dx=\left[\dfrac{1}{6}(1+4x)^{3/2}\right]_0^2$ $=\dfrac{27}{6}-\dfrac{1}{6}=\dfrac{26}{6}=\dfrac{13}{3}$ or $4\dfrac{1}{3}$	M1 A1 A1 M1 A1cao [5]	$k(1 + 4x)^{3/2}$ $\int (1+4x)^{1/2}dx=\dfrac{2}{3}(1+4x)^{3/2}...$ × ¼ substituting limits (dep attempt to integrate)

5(i) period 180°	B1 [1]	condone $0 \le x \le 180°$ or π
(ii) one-way stretch in x-direction scale factor ½ translation in y-direction through $\begin{pmatrix}0\\1\end{pmatrix}$	M1 A1 M1 A1 [4]	[either way round…] condone 'squeeze', 'contract' for M1 stretch used and s.f ½ condone 'move', 'shift', etc for M1 'translation' used, +1 unit $\begin{pmatrix}0\\1\end{pmatrix}$ only is M1 A0
(iii) 	M1 B1 A1 [3]	correct shape, touching x-axis at -90°, 90° correct domain (0, 2) marked or indicated (i.e. amplitude is 2)
6(i) e.g $p = 1$ and $q = -2$ $p > q$ but $1/p = 1 > 1/q = -½$	M1 E1 [2]	stating values of p, q with $p \ge 0$ and $q \le 0$ (but not $p = q = 0$) showing that $1/p > 1/q$ - if 0 used, must state that 1/0 is undefined or infinite
(ii) Both p and q positive (or negative)	B1 [1]	or $q > 0$, 'positive integers'
7(i) $\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{\frac{2}{3}x^{-1/3}}{\frac{2}{3}y^{-1/3}}$ $= -\frac{y^{1/3}}{x^{1/3}} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$ *	M1 A1 M1 E1 [4]	Implicit differentiation (must show = 0) solving for dy/dx www. Must show, or explain, one more step.
(ii) $\frac{dy}{dt} = \frac{dy}{dx}.\frac{dx}{dt}$ $= -\left(\frac{8}{1}\right)^{\frac{1}{3}}.6$ $= -12$	M1 A1 A1cao [3]	any correct form of chain rule

8(i) When $x = 1$ $y = 1^2 - (\ln 1)/8 = 1$ Gradient of PR = $(1 + 7/8)/1 = 1\frac{7}{8}$	B1 M1 A1 [3]	 1.9 or better
(ii) $\frac{dy}{dx} = 2x - \frac{1}{8x}$ When $x = 1$, $dy/dx = 2 - 1/8 = 1\frac{7}{8}$ Same as gradient of PR, so PR touches curve	B1 B1dep E1 [3]	cao 1.9 or better dep 1st B1 dep gradients exact
(iii) Turning points when $dy/dx = 0$ $\Rightarrow \quad 2x - \frac{1}{8x} = 0$ $\Rightarrow \quad 2x = \frac{1}{8x}$ $\Rightarrow \quad x^2 = 1/16$ $\Rightarrow \quad x = ¼\ (x > 0)$ When $x = ¼$, $y = \frac{1}{16} - \frac{1}{8}\ln\frac{1}{4} = \frac{1}{16} + \frac{1}{8}\ln 4$ So TP is $(\frac{1}{4}, \frac{1}{16} + \frac{1}{8}\ln 4)$	M1 M1 A1 M1 A1cao [5]	setting their derivative to zero multiplying through by x allow verification substituting for x in y o.e. but must be exact, not $1/4^2$. Mark final answer.
(iv) $\frac{d}{dx}(x\ln x - x) = x.\frac{1}{x} + 1.\ln x - 1 = \ln x$	M1 A1	product rule $\ln x$
Area = $\int_1^2 (x^2 - \frac{1}{8}\ln x)dx$ $= \left[\frac{1}{3}x^3 - \frac{1}{8}(x\ln x - x)\right]_1^2$ $= (\frac{8}{3} - \frac{1}{4}\ln 2 + \frac{1}{4}) - (\frac{1}{3} - \frac{1}{8}\ln 1 + \frac{1}{8})$ $= \frac{7}{3} + \frac{1}{8} - \frac{1}{4}\ln 2$ $= \frac{59}{24} - \frac{1}{4}\ln 2$ *	M1 M1 A1 M1 E1 [7]	correct integral and limits (soi) – condone no dx $\int \ln x dx = x\ln x - x$ used (or derived using integration by parts) $\frac{1}{3}x^3 - \frac{1}{8}(x\ln x - x)$ – bracket required substituting correct limits must show at least one step

9(i) Asymptotes when $(\sqrt{})(2x - x^2) = 0$ $\Rightarrow$ $x(2 - x) = 0$ $\Rightarrow$ $x = 0$ or 2 so $a = 2$ Domain is $0 < x < 2$	M1 A1 B1ft [3]	or by verification $x > 0$ and $x < 2$, not $\le$
(ii) $y = (2x - x^2)^{-1/2}$ let $u = 2x - x^2$, $y = u^{-1/2}$ $\Rightarrow$ $dy/du = -½\, u^{-3/2}$, $du/dx = 2 - 2x$ $\Rightarrow$ $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = -\frac{1}{2}(2x - x^2)^{-3/2}.(2 - 2x)$ $= \frac{x - 1}{(2x - x^2)^{3/2}}$ *	M1 B1 A1 E1	chain rule (or within correct quotient rule) $-½\, u^{-3/2}$ or $-\frac{1}{2}(2x - x^2)^{-3/2}$ or $\frac{1}{2}(2x - x^2)^{-1/2}$ in quotient rule $\times (2 - 2x)$ www – penalise missing brackets here
$dy/dx = 0$ when $x - 1 = 0$ $\Rightarrow$ $x = 1$, $y = 1/\sqrt{(2 - 1)} = 1$ Range is $y \ge 1$	M1 A1 B1 B1ft [8]	extraneous solutions M0
(iii) (*A*) $g(-x) = \frac{1}{\sqrt{1 - (-x)^2}} = \frac{1}{\sqrt{1 - x^2}} = g(x)$	M1 E1	Expression for g(–*x*) – must have g(–*x*) = g(*x*) seen
(*B*) $g(x - 1) = \frac{1}{\sqrt{1 - (x - 1)^2}}$ $= \frac{1}{\sqrt{1 - x^2 + 2x - 1}} = \frac{1}{\sqrt{2x - x^2}} = f(x)$	M1 E1	must expand bracket
(*C*) f(*x*) is g(*x*) translated 1 unit to the right. But g(*x*) is symmetrical about O*y* So f(*x*) is symmetrical about $x = 1$.	M1 M1 A1	dep both M1s
or $f(1 - x) = g(-x)$, $f(1 + x) = g(x)$ $\Rightarrow$ $f(1 + x) = f(1 - x)$ $\Rightarrow$ f(*x*) is symmetrical about $x = 1$.	M1 E1 A1 [7]	or $f(1-x) = \frac{1}{\sqrt{2 - 2x - (1-x)^2}} = \frac{1}{\sqrt{2 - 2x - 1 + 2x - x^2}} = \frac{1}{\sqrt{1 - x^2}}$ $f(1+x) = \frac{1}{\sqrt{2 + 2x - (1+x)^2}} = \frac{1}{\sqrt{2 + 2x - 1 - 2x - x^2}} = \frac{1}{\sqrt{1 - x^2}}$

4754 (C4) Applications of Advanced Mathematics

Section A

1 $\frac{3x+2}{x(x^2+1)}=\frac{A}{x}+\frac{Bx+C}{(x^2+1)}$ ⇒ $3x+2=A(x^2+1)+(Bx+C)x$ $x=0 \Rightarrow 2=A$ coefft of x^2 : $0=A+B \Rightarrow B=-2$ coefft of x: $3=C$ ⇒ $\frac{3x+2}{x(x^2+1)}=\frac{2}{x}+\frac{3-2x}{(x^2+1)}$	M1 M1 B1 M1 A1 A1 [6]	correct partial fractions equating coefficients at least one of B,C correct
2(i) $(1+2x)^{1/3}=1+\frac{1}{3}.2x+\frac{\frac{1}{3}.(-\frac{2}{3})}{2!}(2x)^2+...$ $=1+\frac{2}{3}x-\frac{2}{18}4x^2+...$ $=1+\frac{2}{3}x-\frac{4}{9}x^2+...$ * Next term $=\frac{\frac{1}{3}.(-\frac{2}{3})(-\frac{5}{3})}{3!}(2x)^3$ $=\frac{40}{81}x^3$ Valid for $-1<2x<1$ ⇒ $-½<x<½$	M1 A1 E1 M1 A1 B1 [6]	binomial expansion correct unsimplified expression simplification www
3 $4\mathbf{j}-3\mathbf{k}=\lambda\,\mathbf{a}+\mu\,\mathbf{b}$ $=\lambda(2\mathbf{i}+\mathbf{j}-\mathbf{k})+\mu(4\mathbf{i}-2\mathbf{j}+\mathbf{k})$ ⇒ $0=2\lambda+4\mu$ $4=\lambda-2\mu$ $-3=-\lambda+\mu$ ⇒ $\lambda=-2\mu,\ 2\lambda=4 \Rightarrow \lambda=2,\ \mu=-1$	M1 M1 A1 A1, A1 [5]	equating components at least two correct equations
4 LHS = $\cot\beta-\cot\alpha$ $=\frac{\cos\beta}{\sin\beta}-\frac{\cos\alpha}{\sin\alpha}$ $=\frac{\sin\alpha\cos\beta-\cos\alpha\sin\beta}{\sin\alpha\sin\beta}$ $=\frac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$ **OR** RHS=$\frac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta}=\frac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta}-\frac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}$ $=\cot\beta-\cot\alpha$	M1 M1 E1 M1 M1 E1 [3]	cot = cos / sin combining fractions www using compound angle formula splitting fractions using cot=cos/sin

5(i) Normal vectors $\begin{pmatrix}2\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\0\\-1\end{pmatrix}$ Angle between planes is θ, where $\cos\theta = \frac{2\times1+(-1)\times0+1\times(-1)}{\sqrt{2^2+(-1)^2+1^2}\sqrt{1^2+0^2+(-1)^2}}$ = 1/√12 ⇒ θ = 73.2° or 1.28 rads	B1 M1 M1 A1 [4]	scalar product finding invcos of scalar product divided by two modulae
(ii) $\mathbf{r}=\begin{pmatrix}2\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$ $=\begin{pmatrix}2+2\lambda\\-\lambda\\1+\lambda\end{pmatrix}$ ⇒ $2(2+2\lambda)-(-\lambda)+(1+\lambda)=2$ ⇒ $5+6\lambda=2$ ⇒ λ = – ½ So point of intersection is (1, ½ , ½)	B1 M1 A1 A1 [4]	
6(i) $\cos\theta+\sqrt{3}\sin\theta = r\cos(\theta-\alpha)$ $= R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 1,\ R\sin\alpha=\sqrt{3}$ ⇒ $R^2 = 1^2+(\sqrt{3})^2 = 4, R = 2$ tan α = √3 ⇒ α = π/3	B1 M1 M1 A1 [4]	$R = 2$ equating correct pairs tan α = √3 o.e.
(ii) derivative of tan θ is $\sec^2\theta$ $\int_0^{\frac{\pi}{3}}\frac{1}{(\cos\theta+\sqrt{3}\sin\theta)^2}d\theta = \int_0^{\frac{\pi}{3}}\frac{1}{4}\sec^2(\theta-\frac{\pi}{3})d\theta$ $=\left[\frac{1}{4}\tan(\theta-\frac{\pi}{3})\right]_0^{\frac{\pi}{3}}$ = ¼ (0 – (–√3)) = √3/4 *	B1 M1 A1 E1 [4]	ft their α $\frac{1}{R^2}$ [tan (θ – π/3] ft their R,α(in radians) www

Section B

7(i) (*A*) 9 / 1.5 = 6 hours (*B*) 18/1.5 = 12 hours	B1 B1 [2]	
(ii) $\frac{d\theta}{dt} = -k(\theta - \theta_0)$ $\Rightarrow \int \frac{d\theta}{\theta - \theta_0} = \int -kdt$ $\Rightarrow \ln(\theta - \theta_0) = -kt + c$ $\theta - \theta_0 = e^{-kt+c}$ $\theta = \theta_0 + Ae^{-kt}$ *	M1 A1 A1 M1 E1 [5]	separating variables $\ln(\theta - \theta_0)$ $-kt + c$ anti-logging correctly(with *c*) $A = e^{c}$
(iii) 98 = 50 + Ae^0 $\Rightarrow$ $A = 48$ Initially $\frac{d\theta}{dt} = -k(98-50) = -48k = -1.5$ $\Rightarrow$ $k = 0.03125$*	M1 A1 M1 E1 [4]	
(iv) (*A*) $89 = 50 + 48e^{-0.03125t}$ $\Rightarrow$ $39/48 = e^{-0.03125t}$ $\Rightarrow$ $t = \ln(39/48)/(-0.03125) = 6.64$ hours (*B*) $80 = 50 + 48e^{-0.03125t}$ $\Rightarrow$ $30/48 = e^{-0.03125t}$ $\Rightarrow$ $t = \ln(30/48)/(-0.03125) = 15$ hours	M1 M1 A1 M1 A1 [5]	equating taking lns correctly for either
(v) Models disagree more for greater temperature loss	B1 [1]	

8(i) $\frac{dy}{d\theta} = 2\cos 2\theta - 2\sin\theta$, $\frac{dx}{d\theta} = 2\cos\theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $\frac{dy}{dx} = \frac{2\cos 2\theta - 2\sin\theta}{2\cos\theta} = \frac{\cos 2\theta - \sin\theta}{\cos\theta}$	B1, B1 M1 A1 [4]	substituting for theirs oe
(ii) When $\theta = \pi/6$, $\frac{dy}{dx} = \frac{\cos\pi/3 - \sin\pi/6}{\cos\pi/6}$ $= \frac{1/2 - 1/2}{\sqrt{3}/2} = 0$ Coords of B: $x = 2 + 2\sin(\pi/6) = 3$ $y = 2\cos(\pi/6) + \sin(\pi/3) = 3\sqrt{3}/2$ BC $= 2\times 3\sqrt{3}/2 = 3\sqrt{3}$	E1 M1 A1,A1 B1ft [5]	for either exact
(iii) (*A*) $y = 2\cos\theta + \sin 2\theta$ $= 2\cos\theta + 2\sin\theta\cos\theta$ $= 2\cos\theta(1 + \sin\theta)$ $= x\cos\theta$ * (*B*) $\sin\theta = ½(x - 2)$ $\cos^2\theta = 1 - \sin^2\theta$ $= 1 - ¼(x - 2)^2$ $= 1 - ¼x^2 + x - 1$ $= (x - ¼x^2)$ * (*C*) Cartesian equation is $y^2 = x^2\cos^2\theta$ $= x^2(x - ¼x^2)$ $= x^3 - ¼x^4$*	M1 E1 B1 M1 E1 M1 E1 [7]	$\sin 2\theta = 2\sin\theta\cos\theta$ squaring and substituting for x
(iv) $V = \int_0^4 \pi y^2 dx$ $= \pi\int_0^4 (x^3 - \frac{1}{4}x^4)dx$ $= \pi\left[\frac{1}{4}x^4 - \frac{1}{20}x^5\right]_0^4$ $= \pi(64 - 51.2)$ $= 12.8\pi = 40.2$ (m^3)	M1 B1 A1 [3]	need limits $\left[\frac{1}{4}x^4 - \frac{1}{20}x^5\right]$ 12.8π or 40 or better.

Comprehension

1	$\frac{400\pi d}{1000}=10$ $d=\frac{25}{\pi}=7.96$	M1 E1	
2	$V= \pi 20^2 h+\frac{1}{2}\left(\pi 20^2 H-\pi 20^2 h\right)$ $=\frac{1}{2}\left(\pi 20^2 H+\pi 20^2 h\right)\text{cm}^3 = 200\pi\left(H+h\right)\ \text{cm}^3$ $=\frac{1}{5}\pi\left(H+h\right)$ litres	M1 M1 E1	divide by 1000
3	$H=5+40\tan 30^\circ$ or $H=h+40\tan\theta$ $V=\frac{1}{5}\pi\left(H+h\right)=\frac{1}{5}\pi\left(10+40\tan 30^\circ\right)$ =20.8 litres	B1 M1 A1	or evaluated including substitution of values
4	$V=\frac{1}{2}\times 80\times\left(40+5\right)$ $\times 30\ \text{cm}^3 = 54\ 000\text{cm}^3$ = 54 litres	M1 M1 A1	×30
5	(i) Accurate algebraic simplification to give $y^2-160y+400=0$ (ii) Use of quadratic formula (or other method) to find other root: $d=157.5$ cm. This is greater than the height of the tank so not possible	B1 M1 A1 E1	
6	y=10 Substitute for y in (4): $V=\frac{1}{1000}\int_0^{100}375\text{d}x$ $V=\frac{1}{1000}\times 37500=37.5$ *	B1 M1 E1 [18]	

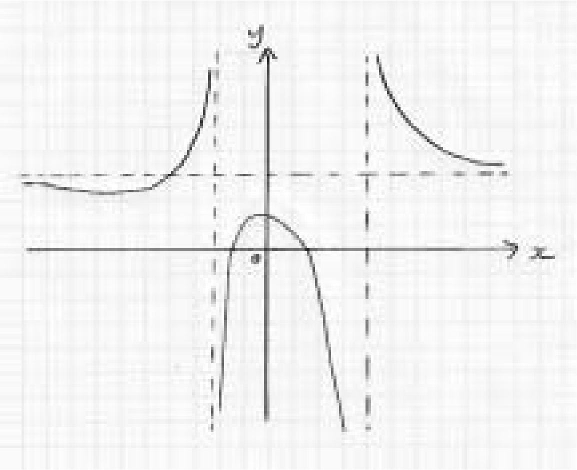
4755 (FP1) Further Concepts for Advanced Mathematics

Section A

1(i)	$z=\dfrac{6\pm\sqrt{36-40}}{2}$ $\Rightarrow z=3+\mathrm{j}$ or $z=3-\mathrm{j}$	M1 A1 **[2]**	Use of quadratic formula/completing the square For both roots
1(ii)	$\lvert 3+\mathrm{j}\rvert=\sqrt{10}=3.16$ (3s.f.) $\arg(3+\mathrm{j})=\arctan\left(\dfrac{1}{3}\right)=0.322$ (3s.f.) $\Rightarrow$ roots are $\sqrt{10}(\cos 0.322+\mathrm{j}\sin 0.322)$ and $\sqrt{10}(\cos 0.322-\mathrm{j}\sin 0.322)$ or $\sqrt{10}(\cos(-0.322)+\mathrm{j}\sin(-0.322))$	M1 M1 A1 **[3]**	Method for modulus Method for argument (both methods must be seen following A0) One mark for both roots in modulus-argument form – accept surd and decimal equivalents and $(r,\ \theta)$ form. Allow $\pm 18.4^{\circ}$ for θ.
2	$2x^2-13x+25=A(x-3)^2-B(x-2)+C$ $\Rightarrow 2x^2-13x+25$ $=Ax^2-(6A+B)x+(2B+C)+9A$ A = 2 B = 1 C = 5	B1 M1 A1 A1 **[4]**	For A=2 Attempt to compare coefficients of x^1 or x^0, or other valid method. For B and C, cao.
3(i)	$\begin{pmatrix}2 & 1\\0 & 2\end{pmatrix}$	B1 **[1]**	
3(ii)	$\begin{pmatrix}2 & -1\\0 & 3\end{pmatrix}\begin{pmatrix}0 & 2 & 3 & 1\\0 & 0 & 2 & 2\end{pmatrix}=\begin{pmatrix}0 & 4 & 4 & 0\\0 & 0 & 6 & 6\end{pmatrix}$ $\Rightarrow$ A″$=(4,\ 0)$, B″$=(4,\ 6)$, C″$=(0,\ 6)$	M1 A1 **[2]**	Applying matrix to column vectors, with a result. All correct
3(iii)	Stretch factor 4 in *x*-direction. Stretch factor 6 in *y*-direction	B1 B1 **[2]**	Both factor and direction for each mark. SC1 for “enlargement”, not stretch.

4	$\arg\left(z-(2-2\mathrm{j})\right)=\frac{\pi}{4}$	B1 B1 B1 **[3]**	Equation involving arg(complex variable). Argument (complex expression) = $\frac{\pi}{4}$ All correct
5	Sum of roots = $\alpha+(-3\alpha)+\alpha+3=3-\alpha=5$ $\Rightarrow \alpha=-2$ Product of roots $= -2\times6\times1=-12$ Product of roots in pairs $=-2\times6+(-2)\times1+6\times1=-8$ $\Rightarrow p=-8$ and $q=12$	M1 A1 M1 M1 A1 A1 **[6]**	Use of sum of roots Attempt to use product of roots Attempt to use sum of products of roots in pairs One mark for each, ft if α incorrect
	Alternative solution $(x-\alpha)(x+3\alpha)(x-\alpha-3)$ $=x^3+(\alpha-3)x^2+(-5\alpha^2-6\alpha)x+3\alpha^3+9\alpha^2$ => $\alpha=-2$, $p=-8$ and $q=12$	M1 M1A1 M1 A1A1 **[6]**	Attempt to multiply factors Matching coefficient of x^2 ,cao. Matching other coefficients One mark for each, ft incorrect α.
6	$\sum_{r=1}^{n}\left[r\left(r^2-3\right)\right]=\sum_{r=1}^{n}r^3-3\sum_{r=1}^{n}r$ $=\frac{1}{4}n^2(n+1)^2-\frac{3}{2}n(n+1)$ $=\frac{1}{4}n(n+1)\left(n(n+1)-6\right)$ $=\frac{1}{4}n(n+1)\left(n^2+n-6\right)=\frac{1}{4}n(n+1)(n+3)(n-2)$	M1 M1 A2 M1 A1 **[6]**	Separate into separate sums. (may be implied) Substitution of standard result in terms of n. For two correct terms (indivisible) Attempt to factorise with $n(n+1)$. Correctly factorised to give fully factorised form

7	When $n = 1$, $6\left(3^n - 1\right) = 12$, so true for $n = 1$ Assume true for $n = k$ $12 + 36 + 108 + ... + (4 \times 3^k) = 6\left(3^k - 1\right)$ $\Rightarrow 12 + 36 + 108 + ... + (4 \times 3^{k+1})$ $= 6\left(3^k - 1\right) + (4 \times 3^{k+1})$ $= 6\left[\left(3^k - 1\right) + \frac{2}{3} \times 3^{k+1}\right]$ $= 6\left[3^k - 1 + 2 \times 3^k\right]$ $= 6\left(3^{k+1} - 1\right)$ But this is the given result with $k + 1$ replacing k. Therefore if it is true for $n = k$, it is true for $n = k + 1$. Since it is true for $n = 1$, it is true for $n = 1, 2, 3$... and so true for all positive integers.	B1 E1 M1 M1 A1 E1 E1 **[7]**	 Assume true for k Add correct next term to both sides Attempt to factorise with a factor 6 c.a.o. with correct simplification Dependent on A1 and first E1 Dependent on B1 and second E1
			Section A Total: 36

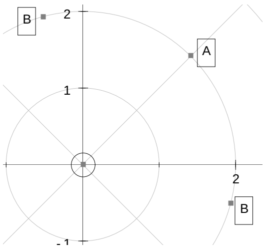
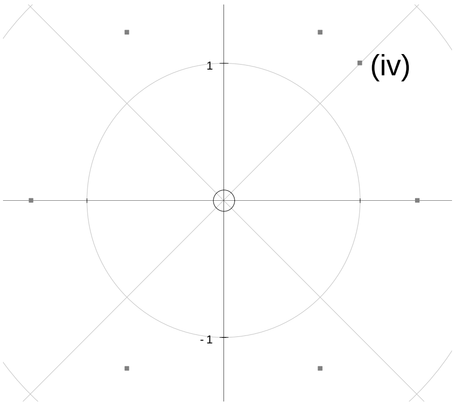
Section B			
8(i)	$\left(\sqrt{3},\ 0\right),\left(-\sqrt{3},\ 0\right)\ \left(0,\ \frac{3}{8}\right)$	B1 B1 **[2]**	Intercepts with *x* axis (both) Intercept with *y* axis SC1 if seen on graph or if $x = \underline{+}\sqrt{3}$, $y = 3/8$ seen without $y = 0$, $x = 0$ specified.
8(ii)	$x = 4,\ x = -2,\ y = 1$	B3 **[3]**	Minus 1 for each error. Accept equations written on the graph.
8(iii)		B1 B1B1 B1 **[4]**	Correct approaches to vertical asymptotes, LH and RH branches LH and RH branches approaching horizontal asymptote On LH branch 0<y<1 as x→-∞.
8(iv)	$-2 < x \le -\sqrt{3}$ and $4 > x \ge \sqrt{3}$	B1 B2 **[3]**	LH interval and RH interval correct (Award this mark even if errors in inclusive/exclusive inequality signs) All inequality signs correct, minus 1 each error

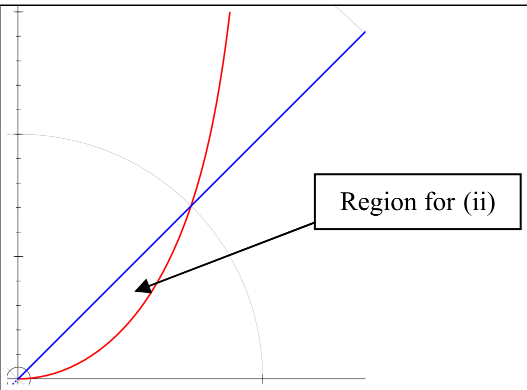
9(i)	$\alpha+\beta=3$ $\alpha\alpha^*=(1+\text{j})(1-\text{j})=2$ $\frac{\alpha+\beta}{\alpha}=\frac{3}{1+\text{j}}=\frac{3(1-\text{j})}{(1+\text{j})(1-\text{j})}=\frac{3}{2}-\frac{3}{2}\text{j}$	B1 M1 A1 M1 A1 **[5]**	 Attempt to multiply $(1+\text{j})(1-\text{j})$ Multiply top and bottom by $1-\text{j}$
9(ii)	$(z-(1+\text{j}))(z-(1-\text{j}))$ $=z^2-2z+2$	M1 A1 **[2]**	Or alternative valid methods (Condone no "=0" here)
9(iii)	$1-\text{j}$ and $2+\text{j}$ Either $(z-(2-\text{j}))(z-(2+\text{j}))$ $=z^2-4z+5$ $(z^2-2z+2)(z^2-4z+5)$ $=z^4-6z^3+15z^2-18z+10$ So equation is $z^4-6z^3+15z^2-18z+10=0$ Or alternative solution Use of $\sum\alpha=6$, $\sum\alpha\beta=15$, $\sum\alpha\beta\gamma=18$ and $\alpha\beta\gamma\delta=10$ to obtain the above equation.	B1 M1 M1 A2 **[5]** M1 A3 **[5]**	For both For attempt to obtain an equation using the product of linear factors involving complex conjugates Using the correct four factors All correct, -1 each error (including omission of "=0") to min of 0 Use of relationships between roots and coefficients. All correct, -1 each error, to min of 0

10(i)	$\alpha = 3\times -5 + 4\times 11 + -1\times 29 = 0$ $\beta = -2\times -7 + 7\times(5+k) + -3\times 7 = 28 + 7k$	B1 M1 A1 **[3]**	 Attempt at row 3 x column 3
10(ii)	$\mathbf{AB} = \begin{pmatrix} 42 & 0 & 0 \\ 0 & 42 & 0 \\ 0 & 0 & 42 \end{pmatrix}$	B2 **[2]**	Minus 1 each error to min of 0
10(iii)	$\mathbf{A}^{-1} = \frac{1}{42}\begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 7 \\ -5 & 29 & 7 \end{pmatrix}$	M1 B1 A1 **[3]**	Use of **B** $\frac{1}{42}$ Correct inverse, allow decimals to 3 sf
10(iv)	$\frac{1}{42}\begin{pmatrix} 11 & -5 & -7 \\ 1 & 11 & 7 \\ -5 & 29 & 7 \end{pmatrix}\begin{pmatrix} 1 \\ -9 \\ 26 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $= \frac{1}{42}\begin{pmatrix} -126 \\ 84 \\ -84 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -2 \end{pmatrix}$ $x = -3,\ y = 2,\ z = -2$	M1 A3 **[4]**	Attempt to pre-multiply by $\mathbf{A}^{-1}$ SC B2 for Gaussian elimination with 3 correct solutions, -1 each error to min of 0 Minus 1 each error
			Section B Total: 36
			Total: 72

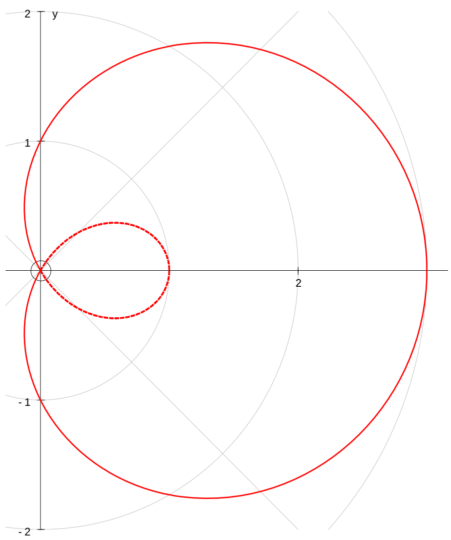
4756 (FP2) Further Methods for Advanced Mathematics

1 (a)(i)	$f(x) = \cos x \qquad f(0) = 1$	M1	Derivatives cos, sin, cos, sin, cos
	$f'(x) = -\sin x \qquad f'(0) = 0$		
	$f''(x) = -\cos x \qquad f''(0) = -1$	A1	Correct signs
	$f'''(x) = \sin x \qquad f'''(0) = 0$		
	$f''''(x) = \cos x \qquad f''''(0) = 1$	A1	Correct values. Dep on previous A1
	$\Rightarrow \quad \cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 ...$	A1 (ag) **4**	www
(ii)	$\cos x \times \sec x = 1$	E1	o.e.
	$\Rightarrow \quad \left(1 - \frac{1}{2}x^2 + \frac{1}{24}x^4\right)\left(1 + ax^2 + bx^4\right) = 1$	M1	Multiply to obtain terms in x^2 and x^4
	$\Rightarrow \quad 1 + \left(a - \frac{1}{2}\right)x^2 + \left(b - \frac{1}{2}a + \frac{1}{24}\right)x^4 = 1$	A1	Terms correct in any form (may not be collected)
	$\Rightarrow \quad a - \frac{1}{2} = 0, \; b - \frac{1}{2}a + \frac{1}{24} = 0$		
	$\Rightarrow \quad a = \frac{1}{2}$	B1	Correctly obtained by any method: must not just be stated
	$b = \frac{5}{24}$	B1 **5**	Correctly obtained by any method
(b)(i)	$y = \arctan \frac{x}{a}$		
	$\Rightarrow \quad x = a \tan y$	M1	(a) tan $y =$ and attempt to differentiate both sides
	$\Rightarrow \quad \frac{dx}{dy} = a \sec^2 y$	A1	Or $\sec^2 y \, \frac{dy}{dx} = \frac{1}{a}$
	$\Rightarrow \quad \frac{dx}{dy} = a(1 + \tan^2 y)$	A1	Use $\sec^2 y = 1 + \tan^2 y$ o.e.
	$\Rightarrow \quad \frac{dy}{dx} = \frac{a}{a^2 + x^2}$	A1 (ag) **4**	www SC1: Use derivative of arctan x and Chain Rule (properly shown)
(ii)(A)	$\int_{-2}^{2} \frac{1}{4 + x^2}\,dx = \left[\frac{1}{2}\arctan\frac{x}{2}\right]_{-2}^{2}$	M1	arctan alone, or any tan substitution
		A1	$\frac{1}{2}$ and $\frac{x}{2}$, or $\int \frac{1}{2}\,d\theta$ without limits
	$= \frac{\pi}{4}$	A1 **3**	Evaluated in terms of π
(ii)(B)	$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{4}{1 + 4x^2}\,dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\frac{1}{4} + x^2}\,dx$	M1	arctan alone, or any tan substitution
	$= \left[2\arctan(2x)\right]_{-\frac{1}{2}}^{\frac{1}{2}}$	A1	2 and $2x$, or $\int 2\,d\theta$ without limits
	$= \pi$	A1 **3**	Evaluated in terms of π **19**

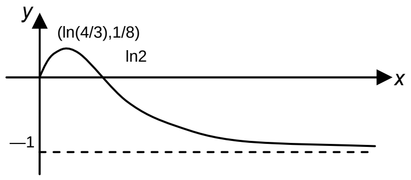
2 (i)	Modulus = 1 Argument = $\frac{\pi}{3}$	B1 B1 **2**	Must be separate Accept 60°, 1.05^c
(ii)	 $a = 2e^{\frac{j\pi}{4}}$ $\arg b = \frac{\pi}{4} \pm \frac{\pi}{3}$ $b = 2e^{-\frac{j\pi}{12}}, 2e^{\frac{7j\pi}{12}}$	G2,1,0 B1 M1 A1ft **5**	G2: A in first quadrant, argument $\approx \frac{\pi}{4}$ B in second quadrant, same mod B′ in fourth quadrant, same mod Symmetry G1: 3 points and at least 2 of above, or B, B′ on axes, or BOB′ straight line, or BOB′ reflex Must be in required form (accept $r = 2$, $\theta = \pi/4$) Rotate by adding (or subtracting) $\pi/3$ to (or from) argument. Must be $\pi/3$ Both. Ft value of r for a. Must be in required form, but don't penalise twice
(iii)	$z_1^6 = \left(\sqrt{2}e^{\frac{j\pi}{3}}\right)^6 = \left(\sqrt{2}\right)^6 e^{2j\pi}$ = 8 Others are $re^{j\theta}$ where $r = \sqrt{2}$ and θ = $-\frac{2\pi}{3}, -\frac{\pi}{3}, 0, \frac{2\pi}{3}, \pi$ 	M1 A1 (ag) M1 A1 G1 G1 **6**	$\left(\sqrt{2}\right)^6 = 8$ or $\frac{\pi}{3} \times 6 = 2\pi$ seen www "Add" $\frac{\pi}{3}$ to argument more than once Correct constant r and five values of θ. Accept θ in [0, 2π] or in degrees 6 points on vertices of regular hexagon Correctly positioned (2 roots on real axis). Ignore scales SC1 if G0 and 5 points correctly plotted
(iv)	$w = z_1 e^{-\frac{j\pi}{12}} = \sqrt{2}e^{\frac{j\pi}{3}} e^{-\frac{j\pi}{12}} = \sqrt{2}e^{\frac{j\pi}{4}}$ $= \sqrt{2}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right)$ $= 1 + j$	M1 A1 G1 **3**	$\arg w = \frac{\pi}{3} - \frac{\pi}{12}$ Or B2 Same modulus as z_1
(v)	$w^6 = \left(\sqrt{2}e^{\frac{j\pi}{4}}\right)^6 = 8e^{\frac{3j\pi}{2}}$ $= -8j$	M1 A1 **2**	Or $z_1^6 e^{-\frac{j\pi}{2}} = 8e^{-\frac{j\pi}{2}}$ cao. Evaluated **18**

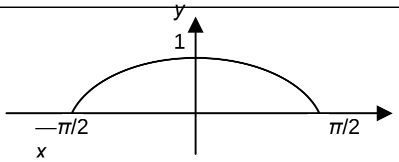
3(a)(i)		G1 G1 G1 **3**	r increasing with θ Correct for $0 \le \theta \le \pi/3$ (ignore extra) Gradient less than 1 at O
(ii)	Area $= \int_0^{\frac{\pi}{4}} \frac{1}{2} r^2 d\theta = \frac{1}{2}a^2 \int_0^{\frac{\pi}{4}} \tan^2\theta \, d\theta$ $= \frac{1}{2}a^2 \int_0^{\frac{\pi}{4}} \sec^2\theta - 1 \, d\theta$ $= \frac{1}{2}a^2 \left[\tan\theta - \theta\right]_0^{\frac{\pi}{4}}$ $= \frac{1}{2}a^2 \left(1 - \frac{\pi}{4}\right)$	M1 M1 A1 A1 G1 **5**	Integral expression involving $\tan^2\theta$ Attempt to express $\tan^2\theta$ in terms of $\sec^2\theta$ $\tan\theta - \theta$ and limits 0, $\frac{\pi}{4}$ A0 if e.g. triangle − this answer Mark region on graph
(b)(i)	Characteristic equation is $(0.2 - \lambda)(0.7 - \lambda) - 0.24 = 0$ $\Rightarrow \lambda^2 - 0.9\lambda - 0.1 = 0$ $\Rightarrow \lambda = 1, -0.1$ When $\lambda = 1$, $\begin{pmatrix} -0.8 & 0.8 \\ 0.3 & -0.3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\Rightarrow -0.8x + 0.8y = 0,\ 0.3x - 0.3y = 0$ $\Rightarrow x - y = 0$, eigenvector is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ o.e. When $\lambda = -0.1$, $\begin{pmatrix} 0.3 & 0.8 \\ 0.3 & 0.8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\Rightarrow 0.3x + 0.8y = 0$ $\Rightarrow$ eigenvector is $\begin{pmatrix} 8 \\ -3 \end{pmatrix}$ o.e.	M1 A1 M1 A1 M1 A1 **6**	$(\mathbf{M} - \lambda\mathbf{I})\mathbf{x} = \mathbf{x}$ M0 below At least one equation relating x and y At least one equation relating x and y
(ii)	**Q** $= \begin{pmatrix} 1 & 8 \\ 1 & -3 \end{pmatrix}$ **D** $= \begin{pmatrix} 1 & 0 \\ 0 & -0.1 \end{pmatrix}$	B1ft B1ft B1 **3**	B0 if **Q** is singular. Must label correctly If order consistent. Dep on B1B1 earned **17**

4 (a)(i)	$\cosh^2 x = \left[\tfrac{1}{2}\left(e^x+e^{-x}\right)\right]^2 = \tfrac{1}{4}\left(e^{2x}+2+e^{-2x}\right)$ $\sinh^2 x = \left[\tfrac{1}{2}\left(e^x-e^{-x}\right)\right]^2 = \tfrac{1}{4}\left(e^{2x}-2+e^{-2x}\right)$ $\cosh^2 x - \sinh^2 x = \tfrac{1}{4}(2+2) = 1$	M1 A1 (ag) **2**	Both expressions (M0 if no “middle” term) and subtraction www
	OR $\cosh x + \sinh x = e^x$ $\cosh x - \sinh x = e^{-x}$ $\cosh^2 x - \sinh^2 x = e^x \times e^{-x} = 1$ A1		Both, and multiplication Completion
(ii)(A)	$\cosh x = \sqrt{1+\sinh^2 x} = \sqrt{1+\tan^2 y}$ $= \sec y$ $\Rightarrow \tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{\tan y}{\sec y} = \sin y$	M1 A1 A1 (ag) **3**	Use of $\cosh^2 x = 1 + \sinh^2 x$ and $\sinh x = \tan y$ www
(ii)(B)	$\operatorname{arsinh} x = \ln(x + \sqrt{1+x^2}\,)$ $\Rightarrow \operatorname{arsinh}(\tan y) = \ln(\tan y + \sqrt{1+\tan^2 y}\,)$ $\Rightarrow x = \ln(\tan y + \sec y)$	M1 A1 A1 (ag) **3**	Attempt to use ln form of arsinh www
	OR $\sinh x = \tan y \Rightarrow \dfrac{e^x - e^{-x}}{2} = \tan y$ $\Rightarrow e^{2x} - 2e^x \tan y - 1 = 0$ M1 $\Rightarrow e^x = \tan y \pm \sqrt{\tan^2 y + 1}$ A1 $\Rightarrow x = \ln(\tan y + \sec y)$ A1		Arrange as quadratic and solve for e^x o.e. www
(b)(i)	$y = \operatorname{artanh} x \Rightarrow x = \tanh y$ $\Rightarrow \dfrac{dx}{dy} = \operatorname{sech}^2 y$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\operatorname{sech}^2 y} = \dfrac{1}{1-\tanh^2 y} = \dfrac{1}{1-x^2}$ Integral $= \left[\operatorname{artanh} x\right]_{-\frac{1}{2}}^{\frac{1}{2}}$ $= 2 \operatorname{artanh} \dfrac{1}{2}$	M1 A1 M1 A1 (ag) **4**	$\tanh y =$ and attempt to differentiate Or $\operatorname{sech}^2 y \dfrac{dy}{dx} = 1$ Or B2 for $\dfrac{1}{1-x^2}$ www artanh or any tanh substitution www
(ii)	$\dfrac{1}{1-x^2} = \dfrac{1}{(1-x)(1+x)} = \dfrac{A}{1-x} + \dfrac{B}{1+x}$ $\Rightarrow 1 = A(1+x) + B(1-x)$ $\Rightarrow A = ½, B = ½$ $\Rightarrow \displaystyle\int \dfrac{1}{1-x^2}dx = \int \dfrac{\frac{1}{2}}{1-x} + \dfrac{\frac{1}{2}}{1+x}dx$ $= -\tfrac{1}{2}\ln\lvert 1-x\rvert + \tfrac{1}{2}\ln\lvert 1+x\rvert + c$ or $\tfrac{1}{2}\ln\left\lvert\tfrac{1+x}{1-x}\right\rvert + c$ o.e.	M1 A1 M1 A1 **4**	Correct form of partial fractions and attempt to evaluate constants Log integrals www. Condone omitted modulus signs and constant After 0 scored, SC1 for correct answer
(iii)	$\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \dfrac{1}{1-x^2}dx = \left[-\tfrac{1}{2}\ln\lvert 1-x\rvert + \tfrac{1}{2}\ln\lvert 1+x\rvert\right]_{-\frac{1}{2}}^{\frac{1}{2}} = \ln 3$ $\Rightarrow 2 \operatorname{artanh} \dfrac{1}{2} = \ln 3 \Rightarrow \operatorname{artanh} \dfrac{1}{2} = \dfrac{1}{2} \ln 3$	M1 A1 (ag) **2**	Substitution of ½ **and** –½ seen anywhere (or correct use of 0, ½) www **18**

5 (i)		G1 G1 G1	Symmetry in horizontal axis (3, 0) to (0, 0) (0, 0) to (0, 1)
		3	
(ii)(A)	$a > 0.5$	B1	
	$a < -0.5$	B1	
(ii)(B)	Circle: r is constant	B1	Shape and reason
(ii)(C)	The two loops get closer together	B1	
	The shape becomes more nearly circular	B1	
(ii)(D)	Cusp	B1	
	$a = -0.5$	B1	
		7	
(iii)	$1 + 2a\cos\theta = 0 \Rightarrow \cos\theta = -\dfrac{1}{2a}$	B1	Equation
	If $a > 0.5$, $-1 < -\dfrac{1}{2a} < 0$ and there are two values of θ in $[0, 2\pi]$,		
	$\pi - \arccos\left(\dfrac{1}{2a}\right)$ and $\pi + \arccos\left(\dfrac{1}{2a}\right)$	M1	
	These differ by 2 $\arccos\left(\dfrac{1}{2a}\right)$	A1 (ag)	
	$\arccos\left(\dfrac{1}{2a}\right) = \arctan\sqrt{4a^2 - 1}$	M1 A1	Relating arccos to arctan by triangle or $\tan^2\theta = \sec^2\theta - 1$
	Tangents are $y = x\sqrt{4a^2 - 1}$	A1	
	and $y = -x\sqrt{4a^2 - 1}$	A1ft	Negative of above
	$\sqrt{4a^2 - 1}$ is real for $a > 0.5$ if $a > 0$	E1	
		8	**18**

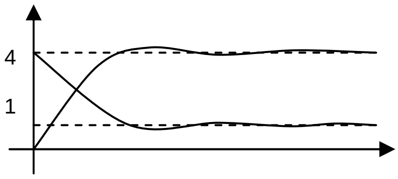
4758 Differential Equations

1(i)	$\alpha^3+2\alpha^2-\alpha-2=0$	B1		
	$(-2)^3+2(-2)^2-(-2)-2=0$	E1	Or factorise	
	$(\alpha+2)(\alpha^2-1)=0$	M1	Solve	
	$\alpha=-2,\pm1$	A1		
	$y=Ae^{-2x}+Be^{-x}+Ce^{x}$	M1	Attempt CF	
		F1	CF for their three roots	
				6
(ii)	PI $y=\frac{2}{-2}=-1$	M1	Constant PI	
		A1	Correct PI	
	GS $y=-1+Ae^{-2x}+Be^{-x}+Ce^{x}$	F1	GS = PI + CF	
				3
(iii)	$e^{x}\to\infty$ as $x\to\infty$	M1	Consider as $x\to\infty$	
	so finite limit $\Rightarrow C=0$	F1	Must be shown, not just stated	
	$x=0$, $y=0\Rightarrow 0=-1+A+B$	M1	Use condition	
	$x=\ln 2, y=0\Rightarrow 0=-1+\frac{1}{4}A+\frac{1}{2}B$	M1	Use condition	
	Solving gives $A=-2$, $B=3$	M1		
	$y=-2e^{-2x}+3e^{-x}-1$	E1	Convincingly shown	
				6
(iv)	$y=-(2e^{-x}-1)(e^{-x}-1)$			
	$y=0\Leftrightarrow e^{-x}=\frac{1}{2}$ or 1	M1	Solve	
	$\Leftrightarrow x=\ln 2$ or 0	E1	Convincingly show no other roots	
	$\frac{dy}{dx}=4e^{-2x}-3e^{-x}=e^{-x}(4e^{-x}-3)$			
	$\frac{dy}{dx}=0\Leftrightarrow e^{-x}=\frac{3}{4}$ as $e^{-x}\neq 0$	M1	Solve	
	$\Leftrightarrow x=\ln\frac{4}{3}$	E1	Show only one root	
	Stationary point at $(\ln\frac{4}{3},\frac{1}{8})$	A1		
				5
(v)		B1	Through (0, 0)	
		B1	Through (ln 2, 0)	
		B1	Stationary point at their answer to (iv)	
		B1	$y\to-1$ as $x\to\infty$	
				4

2(i)	$\frac{dy}{dx}+y\tan x = x\cos x$	M1	Rearrange	
	$I=\exp\int \tan x dx$	M1	Attempt IF	
	$=\exp\ln\sec x$	A1	Correct IF	
	$=\sec x$	A1	Simplified	
	$\frac{d}{dx}(y\sec x)=x$	M1	Multiply and recognise derivative	
	$y\sec x=\frac{1}{2}x^2+A$	M1	Integrate	
		A1	RHS	
	$y=(\frac{1}{2}x^2+A)\cos x$	F1	Divide by their IF (must divide constant)	
	$x=0, y=1 \Rightarrow A=1$	M1	Use condition	
	$y=(\frac{1}{2}x^2+1)\cos x$	F1	Follow their non-trivial GS	10
(ii)		B1	Shape correct for $-\frac{1}{2}\pi<x<\frac{1}{2}\pi$	
		B1	Through (0,1)	2
(iii)	$y'=\frac{x\cos x\sin x - y\sin x}{\cos x}$	M1	Rearrange	
	$y'(0)=0$	B1		
	$y(0.1)=1$	B1		
	$y'(0.1)=-0.090351$	B1		
	$y(0.2)=1+0.1\times -0.090351=0.990965$	M1	Use of algorithm for second step	
		A1	3sf or better	6
(iv)	$I=\sec x$	M1	Same IF as in (i) or attempt from scratch	
	$\frac{d}{dx}(y\sec x)=x\tan x$	A1		
	$[y\sec x]_{x=0}^{x=0.2}=\int_0^{0.2} x\tan x dx$	M1	Integrate	
		A1	Accept no limits	
	$y(0.2)\sec(0.2)-1\times\sec 0\approx 0.002688$	M1	Substitute limits (both sides)	
	$\Rightarrow y(0.2)\approx 0.982701$	A1	Awrt 0.983	6

3(i)	$60v\frac{dv}{dx}=60g-\frac{1}{4}v^2$	M1	N2L	
		A1	Correct N2L equation	
	$\frac{v}{240g-v^2}\frac{dv}{dx}=\frac{1}{240}$	E1	Convincingly shown	
	$\int\frac{v}{240g-v^2}dv=\int\frac{1}{240}dx$	M1	Integrate	
	$-\frac{1}{2}\ln\lvert 240g-v^2\rvert=\frac{1}{240}x+c$	A1	$\ln\lvert 240g-v^2\rvert$ seen	
		A1	RHS	
	$240g-v^2=Ae^{\frac{x}{120}}$	M1	Rearrange, dealing properly with constant	
	$x=0,v=0\Rightarrow A=240g$	M1	Use condition	
	$v^2=240g(1-e^{-\frac{x}{120}})$	A1	Cao	9
(ii)	$x=10\Rightarrow v=\sqrt{240g(1-e^{-\frac{10}{120}})}\approx 13.71$	E1	Convincingly shown	1
(iii)	$60\frac{dv}{dt}=60g-60v-90g$	M1	N2L	
	$\frac{dv}{dt}=-\frac{1}{2}g-v$ or $\frac{dv}{dt}+v=-\frac{1}{2}g$	A1	Correct DE	
	Solving DE (three alternative methods):			
	$\int\frac{dv}{v+\frac{1}{2}g}=\int-dt$	M1	Separate	
	$\ln\lvert v+\frac{1}{2}g\rvert=-t+k$	M1	Integrate	
		A1	LHS	
	$v+\frac{1}{2}g=Ae^{-t}$	M1	Rearrange, dealing properly with constant	
	or			
	$\alpha+1=0\Rightarrow\alpha=-1$		*M1 Solve auxiliary equation*	
	CF Ae^{-t}		*M1 CF for their root*	
	PI $-\frac{1}{2}g$		*M1 Attempt to find constant PI*	
	$v=Ae^{-t}-\frac{1}{2}g$		*A1 All correct*	
	or			
	$I=e^{t}$		*M1 Attempt integrating factor*	
	$\frac{d}{dt}(e^{t}v)=-\frac{1}{2}ge^{t}$		*M1 Multiply*	
	$e^{t}v=-\frac{1}{2}ge^{t}+A$		*M1 Integrate*	
	$v=Ae^{-t}-\frac{1}{2}g$		*A1 All correct*	
	$v=13.71,t=0\Rightarrow 13.71=A-\frac{1}{2}g\Rightarrow A=18.61$	M1	Use condition	
	$v=18.61e^{-t}-4.9$	E1	Complete argument	8

(iv)	At greatest depth, $v = 0$	M1	Set velocity to zero and attempt to solve	
	$\Rightarrow e^{-t} = \frac{4.9}{18.61} \Rightarrow t = 1.3345$	A1		
	Depth $= \int_0^{1.3345} (18.61e^{-t} - 4.9)dt$	M1	Integrate	
	$= \left[-18.61e^{-t} - 4.9t\right]_0^{1.3345}$	A1	Ignore limits	
		M1	Use limits (or evaluate constant and substitute for *t*)	
	= 7.17 m	A1	All correct	6

4(i)	$\left.\begin{matrix} -3x - y + 7 = 0 \\ 2x - y + 2 = 0 \end{matrix}\right\} \Leftrightarrow \begin{matrix} x = 1 \\ y = 4 \end{matrix}$	B1 B1		2
(ii)	$\ddot{x} = -3\dot{x} - \dot{y}$	M1	Differentiate	
	$= -3\dot{x} - (2x - y + 2)$	M1	Substitute for $\dot{y}$	
	$y = -3x + 7 - \dot{x}$	M1	y in terms of $x, \dot{x}$	
	$\ddot{x} = -3\dot{x} - 2x - 3x + 7 - \dot{x} - 2$	M1	Substitute for y	
	$\Rightarrow \ddot{x} + 4\dot{x} + 5x = 5$	E1	Complete argument	5
(iii)	$\alpha^2 + 4\alpha + 5 = 0$	M1	Auxiliary equation	
	$\Rightarrow \alpha = -2 \pm i$	A1		
		M1	CF for complex roots	
	CF $e^{-2t}(A\cos t + B\sin t)$	F1	CF for their roots	
	PI $x = \frac{5}{5} = 1$	B1		
	GS $x = 1 + e^{-2t}(A\cos t + B\sin t)$	F1	GS = PI + CF with two arbitrary constants	6
(iv)	$y = -3x + 7 - \dot{x}$	M1	y in terms of $x, \dot{x}$	
	$\dot{x} = -2e^{-2t}(A\cos t + B\sin t) + e^{-2t}(-A\sin t + B\cos t)$	M1	Differentiate their *x* (product rule)	
	$y = 4 + e^{-2t}((A - B)\sin t - (A + B)\cos t)$	A1	Constants must correspond	3
(v)	$1 + A = 4$ $4 - A - B = 0$	M1 M1	Use condition on *x* Use condition on *y*	
	$A = 3, B = 1$ $x = 1 + e^{-2t}(3\cos t + \sin t)$ $y = 4 + e^{-2t}(2\sin t - 4\cos t)$	A1	Both solutions	3
(vi)		B1 B1 B1 B1	(0, 4) $\to 1$ (0, 0) $\to 4$	
	As the solutions approach the asymptotes, the gradients approach zero.	B1	Must refer to gradients	5

4761 Mechanics 1

Q 1		Mark	Comment	Sub
(i)	6 m s^{-1} 4 m s^{-2}	B1 B1	Neglect units. Neglect units.	2
(ii)	$v(5) = 6 + 4 \times 5 = 26$ $s(5) = 6 \times 5 + 0.5 \times 4 \times 25 = 80$ so 80 m	B1 M1 A1	 Or equiv. FT (i) and **their** v(5) where necessary. cao	3
(iii)	distance is 80 + $26 \times (15 - 5) + 0.5 \times 3 \times (15 - 5)^2$ = 490 m	M1 M1 A1	Their 80 + attempt at distance with a = 3 Appropriate *uvast* . Allow t = 15. FT **their** v(5). cao	3
		8		

Q 2		Mark	Comment	Sub
(i)	 When t = 2, velocity is $6 + 4 \times 2 = 14$	M1 A1	Recognising that areas under graph represent changes in velocity in (i) or (ii) or equivalent *uvast.*	2
(ii)	Require velocity of – 6 so must inc by – 20 $-8 \times (t - 2) = -20$ so $t = 4.5$	M1 F1	FT ±(6 + **their** 14) used in any attempt at area/ *uvast* FT **their** 14 [Award SC2 for 4.5 WW and SC1 for 2.5 WW]	2
		4		

Q 3		Mark	Comment	Sub
(i)	$\mathbf{F} + \begin{pmatrix} -4 \\ 8 \end{pmatrix} = 6\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\mathbf{F} = \begin{pmatrix} 16 \\ 10 \end{pmatrix}$	M1 B1 B1 A1	N2L. $F = ma$. All forces present Addition to get resultant. May be implied. For **F** $\pm\begin{pmatrix} -4 \\ 8 \end{pmatrix} = 6\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. SC4 for $\mathbf{F} = \begin{pmatrix} 16 \\ 10 \end{pmatrix}$ WW. If magnitude is given, final mark is lost unless vector answer is clearly intended.	4
(ii)	$\arctan\left(\frac{16}{10}\right)$ 57.994… so 58.0° (3 s. f.)	M1 A1	Accept equivalent and FT **their F** only. Do not accept wrong angle. Accept 360 - $\arctan\left(\frac{16}{10}\right)$ cao. Accept 302° (3 s.f.)	2
		6		

Q4		Mark	Comment	Sub
	either We need $3.675 = 9.8t - 4.9t^2$	*M1	Equating given expression or **their** attempt at y to ± 3.675 . If **they** attempt y, allow sign errors, $g = 9.81$ etc. and $u = 35$.	
	Solving $4t^2 - 8t + 3 = 0$	M1*	Dependent. Any method of solution of a 3 term quadratic.	
	gives $t = 0.5$ or $t = 1.5$	A1 F1	cao. Accept only the larger root given Both roots shown and larger chosen provided both +ve. Dependent on 1st M1. [Award M1 M1 A1 for 1.5 seen WW]	
	or	M1	Complete method for total time from motion in separate parts. Allow sign errors, $g = 9.81$ etc. Allow $u = 35$ initially only.	
	Time to greatest height $0 = 35\times 0.28 - 9.8t$ so $t = 1$	A1	Time for 1st part	
	Time to drop is 0.5	A1	Time for 2nd part	
	total is 1.5 s	A1	cao	
	then Horiz distance is $35\times 0.96t$	B1	Use of $x = u\cos\alpha t$. May be implied.	
	So distance is $35\times 0.96\times 1.5 = 50.4$ m	F1	FT **their** quoted t provided it is positive.	6
		6		

Q5		Mark	Comment	Sub
(i)	For the parcel	M1	Applying N2L to the parcel. Correct mass. Allow $F = mga$. Condone missing force but do not allow spurious forces.	
	↑ N2L $55 - 5g = 5a$	A1	Allow only sign error(s).	
	$a = 1.2$ so 1.2 m s $^{-2}$	A1	Allow –1.2 only if sign convention is clear.	3
(ii)	$R - 80g = 80\times 1.2$ or $R - 75g - 55 = 75\times 1.2$	M1	N2L. Must have correct mass. Allow only sign errors. FT **their** a	
	$R = 880$ so 880 N	A1	cao [NB beware spurious methods giving 880 N]	2
		5		

Q6		Mark	Comment	Sub
	Method 1			
	$\uparrow\ v_A = 29.4 - 9.8T \qquad \downarrow\ v_B = 9.8T$	M1	Either attempted. Allow sign errors and $g = 9.81$ etc	
		A1	Both correct	
	For same speed $29.4 - 9.8T = 9.8T$	M1	Attempt to equate. Accept sign errors and $T = 1.5$ substituted in both.	
	so $T = 1.5$	E1	If 2 subs there must be a statement about equality	
	and $V = 14.7$	F1	FT T or V, whichever is found second	
	$H = 29.4\times1.5 - 0.5\times9.8\times1.5^2$ $+ 0.5\times9.8\times1.5^2$	M1	Sum of the distance travelled by each attempted	
	$= 44.1$	A1	cao	
	Method 2			
	$V^2 = 29.4^2 - 2\times9.8\times x = 2\times9.8\times(H-x)$	M1	Attempts at V^2 for each particle equated. Allow sign errors, 9.81 etc Allow h_1, h_2 without $h_1 = H - h_2$	
		B1	Both correct. Require $h_1 = H - h_2$ but not an equation.	
	$29.4^2 = 19.6H$ so $H = 44.1$	A1	cao	
	Relative velocity is 29.4 so	M1	Any method that leads to T or V	
	$T = \dfrac{44.1}{29.4}$	E1		
	Using $v = u + at$	M1	Any method leading to the other variable	
	$V = 0 + 9.8\times1.5 = 14.7$	F1		
			Other approaches possible. If 'clever' ways seen, reward according to weighting above.	7
		7		

Q7		Mark	Comment	Sub
(i)	Diagram	B1 B1	Weight, friction and 121 N present with arrows. All forces present with suitable labels. Accept *W*, *mg*, 100*g* and 980. No extra forces.	
	Resolve → $121\cos 34 - F = 0$ $F = 100.313\ldots$ so 100 N (3 s. f.)	M1 E1	Resolving horiz. Accept s ↔ c . Some evidence required for the *show*, e.g. at least 4 figures. Accept ± .	
	Resolve ↑ $R + 121\sin 34 - 980 = 0$	M1 B1	Resolve vert. Accept s ↔ c and sign errors. All correct	
	$R = 912.337\ldots$ so 912 N (3 s. f.)	A1		7
(ii)	It will continue to move at a constant speed of 0.5 m s^{-1}.	E1 E1	Accept no reference to direction Accept no reference to direction [Do not isw: conflicting statements get zero]	2
(iii)	Using N2L horizontally $155\cos 34 - 95 = 100a$	M1	Use of N2L. Allow $F = mga$, *F* omitted and 155 not resolved.	
		A1	Use of $F = ma$ with resistance and T resolved. Allow $s \leftrightarrow c$ and signs as the **only** errors.	
	$a = 0.335008\ldots$ so 0.335 m s^{-2} (3 s. f.)	A1		3
(iv)	$a = 5 \div 2 = 2.5$	M1 A1	Attempt to find *a* from information	
	N2L down the slope $100g\sin 26 - F = 100 \times 2.5$	M1	$F = ma$ using **their** "new" *a*. All forces present. No extras. Require attempt at wt cpt. Allow $s \leftrightarrow c$ and sign errors.	
		B1	Weight term resolved correctly, seen in an equn or on a diagram.	
	$F = 179.603\ldots$ so 180 N (3 s. f.)	A1	cao. Accept – 180 N if consistent with direction of *F* on their diagram	5
		17		

Q8		Mark	Comment	Sub
(i)	$v_x = 8 - 4t$	M1	**either** Differentiating **or** Finding '*u*' and '*a*' from *x* and use of *v* = *u* + *at*	
		A1		
	$v_x = 0 \Leftrightarrow t = 2$ so at $t = 2$	F1	FT **their** $v_x = 0$	3
(ii)	$y = \int \left(3t^2 - 8t + 4\right) \mathrm{d}t$	M1	Integrating v_y with at least one correct integrated term.	
	$= t^3 - 4t^2 + 4t + c$	A1	All correct. Accept no arbitrary constant.	
	$y = 3$ when $t = 1$ so $3 = 1 - 4 + 4 + c$	M1	Clear evidence	
	so $c = 3 - 1 = 2$ and $y = t^3 - 4t^2 + 4t + 2$	E1	Clearly shown and stated	4
(iii)	We need $x = 0$ so $8t - 2t^2 = 0$	M1	May be implied.	
	so $t = 0$ or $t = 4$	A1	Must have both	
	$t = 0$ gives $y = 2$ so 2 m	A1	Condone 2**j**	
	$t = 4$ gives $y = 4^3 - 4^3 + 16 + 2 = 18$ so 18 m	A1	Condone 18**j**	4
(iv)	We need $v_x = v_y = 0$	M1	**either** Recognises $v_x = 0$ when $t = 2$ **or** Finds time(s) when $v_y = 0$ **or** States or implies $v_x = v_y = 0$	
	From above, $v_x = 0$ only when $t = 2$ so evaluate $v_y(2)$	M1	Considers $v_x = 0$ **and** $v_y = 0$ with **their** time(s)	
	$v_y(2) = 0$ [$(t - 2)$ is a factor] so yes only at $t = 2$	A1	$t = 2$ recognised as only value (accept as evidence only $t = 2$ used below). For the last 2 marks, no credit lost for reference to $t = \frac{2}{3}$.	
	At $t = 2$, the position is (8, 2)	B1	May be implied	
	Distance is $\sqrt{8^2 + 2^2} = \sqrt{68}$ m (8.25 3 s.f.)	B1	FT from **their** position. Accept one position followed through correctly.	5
(v)	$t = 0, 1$ give (0, 2) and (6, 3)	B1	At least one value $0 \le t < 2$ correctly calc. This need not be plotted	
		B1	Must be *x*-*y* curve. Accept sketch. Ignore curve outside interval for *t*. Accept unlabelled axes. Condone use of line segments.	
		B1	At least three correct points **used** in *x*-*y* graph or sketch. General shape correct. Do not condone use of line segments.	3
		19		

4762 Mechanics 2

Q 1		Mark		Sub
(i)	**either**			
		M1	Use of $I = Ft$	
	$m \times 2u = 5F$	A1		
	so $F = 0.4mu$ in direction of the velocity	A1	Must have reference to direction. Accept diagram.	
	or			
		M1	Use of *suvat* **and** N2L	
	$a = \frac{2u}{5}$	A1	May be implied	
	so $F = 0.4mu$ in direction of the velocity	A1	Must have reference to direction. Accept diagram.	3
(ii)				
		M1	For 2 equns considering PCLM, NEL or Energy	
	PCLM → $2um + 3um = mv_P + 3mv_Q$			
	NEL → $v_Q - v_P = 2u - u = u$			
	Energy $\frac{1}{2}m \times (2u)^2 + \frac{1}{2}(3m) \times u^2$			
	$= \frac{1}{2}m \times v_P{}^2 + \frac{1}{2}(3m) \times v_Q{}^2$			
		A1	One correct equation	
		A1	Second correct equation	
	Solving to get both velocities	M1	Dep on 1st M1. Solving pair of equations.	
	$v_Q = \frac{3u}{2}$	E1	If Energy equation used, allow 2nd root discarded without comment.	
	$v_P = \frac{u}{2}$	A1		
			[If AG subst in one equation to find other velocity, and no more, max SC3]	6
(iii)	**either**			
	After collision with barrier $v_Q = \frac{3eu}{2} \leftarrow$	B1	Accept no direction indicated	
	so $\rightarrow \; m\frac{u}{2} - 3m\frac{3eu}{2} = -4m\frac{u}{4}$	M1	PCLM	
		A1	LHS Allow sign errors. Allow use of $3mv_Q$.	
		A1	RHS Allow sign errors	
	so $e = \frac{1}{3}$	A1		
	At the barrier the impulse on Q is given by			
	$\rightarrow \; 3m\left(-\frac{3u}{2} \times \frac{1}{3} - \frac{3u}{2}\right)$	M1	Impulse is $m(v - u)$	
		F1	$\pm\frac{3u}{2} \times \frac{1}{3}$	
	so impulse on Q is $-6mu \rightarrow$	F1	Allow ± and direction not clear. FT only *e*.	
	so impulse on the barrier is $6mu \rightarrow$	A1	cao. Direction must be clear. Units not required.	9
		18		

Q 1	continued	**mark**		**sub**
(iii)	**or**			
	After collision with barrier $v_Q = \frac{3eu}{2} \leftarrow$	B1		
	Impulse – momentum overall for Q			
	$\rightarrow \quad 2mu + 3mu + I = -4m \times \frac{u}{4}$	M1	All terms present	
		A1	All correct except for sign errors	
	$I = -6mu$	A1		
	so impulse of $6mu$ on the barrier $\rightarrow$	A1	Direction must be clear. Units not required.	
	Consider impact of Q with the barrier to give speed v_Q after impact			
	$\rightarrow \quad \frac{3u}{2} \times 3m - 6mu = 3mv_Q$	M1	Attempt to use I - M	
		F1		
	so $v_Q = -\frac{u}{2}$	F1		
	$e = \frac{u}{2} \div \frac{3u}{2} = \frac{1}{3}$	A1	cao	
				9

Q 2		Mark		Sub
(i)	$R = 80g\cos\theta$ or $784\cos\theta$	B1	Seen	
	$F_{\max} = \mu R$	M1		
	so $32g\cos\theta$ or $313.6\cos\theta$ N	A1		3
(ii)	Distance is $\dfrac{1.25}{\sin\theta}$	B1		
	WD is $F_{max}\, d$	M1		
	so $32g\cos\theta \times \dfrac{1.25}{\sin\theta}$	E1	Award for this or equivalent seen	
	$= \dfrac{392}{\tan\theta}$			3
(iii)	Δ GPE is mgh	M1		
	so $80 \times 9.8 \times 1.25 = 980$ J	A1	Accept 100g J	2
(iv)	**either**			
	$P = Fv$	M1		
	so $(80g\sin 35 + 32g\cos 35) \times 1.5$	B1	Weight term	
		A1	All correct	
	= 1059.85… so 1060 W (3 s. f.)	A1	cao	
	or			
	$P = \dfrac{\text{WD}}{\Delta t}$	M1		
	so $\dfrac{980 + \dfrac{392}{\tan 35}}{\left(\dfrac{1.25}{\sin 35}\right) \div 1.5}$	B1	Numerator FT **their** GPE	
		B1	Denominator	
	= 1059.85… so 1060 W (3 s. f.)	A1	cao	4
(v)	**either**			
	Using the W-E equation	M1	Attempt speed at ground or dist to reach required speed. Allow only init KE omitted	
	$0.5 \times 80 \times v^2 - 0.5 \times 80 \times \left(\dfrac{1}{2}\right)^2 = 980 - \dfrac{392}{\tan 35}$	B1	KE terms. Allow sign errors. FT from (iv).	
		B1	Both WD against friction and GPE terms. Allow sign errors. FT from parts above.	
		A1	All correct	
	$v = 3.2793..$ so yes	A1	CWO	
	or			
	N2L down slope	M1	All forces present	
	$a = 2.409973…$	A1		
	distance slid, using *uvast* is 1.815372…	A1		
	vertical distance is 1.815372…× sin35	M1	valid comparison	
	= 1.0412… < 1.25 so yes	A1	CWO	5
		17		

Q 3		Mark		Sub
(i)		M1 B1 M1	Correct method for $\bar{y}$ or $\bar{z}$ Total mass correct $15\cos\alpha$ or $15\sin\alpha$ attempted either part	
	$\bar{y}: \quad 250\times 4+125\left(8+\frac{30}{2}\cos\alpha\right)=375\bar{y}$	B1 B1	$\left(8+\frac{30}{2}\cos\alpha\right)$ 250×4	
	$\bar{y}=\frac{28}{3}=9\frac{1}{3}$	E1	Accept any form	
	$\bar{z}: \quad (250\times 0+)\ 125\times\frac{30}{2}\sin\alpha=375\bar{z}$	B1	LHS	
	$\bar{z}=3$	E1		8
(ii)	Yes. Take moments about CD. c.w moment from weight; no a.c moment from table	E1 E1	[Award E1 for $9\frac{1}{3}>8$ seen or 'the line of action of the weight is outside the base]	2
(iii)	c.m. new part is at (0, 8 + 20, 15)	M1 M1	Either *y* or *z* coordinate correct Attempt to 'add' to (i) or start again. Allow mass error.	
	$375\times\frac{28}{3}+125\times 28=500\bar{y}$ so $\bar{y}=14$	E1		
	$375\times 3+125\times 15=500\bar{z}$ so $\bar{z}=6$	E1		4
(iv)	Diagram	B1 B1	Roughly correct diagram Angle identified (may be implied)	
	Angle is $\arctan\frac{6}{14}$	M1	Use of tan. Allow use of 14/6 or equivalent.	
	$= 23.1985\ldots$ so 23.2° (3 s. f.)	A1	cao	4
		18		

Q 4		mark		sub
(a)				
	Let the ↑ forces at P and Q be R_P and R_Q			
(i)	c.w. moments about P	M1	Moments taken about a named point.	
	$2\times600-3R_Q=0$ so force of 400 N ↑ at Q	A1		
	a.c. moments about Q or resolve	M1		
	$R_P=200$ so force of 200 N ↑ at P	A1		4
(ii)	$R_P=0$	B1	Clearly recognised or used.	
	c.w. moments about Q	M1	Moments attempted with all forces. Dep on $R_P=0$ or R_P not evaluated.	
	$2L-1\times600=0$ so $L=300$	A1		3
(b) (i)				
	$\cos\alpha=\frac{15}{17}$ or $\sin\alpha=\frac{8}{17}$ or $\tan\alpha=\frac{8}{15}$	B1	Seen here or below or implied by use.	
	c.w moments about A	M1	Moments. All forces must be present and appropriate resolution attempted.	
	$16\times340\cos\alpha-8R=0$	A1		
	so $R=600$	E1	Evidence of evaluation.	4
(ii)	Diagram	B1	Must have 600 (or R) and 340 N and reactions at A.	
		B1	All internal forces clearly marked as tension or thrust. Allow mixture. [Max of B1 if extra forces present]	
	(Solution below assumes all internal forces set as tensions)			2
(iii)		M1	Equilibrium at a pin-joint	
	B ↓ $340\cos\alpha+T_{BC}\cos\alpha=0$			
	so $T_{BC}=-340$ (Thrust of) 340 N in BC	A1		
	C → $T_{BC}\sin\alpha-T_{AC}\sin\alpha=0$			
	so $T_{AC}=-340$ (Thrust of) 340 N in AC	F1		
	B ← $T_{AB}+T_{BC}\sin\alpha-340\sin\alpha=0$	M1	Method for T_{AB}	
	so $T_{AB}=320$ (Tension of) 320 N in AB	A1		
	Tension/ Thrust all consistent with working	F1		
			[Award a max of 4/6 if working inconsistent with diagram]	6
		19		

4763 Mechanics 3

1 (i)	[Force] $=\mathrm{M\,L\,T^{-2}}$ [Density] $=\mathrm{M\,L^{-3}}$	B1 B1 **2**	
(ii)	$[\eta]=\dfrac{[F][d]}{[A][v_2-v_1]}=\dfrac{(\mathrm{M\,L\,T^{-2}})(\mathrm{L})}{(\mathrm{L^2})(\mathrm{L\,T^{-1}})}$ $=\mathrm{M\,L^{-1}\,T^{-1}}$	B1 M1 A1 **3**	for $[A]=\mathrm{L^2}$ and $[v]=\mathrm{L\,T^{-1}}$ Obtaining the dimensions of η
(iii)	$\left[\dfrac{2a^2\rho g}{9\eta}\right]=\dfrac{\mathrm{L^2\,(M\,L^{-3})(L\,T^{-2})}}{\mathrm{M\,L^{-1}\,T^{-1}}}=\mathrm{L\,T^{-1}}$ which is same as the dimensions of v	B1 M1 E1 **3**	For $[g]=\mathrm{L\,T^{-2}}$ Simplifying dimensions of RHS Correctly shown
(iv)	$(\mathrm{M\,L^{-3}})\mathrm{L}^{\alpha}(\mathrm{L\,T^{-1}})^{\beta}(\mathrm{M\,L^{-1}\,T^{-1}})^{\gamma}$ is dimensionless $\gamma=-1$ $-\beta-\gamma=0$ $-3+\alpha+\beta-\gamma=0$ $\alpha=1,\quad \beta=1$	B1 cao M1 M1A1 A1 cao **5**	
(v)	$R=\dfrac{\rho wv}{\eta}=\dfrac{0.4\times25\times150}{1.6\times10^{-5}}\quad(=9.375\times10^{7})$ $=\dfrac{1.3\times5v}{1.8\times10^{-5}}$ Required velocity is $260\ \mathrm{m\,s^{-1}}$	M1 A1 A1 cao **3**	Evaluating R Equation for v

2 (a)(i)	$T\cos\alpha = T\cos\beta + 0.27\times 9.8$ $\sin\alpha = \frac{1.2}{2.0} = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$ $(\alpha = 36.87°)$ $\sin\beta = \frac{1.2}{1.3} = \frac{12}{13}$, $\cos\beta = \frac{5}{13}$ $(\beta = 67.38°)$ $\frac{27}{65}T = 2.646$ Tension is 6.37 N	M1 A1 B1 M1 E1 **5**	Resolving vertically (weight and at least one resolved tension) Allow T_1 and T_2 For $\cos\alpha$ and $\cos\beta$ [or α and β] Obtaining numerical equation for T e.g. $T(\cos 36.9 - \cos 67.4) = 0.27\times 9.8$ *(Condone 6.36 to 6.38)*
(ii)	$T\sin\alpha + T\sin\beta = 0.27\times\frac{v^2}{1.2}$ $6.37\times\frac{3}{5} + 6.37\times\frac{12}{13} = 0.27\times\frac{v^2}{1.2}$ $v^2 = 43.12$ Speed is $6.57\ \mathrm{m\,s^{-1}}$	M1 A1 M1 A1 **4**	Using $v^2/1.2$ Allow T_1 and T_2 Obtaining numerical equation for v^2
(b)(i)	$0.2\times 9.8 = 0.2\times\frac{u^2}{1.25}$ $u^2 = 9.8\times 1.25 = 12.25$ Speed is $3.5\ \mathrm{m\,s^{-1}}$	M1 E1 **2**	Using acceleration $u^2/1.25$
(ii)	$\frac{1}{2}m(v^2 - 3.5^2) = mg(1.25 - 1.25\cos 60)$ $v^2 = 24.5$ Radial component is $\frac{24.5}{1.25}$ $= 19.6\ \mathrm{m\,s^{-2}}$ Tangential component is $g\sin 60$ $= 8.49\ \mathrm{m\,s^{-2}}$	M1 A1 M1 A1 M1 A1 **6**	Using conservation of energy With numerical value obtained by using energy (M0 if mass, or another term, included) For sight of $(m)g\sin 60°$ with no other terms
(iii)	$T + 0.2\times 9.8\cos 60 = 0.2\times 19.6$ Tension is 2.94 N	M1 A1 cao **2**	Radial equation (3 terms) *This M1 can be awarded in (ii)*

3 (i)	$\frac{980}{25}y = 5\times 9.8$ Extension is 1.25 m	M1 A1 **2**	Using $\frac{\lambda y}{l_0}$ (Allow M1 for $980y = mg$)
(ii)	$T = \frac{980}{25}(1.25 + x)$ $5\times 9.8 - 39.2(1.25 + x) = 5\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $-39.2x = 5\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$ $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -7.84x$	B1 (ft) M1 F1 E1 **4**	*(ft) indicates ft from previous parts as for A marks* Equation of motion with three terms Must have $\ddot{x}$ In terms of x only
(iii)	$8.4^2 = 7.84(A^2 - 1.25^2)$ Amplitude is 3.25 m	M2 A1 A1 **4**	Using $v^2 = \omega^2(A^2 - x^2)$
	OR M2 $\frac{980}{2\times 25}y^2 = 5\times 9.8y + \tfrac{1}{2}\times 5\times 8.4^2$ A1 $y = 4.5$ Amplitude is $4.5 - 1.25 = 3.25$ m A1		Equation involving EE, PE and KE
	OR $x = A\sin 2.8t + B\cos 2.8t$ $x = -1.25,\ v = 8.4$ when $t = 0$ M2 $\Rightarrow A = 3,\ B = -1.25$ A1 Amplitude is $\sqrt{A^2 + B^2} = 3.25$ A1		Obtaining A and B Both correct
(iv)	Maximum speed is $A\omega = 3.25\times 2.8$ $= 9.1\ \mathrm{m\,s^{-1}}$	M1 A1 **2**	or equation involving EE, PE and KE ft only if answer is greater than 8.4
(v)	$x = 3.25\cos 2.8t$ $-1.25 = 3.25\cos 2.8t$ Time is 0.702 s	B1 (ft) M1 M1 A1 cao **4**	or $x = 3.25\sin 2.8t$ or $v = 9.1\cos 2.8t$ or $v = 9.1\sin 2.8t$ or $x = 3.25\sin(2.8t + \varepsilon)$ etc or $x = \pm 3\sin 2.8t \pm 1.25\cos 2.8t$ Obtaining equation for t or ε by setting $x = (\pm)1.25$ or $v = (\pm)8.4$ or solving $\pm 3\sin 2.8t \pm 1.25\cos 2.8t = 3.25$ Strategy for finding the required time e.g. $\frac{1}{2.8}\sin^{-1}\frac{1.25}{3.25} + \frac{1}{4}\times\frac{2\pi}{2.8}$ $2.8t - 0.3948 = \tfrac{1}{2}\pi$ or $2.8t - 1.966 = 0$

(vi)	e.g. Rope is light Rock is a particle No air resistance / friction / external forces Rope obeys Hooke's law / Perfectly elastic / Within elastic limit / No energy loss in rope	B1B1B1 **3**	Three modelling assumptions

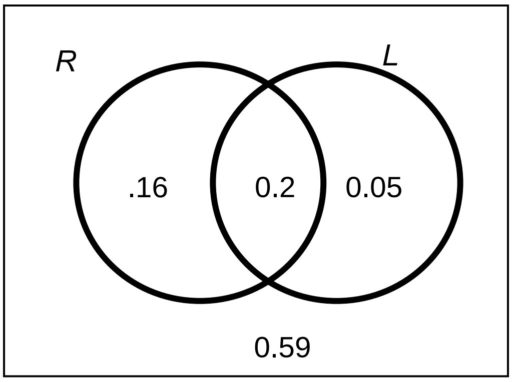
4 (a)	$\int \frac{1}{2}y^2\,\mathrm{d}x = \int_{-a}^{a} \frac{1}{2}(a^2-x^2)\,\mathrm{d}x$ $= \left[\ \frac{1}{2}(a^2x-\frac{1}{3}x^3)\ \right]_{-a}^{a}$ $= \frac{2}{3}a^3$ $\bar{y} = \dfrac{\frac{2}{3}a^3}{\frac{1}{2}\pi a^2}$ $= \dfrac{4a}{3\pi}$	M1 A1 M1 E1 **4**	For integral of (a^2-x^2) *Dependent on previous M1*
(b)(i)	$V = \int \pi y^2\,\mathrm{d}x = \int_{0}^{h} \pi(mx)^2\,\mathrm{d}x$ $= \left[\ \frac{1}{3}\pi m^2x^3\ \right]_0^h = \frac{1}{3}\pi m^2h^3$ $\int \pi xy^2\,\mathrm{d}x = \int_0^h \pi x(mx)^2\,\mathrm{d}x$ $= \left[\ \frac{1}{4}\pi m^2x^4\ \right]_0^h = \frac{1}{4}\pi m^2h^4$ $\bar{x} = \dfrac{\frac{1}{4}\pi m^2h^4}{\frac{1}{3}\pi m^2h^3}$ $= \frac{3}{4}h$	M1 A1 M1 A1 M1 E1 **6**	*π may be omitted throughout* For integral of x^2 or use of $V=\frac{1}{3}\pi r^2h$ and $r=mh$ For integral of x^3 *Dependent on M1 for integral of x^3*
(ii)	$m_1 = \frac{1}{3}\pi\times0.7^2\times2.4\rho = \frac{1}{3}\pi\rho\times1.176$ $\mathrm{VG}_1 = 1.8$ $m_2 = \frac{1}{3}\pi\times0.4^2\times1.1\rho = \frac{1}{3}\pi\rho\times0.176$ $\mathrm{VG}_2 = 1.3+\frac{3}{4}\times1.1 = 2.125$ $(m_1-m_2)(\mathrm{VG})+m_2(\mathrm{VG}_2) = m_1(\mathrm{VG}_1)$ $(\mathrm{VG})+0.176\times2.125 = 1.176\times1.8$ Distance (VG) is 1.74 m	B1 B1 M1 F1 A1 **5**	For m_1 and m_2 (or volumes) or $\frac{1}{4}\times1.1$ from base Attempt formula for composite body
(iii)	VQG is a right-angle $\mathrm{VQ} = \mathrm{VG}\cos\theta$ where $\tan\theta = \dfrac{0.7}{2.4}$ $(\theta = 16.26°)$ $\mathrm{VQ} = 1.7428\times\dfrac{24}{25}$ $= 1.67$ m	M1 M1 A1 **3**	ft is $\mathrm{VG}\times0.96$

4766 Statistics 1

Section A

Q1 (i)	(With $\sum fx = 7500$ and $\sum f = 10000$ then arriving at the mean) (i) £0.75 scores (B1, B1) (ii) 75p scores (B1, B1) (iii) 0.75p scores (B1, B0) (incorrect units) (iv) £75 scores (B1, B0) (incorrect units) **After B0, B0** then sight of $\frac{7500}{10000}$ scores SC1. SC1or an answer in the range £0.74 - £0.76 or 74p – 76p (both inclusive) scores SC1 (units essential to gain this mark)	B1 for numerical mean (0.75 or 75 seen) B1dep for correct units attached	
	Standard Deviation: (CARE NEEDED here with close proximity of answers) • 50.2(0) using divisor 9999 scores B2 (50.20148921) • 50.198 (= 50.2) using divisor 10000 scores B1(*rmsd*) • If divisor is not shown (or calc used) and only an answer of 50.2 (i.e. not coming from 50.198) is seen then award B2 on b.o.d. (default)	B2 correct s.d. (B1) correct rmsd (B2) default	
	After B0 scored then an attempt at S_{xx} as evident by either $S_{xx} = (5000+200000+25000000) - \frac{7500^2}{10000} \quad (=25199375)$ or $S_{xx} = (5000 + 200000 + 25000000) - 10000(0.75)^2$ **scores (M1) or M1ft 'their 7500^2' or 'their 0.75^2'** NB The structure must be correct in both above cases with a max of 1 slip only after applying the f.t.	**$\sum fx^2 = 25,205,000$** **Beware $\sum x^2$=25, 010, 100** **After B0** scored then (M1) or M1f.t. for attempt at S_{xx} *NB full marks for correct results from recommended method which is use of calculator functions*	**4**

(ii)	P(Two £10 or two £100) $= \frac{50}{10000}\times\frac{49}{9999}+\frac{20}{10000}\times\frac{19}{9999}$ = 0.0000245 + 0.0000038 = (0.00002450245 + 0.00000380038) = 0.000028(3) o.e. = (0.00002830283) <u>After M0, M0</u> then $\frac{\mathbf{50}}{\mathbf{10000}}\times\frac{\mathbf{50}}{\mathbf{10000}}+\frac{\mathbf{20}}{\mathbf{10000}}\times\frac{\mathbf{20}}{\mathbf{10000}}$ o.e. Scores SC1 (ignore final answer but SC1 may be implied by sight of 2.9×10^{-5} o.e.) Similarly, $\frac{\mathbf{50}}{\mathbf{10000}}\times\frac{\mathbf{49}}{\mathbf{10000}}+\frac{\mathbf{20}}{\mathbf{10000}}\times\frac{\mathbf{19}}{\mathbf{10000}}$ scores SC1	M1 for either correct product seen (ignore any multipliers) M1 sum of both correct (ignore any multipliers) A1 CAO (as opposite with no rounding) (SC1 case #1) (SC1 case #2) <u>**CARE**</u> answer is also 2.83×10^{-5}	**3**
		TOTAL	**7**
Q2 **(i)**	Either $\text{P(all correct)} = \frac{1}{6}\times\frac{1}{5}\times\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}\times\frac{1}{1}=\frac{1}{720}$ or $\text{P(all correct)} = \frac{1}{6!}=\frac{1}{720}=0.00139$	M1 for 6! Or 720 (sioc) or product of fractions A1 CAO (accept 0.0014)	**2**
(ii)	Either $\text{P(picks T, O, M)} = \frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}=\frac{1}{20}$ or $\text{P(picks T, O, M)} = \frac{1}{6}\times\frac{1}{5}\times\frac{1}{4}\times 3!=\frac{1}{20}$ or $\text{P(picks T, O, M)} = \frac{1}{\binom{6}{3}}=\frac{1}{20}$	M1 for denominators M1 for numerators or 3! A1 CAO Or M1 for $\binom{6}{3}$ or 20 <u>sioc</u> M1 for $1/\binom{6}{3}$ A1 CAO	**3**
		TOTAL	**5**
Q3 **(i)**	$p = 0.55$	B1 cao	**1**
(ii)	E(X) = $0\times0.55+1\times0.1+2\times0.05+3\times0.05+4\times0.25=1.35$ $\text{E}(X^2) = 0\times0.55+1\times0.1+4\times0.05+9\times0.05+16\times0.25$ = 0 + 0.1 + 0.2 + 0.45 + 4 = (4.75) Var(X) = ‘their’ 4.75 – 1.35^2 = 2.9275 awfw (2.9275 – 2.93)	M1 for Σrp (at least 3 non zero terms correct) A1 CAO(no ‘n’ or ‘n-1’ divisors) M1 for Σr^2p (at least 3 non zero terms correct) M1dep for – their E(X)² provided Var(X) > 0 A1 cao (no ‘n’ or ‘n-1’ divisors)	**5**
(iii)	P(At least 2 both times) = $(0.05+0.05+0.25)^2$ = 0.1225 o.e.	M1 for $(0.05+0.05+0.25)^2$ or 0.35^2 seen A1cao: awfw (0.1225 - 0.123) or 49/400	**2**
		TOTAL	**8**

Q4 **(i)**	$X \sim \mathrm{B}(50, 0.03)$ (***A***) $\mathrm{P}(X=1) = \binom{50}{1} \times 0.03 \times 0.97^{49} = 0.3372$	M1 0.03×0.97^{49} or 0.0067(4)…. M1 $\binom{50}{1} \times pq^{49}$ (p+q =1) A1 CAO **(awfw 0. 337 to 0. 3372) or 0.34(2s.f.) or 0.34(2d.p.) but not just 0.34**	**3**
	(***B***) $\mathrm{P}(X=0) = 0.97^{50} = 0.2181$ $\boldsymbol{P}(\boldsymbol{X} > 1) = 1 - 0.2181 - 0.3372 = 0.4447$	B1 for 0.97^{50} or 0.2181 (awfw 0.218 to 0.2181) M1 for 1 – (‘their’ p (X = 0) + ‘their’ p(X = 1)) must have both probabilities A1 CAO (awfw 0.4447 to 0.445)	**3**
(ii)	Expected number = np = 240 × 0.3372 = 80.88 – 80.93 = (81) *Condone 240 × 0.34 = 81.6 = (82) but for M1 A1f.t.*	M1 for 240 × prob (A) A1FT	**2**
		TOTAL	**8**
Q5 **(i)**	P(R) × P(*L*) = 0.36 × 0.25 = 0.09 ≠ P(*R*∩*L*) Not equal so not independent. (Allow 0.36 × 0.25 ≠ 0.2or 0.09 ≠ 0.2 or ≠ p(R ∩ L) so not independent)	M1 for 0.36 × 0.25 or 0.09 seen A1 (numerical justification needed)	**2**
(ii)		G1 for two overlapping circles labelled G1 for 0.2 and either 0.16 *or* 0.05 in the **correct places** G1 for all 4 **correct** probs in the **correct** places (including the 0.59) The last two G marks are independent of the labels	**3**
(iii)	$P(L \mid R) = \frac{P(L \cap R)}{P(R)} = \frac{0.2}{0.36} = \frac{5}{9} = 0.556$ (awrt 0.56) This is the probability that Anna is late given that it is raining. (must be in context) Condone ‘if ’ or ‘when’ or ‘on a rainy day’ for ‘given that’ but **<u>not</u>** the words ‘and’ or ‘because’ or ‘due to’	M1 for 0.2/0.36 o.e. A1 cao E1 (indep of M1A1) **Order/structure <u>must</u> be correct i.e. no reverse statement**	**3**
		TOTAL	**8**

Section B

Q6 (i)	Median = 4.06 – 4.075 (inclusive) Q_1 = 3.8 Q_3 = 4.3 Inter-quartile range = 4.3 – 3.8 = 0.5	B1cao B1 for Q_1 (cao) B1 for Q_3 (cao) B1 ft for IQR must be using t-values not locations to earn this mark	**4**
(ii)	Lower limit 'their 3.8' – 1.5 × 'their 0.5' = (3.05) Upper limit 'their 4.3' + 1.5 × 'their 0.5' = (5.05) Very few if any temperatures below 3.05 (but not zero) None above 5.05 'So few, if any outliers' scores SC1	B1ft: must have -1.5 B1ft: must have +1.5 E1ft dep on -1.5 and Q_1 E1ft dep on+1.5 and Q_3 Again, must be using t-values NOT locations to earn these 4 marks	**4**
(iii)	Valid argument such as 'Probably not, because there is nothing to suggest that they are not genuine data items; (they do not appear to form a separate pool of data.') Accept: exclude outlier – 'measuring equipment was wrong' or 'there was a power cut' or ref to hot / cold day [Allow suitable valid alternative arguments]	E1	**1**
(iv)	Missing frequencies 25, 125, 50	B1, B1, B1 (all cao)	**3**
(v)	Mean = (3.2×25 + 3.6×125 + 4.0×243 + 4.4×157 + 4.8×50)/600 = 2432.8/600 = 4.05(47)	M1 for at least 4 midpoints correct and being used in attempt to find $\sum ft$ A1cao: awfw (4.05 – 4.055) ISW or rounding	**2**
(vi)	New mean = 1.8 × 'their 4.05(47)' + 32 = 39.29(84) to 39.3 New s = 1.8 × 0.379 = 0.682	B1 FT M1 for 1.8 × 0.379 A1 CAO awfw (0.68 – 0.6822)	**3**
		TOTAL	**17**

Q7 **(i)**	$X \sim B(10, 0.8)$ (**A**) *Either* $P(\mathbf{X} = 8) = \binom{10}{8} \times 0.8^8 \times 0.2^2 = 0.3020$ (awrt) *or* $P(X = 8) = P(X \le 8) - P(X \le 7)$ $= 0.6242 - 0.3222 = 0.3020$	M1 $0.8^8 \times 0.2^2$ or 0.00671… M1 $\binom{10}{8} \times p^8 q^2$; (p +q =1) Or $45 \times p^8 q^2$; (p +q =1) A1 CAO **(0.302) not 0.3** OR: M2 for 0.6242 – 0.3222 A1 CAO	3
	(**B**) *Either* $P(X \ge 8) = 1 - P(X \le 7)$ $= 1 - 0.3222 = 0.6778$ *or* $P(X \ge 8) = P(X = 8) + P(X = 9) + P(X = 10)$ $= 0.3020 + 0.2684 + 0.1074 = 0.6778$	M1 for 1 – 0.3222 (s.o.i.) A1 CAO awfw 0.677 – 0.678 or M1 for sum of 'their' p(X=8) plus correct expressions for p(x=9) and p(X=10) A1 CAO awfw 0.677 – 0.678	2
(ii)	Let $X \sim B(18, p)$ Let p = probability of delivery (within 24 hours) (for population)	B1 for definition of p	
	H_0: $p = 0.8$ H_1: $p < 0.8$	B1 for H_0 B1 for H_1	
	$P(X \le 12) = 0.1329 > 5\%$ ref: [pp =0.0816]	M1 for probability 0.1329	
		M1dep strictly for comparison of 0.1329 with 5% (seen or clearly implied)	
	So not enough evidence to reject H_0	A1dep on both M's	
	Conclude that there is not enough evidence to indicate that less than 80% of orders will be delivered within 24 hours	E1dep on M1,M1,A1 for conclusion in context	
	Note: use of critical region method scores M1 for region {0,1,2,…,9, 10} M1dep for 12 does not lie in critical region then A1dep E1dep as per scheme		7

(iii)	Let $X \sim B(18, 0.8)$ H_1: $p \neq 0.8$ LOWER TAIL $P(X \leq 10) = 0.0163 < 2.5\%$ $P(X \leq 11) = 0.0513 > 2.5\%$ UPPER TAIL $P(X \geq 17) = 1 - P(X \leq 16) = 1 - 0.9009 = 0.0991 > 2.5\%$ $P(X \geq 18) = 1 - P(X \leq 17) = 1 - 0.9820 = 0.0180 < 2.5\%$ So critical region is $\{\underline{0},1,2,3,4,5,6,7,8,9,10,18\}$ o.e. Condone $X \leq 10$ and $X \geq 18$ or $X = 18$ but **not** $p(X \leq 10)$ and $p(X \geq 18)$ Correct CR without supportive working scores SC2 max after the 1st B1 (SC1 for each fully correct tail of CR)	B1 for H_1 B1 for 0.0163 or 0.0513 seen M1dep for either correct comparison with **2.5% (not 5%)** (seen or clearly implied) A1dep for correct lower tail CR (must have zero) B1 for 0.0991 or 0.0180 seen M1dep for either correct comparison with **2.5% (not 5%)** (seen or clearly implied) A1dep for correct upper tail CR	**7**
		TOTAL	**19**

4767 Statistics 2

Question 1

<table>
<tr>
<td>(i)</td>
<td>
<table>
<tr><td>x</td><td>18</td><td>43</td><td>52</td><td>94</td><td>98</td><td>206</td><td>784</td><td>1530</td></tr>
<tr><td>y</td><td>1.15</td><td>0.97</td><td>1.26</td><td>1.35</td><td>1.28</td><td>1.42</td><td>1.32</td><td>1.64</td></tr>
<tr><td>Rank x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>Rank y</td><td>2</td><td>1</td><td>3</td><td>6</td><td>4</td><td>7</td><td>5</td><td>8</td></tr>
<tr><td>d</td><td>-1</td><td>1</td><td>0</td><td>-2</td><td>1</td><td>-1</td><td>2</td><td>0</td></tr>
<tr><td>d^2</td><td>1</td><td>1</td><td>0</td><td>4</td><td>1</td><td>1</td><td>4</td><td>0</td></tr>
</table>

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)} = 1 - \frac{6 \times 12}{8 \times 63}$$

= 0.857 (to 3 s.f.) [*allow* 0.86 to 2 s.f.]
</td>
<td>M1 for attempt at ranking (allow all ranks reversed)
M1 for d^2
A1 for $\Sigma d^2 = 12$
M1 for method for r_s
A1 f.t. for $|r_s| < 1$
NB No ranking scores zero</td>
<td>5</td>
</tr>
<tr>
<td>(ii)</td>
<td>H_0: no association between X and Y in the population
H_1: some association between X and Y in the population
Two tail test critical value at 5% level is 0.7381
Since 0.857> 0. 7381, there is sufficient evidence to reject H_0,
i.e. conclude that the evidence suggests that there is association between population size X and average walking speed Y.</td>
<td>B1 for H_0
B1 for H_1
B1 for population SOI
NB H_0 H_1 not ito ρ
B1 for $\pm$0. 7381
M1 for sensible comparison with c.v., provided $|r_s| < 1$
A1 for conclusion in words f.t. their r_s and sensible cv</td>
<td>6</td>
</tr>
<tr>
<td>(iii)</td>
<td>
$\bar{t} = 45, \quad \bar{w} = 2.2367$

$$b = \frac{Stw}{Stt} = \frac{584.6 - 270 \times 13.42/6}{13900 - 270^2/6} = \frac{-19.3}{1750} = \text{-0.011}$$

$$\text{OR } b = \frac{584.6/6 - 45 \times 2.2367}{13900/6 - 45^2} = \frac{-3.218}{291.6667} = \text{-0.011}$$

hence least squares regression line is:

$$w - \bar{w} = b(t - \bar{t})$$
$$\Rightarrow w - 2.2367 = -0.011(t - 45)$$
$$\Rightarrow w = -0.011t + 2.73$$
</td>
<td>B1 for $\bar{t}$ and $\bar{w}$ used (SOI)
M1 for attempt at gradient (b)
A1 CAO for -0.011
M1 for equation of line
A1 FT for complete equation</td>
<td>5</td>
</tr>
</table>

(iv)	(*A*) For $t = 80$, predicted speed $= -0.011 \times 80 + 2.73 = 1.85$ (*B*) The relationship relates to adults, but a ten year old will not be fully grown so may walk more slowly. NB Allow E1 for comment about extrapolation not in context	M1 A1 FT provided $b < 0$ E1 extrapolation o.e. E1 sensible contextual comment	**4**
		TOTAL	**20**

Question 2

(i)	Binomial(5000,0.0001)	B1 for binomial B1 dep, for parameters	**2**
(ii)	*n* is large and *p* is small $\lambda = 5000 \times 0.0001 = 0.5$	B1, B1 (Allow appropriate numerical ranges) B1	**3**
(iii)	$\text{P}(X \geq 1) = 1 - \text{e}^{-\square}\dfrac{0.5^0}{0!} = 1 - 0.6065 = 0.3935$ or from tables $= 1 - 0.6065 = 0.3935$	M1 for correct calculation or correct use of tables A1	**2**
(iv)	P(9 of 20 contain at least one) $= \begin{pmatrix} 20 \\ 9 \end{pmatrix} \times 0.3935^9 \times 0.6065^{11}$ $= 0.1552$	M1 for coefficient M1 for $p^9 \times (1-p)^{11}$, *p* from part (iii) A1	**3**
(v)	Expected number $= 20 \times 0.3935 = 7.87$	M1 A1 FT	**2**
(vi)	Mean $= \dfrac{\Sigma xf}{n} = \dfrac{7+4}{20} = \dfrac{11}{20} = 0.55$ Variance $= \dfrac{1}{n-1}\left(\Sigma fx^2 - n\bar{x}^2\right)$ $= \dfrac{1}{19}\left(15 - 20 \times 0.55^2\right) = 0.471$	B1 for mean M1 for calculation A1 CAO	**3**
(vii)	Yes, since the mean is close to the variance, and also as the expected frequency for 'at least one', i.e. 7.87, is close to the observed frequency of 9.	B1 E1 for sensible comparison B1 for observed frequency $= 7 + 2 = 9$	**3**
		TOTAL	**18**

Question 3

(i)	(A) $\quad \mathrm{P}(X < 120) = \mathrm{P}\left(Z < \dfrac{120-115.3}{21.9}\right)$ $= \mathrm{P}(Z < 0.2146)$ $= \Phi(0.2146) = 0.5849$	M1 for standardizing A1 for $z = 0.2146$ A1 CAO (min 3 sf, to include use of difference column)	**3**
	(B) $\quad \mathrm{P}(100 < X < 110) =$ $\mathrm{P}\left(\dfrac{100-115.3}{21.9} < Z < \dfrac{110-115.3}{21.9}\right)$ $= \mathrm{P}(-0.6986 < Z < -0.2420)$ $= \Phi(0.6986) - \Phi(0.2420)$ $= 0.7577 - 0.5956$ $= 0.1621$	M1 for standardizing both 100 & 110 M1 for correct structure in calc^n A1 CAO	**3**
	(C) From tables $\Phi^{-1}(0.1) = -1.282$ $\dfrac{k-115.3}{21.9} = -1.282$ $k = 115.3 - 1.282 \times 21.9 = 87.22$	B1 for ±1.282 seen M1 for equation in k and negative z-value A1 CAO	**3**
(ii)	From tables, $\Phi^{-1}(0.70) = 0.5244, \Phi^{-1}(0.15) = -1.036$ $180 = \mu + 0.5244\sigma$ $140 = \mu - 1.036\sigma$ $40 = 1.5604\sigma$ $\sigma = 25.63, \mu = 166.55$	B1 for 0.5244 or ±1.036 seen M1 for at least one equation in μ and σ and Φ^{-1} value M1 dep for attempt to solve two equations A1 CAO for both	**4**
(iii)	$\Phi^{-1}(0.975) = 1.96$ $a = 166.55 - 1.96 \times 25.63 = 116.3$ $b = 166.55 + 1.96 \times 25.63 = 216.8$	B1 for ±1.96 seen M1 for either equation A1 A1 [Allow other correct intervals]	**4**
		TOTAL	**17**

Question 4

<table>
<tr>
<td>(i)</td>
<td>H_0: no association between growth and type of plant;
H_1: some association between growth and type of plant;
<table>
<tr><th>EXPECTED</th><th>Good</th><th>Average</th><th>Poor</th></tr>
<tr><td>Coriander</td><td>12.10</td><td>24.93</td><td>17.97</td></tr>
<tr><td>Aster</td><td>10.56</td><td>21.76</td><td>15.68</td></tr>
<tr><td>Fennel</td><td>10.34</td><td>21.31</td><td>15.35</td></tr>
</table>
<table>
<tr><th>CONTRIBUTION</th><th>Good</th><th>Average</th><th>Poor</th></tr>
<tr><td>Coriander</td><td>0.0008</td><td>0.3772</td><td>0.4899</td></tr>
<tr><td>Aster</td><td>1.2002</td><td>0.6497</td><td>3.4172</td></tr>
<tr><td>Fennel</td><td>1.2955</td><td>0.0226</td><td>1.2344</td></tr>
</table>
$X^2 = 8.69$
Refer to χ_4^2
Critical value at 5% level = 9.488
Result is not significant
There is not enough evidence to suggest that there is some association between reported growth and type of plant;
NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1or final A1</td>
<td>B1 (in context)
M1 A2 for expected values (to 2 dp)
(allow A1 for at least one row or column correct)
M1 for valid attempt at $(O-E)^2/E$
A1 for all correct
NB These M1A1 marks cannot be implied by a correct final value of X^2
M1 for summation
A1 for X^2 CAO
B1 for 4 d.o.f.
B1 CAO for cv
M1
A1</td>
<td>12</td>
</tr>
<tr>
<td>(ii)</td>
<td>Test statistic = $\frac{49.2-47}{8.5/\sqrt{50}} = \frac{2.2}{1.202} = 1.830$
1% level 1 tailed critical value of z = 2.326
$1.830 < 2.326$ so not significant.
There is not sufficient evidence to reject H_0
There is insufficient evidence to conclude that the flowers are larger.</td>
<td>M1 correct denominator
A1
B1 for 2.326
M1 (dep on first M1) for sensible comparison leading to a conclusion
A1 for fully correct conclusion in words in context</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TOTAL</td>
<td>17</td>
</tr>
</table>

4768 Statistics 3

<table>
<tr><td>Q1
(a)</td><td>$\mathrm{f}(x)=\lambda x^{c},\ \ 0\le x\le 1,\ \ \lambda>1$</td><td></td><td></td><td></td></tr>
<tr><td>(i)</td><td>$\int_0^1 \lambda x^{c}\,\mathrm{d}x=1$
$\therefore\left[\frac{\lambda x^{c+1}}{c+1}\right]_0^1=1$
$\therefore\frac{\lambda}{c+1}=1 \qquad \therefore c=\lambda-1$</td><td>M1
M1
A1</td><td>Correct integral, with limits (possibly appearing later), set equal to 1.
Integration correct and limits used.
c.a.o.</td><td>3</td></tr>
<tr><td>(ii)</td><td>$\mathrm{E}(X)=\int_0^1 \lambda x^{\lambda}\,\mathrm{d}x$
$=\left[\frac{\lambda x^{\lambda+1}}{\lambda+1}\right]_0^1 \quad =\frac{\lambda}{\lambda+1}.$</td><td>M1
M1
A1</td><td>Correct form of integral for $\mathrm{E}(X)$. Allow c’s expression for c.
Integration correct and limits used. ft c’s c.</td><td>3</td></tr>
<tr><td>(iii)</td><td>$\mathrm{E}(X^2)=\int_0^1 \lambda x^{\lambda+1}\,\mathrm{d}x$
$=\left[\frac{\lambda x^{\lambda+2}}{\lambda+2}\right]_0^1=\frac{\lambda}{\lambda+2}.$
$\mathrm{Var}(X)=\frac{\lambda}{\lambda+2}-\left(\frac{\lambda}{\lambda+1}\right)^2=\frac{\lambda(\lambda+1)^2-\lambda^2(\lambda+2)}{(\lambda+2)(\lambda+1)^2}$
$=\frac{\lambda^3+2\lambda^2+\lambda-\lambda^3-2\lambda^2}{(\lambda+2)(\lambda+1)^2}=\frac{\lambda}{(\lambda+2)(\lambda+1)^2}.$</td><td>M1
A1
M1
A1</td><td>Correct form of integral for $\mathrm{E}(X^2)$. Allow c’s expression for c.

Use of $\mathrm{Var}(X)=\mathrm{E}(X^2)-\mathrm{E}(X)^2$. Allow c’s $\mathrm{E}(X^2)$ and $\mathrm{E}(X)$.
Algebra shown convincingly. Beware printed answer.</td><td>4</td></tr>
<tr><td>(b)</td><td><table>
<tr><th>Times</th><th>− 32</th><th>Rank of |diff|</th></tr>
<tr><td>40</td><td>8</td><td>4</td></tr>
<tr><td>20</td><td>−12</td><td>7</td></tr>
<tr><td>18</td><td>−14</td><td>8</td></tr>
<tr><td>11</td><td>−21</td><td>12</td></tr>
<tr><td>47</td><td>15</td><td>9</td></tr>
<tr><td>36</td><td>4</td><td>2</td></tr>
<tr><td>38</td><td>6</td><td>3</td></tr>
<tr><td>35</td><td>3</td><td>1</td></tr>
<tr><td>22</td><td>−10</td><td>5</td></tr>
<tr><td>14</td><td>−18</td><td>10</td></tr>
<tr><td>12</td><td>−20</td><td>11</td></tr>
<tr><td>21</td><td>−11</td><td>6</td></tr>
</table>
$W_+=1+2+3+4+9=19$
Refer to Wilcoxon single sample tables for $n=12$.
Lower (or upper if 59 used) 5% tail is 17 (or 61 if 59 used).
Result is not significant.
Seems that there is no evidence that Godfrey’s times have decreased.</td><td>M1
M1
A1
B1
M1
A1
A1
A1</td><td>H_0: $m=32$, H_1: $m<32$, where m is the population median time.
for subtracting 32.
for ranks.
ft if ranks wrong.
(or $W_-=5+6+7+8+10+11+12=59$)
No ft from here if wrong.
i.e. a 1-tail test. No ft from here if wrong.
ft only c’s test statistic.
ft only c’s test statistic.</td><td>8</td></tr>
<tr><td></td><td></td><td></td><td></td><td>18</td></tr>
</table>

Q2	$V_G \sim N(56.5,\ 2.9^2)$ $V_W \sim N(38.4,\ 1.1^2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables penalise the first occurrence only.	
(i)	$P(V_G < 60) = P(Z < \frac{60 - 56.5}{2.9} = 1.2069)$ $= 0.8862$	M1 A1 A1	For standardising. Award once, here or elsewhere.	3
(ii)	$V_T \sim N(56.5 + 38.4 = 94.9,$ $2.9^2 + 1.1^2 = 9.62)$ $P(\text{this} > 100) = P(Z > \frac{100 - 94.9}{3.1016} = 1.6443)$ $= 1 - 0.9499 = 0.0501$	B1 B1 A1	Mean. Variance. Accept sd (= 3.1016). c.a.o.	3
(iii)	$W_T \sim N(3.1 \times 56.5 + 0.8 \times 38.4 = 205.87,$ $3.1^2 \times 2.9^2 + 0.8^2 \times 1.1^2 = 81.5945)$ $P(200 < \text{this} < 220)$ $= P(\frac{200 - 205.87}{9.0330} < Z < \frac{220 - 205.87}{9.0330})$ $= P(-0.6498 < Z < 1.5643)$ $= 0.9411 - (1 - 0.7422) = 0.6833$	M1 A1 M1 A1 M1 A1	Use of "mass = density × volume" Mean. Variance. Accept sd (= 9.0330). Formulation of requirement. c.a.o.	6
(iv)	Given $\bar{x} = 205.6 \quad s_{n-1} = 8.51$ H_0: $\mu = 200$, H_1: $\mu > 200$ Test statistic is $\frac{205.6 - 200}{\frac{8.51}{\sqrt{10}}}$ $= 2.081.$ Refer to t_9. Single-tailed 5% point is 1.833. Significant. Seems that the required reduction of the mean weight has not been achieved.	M1 A1 M1 A1 A1 A1	Allow alternative: 200 + (c's 1.833) $\times \frac{8.51}{\sqrt{10}}$ (= 204.933) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ – (c's 1.833) × $\frac{8.51}{\sqrt{10}}$ (= 200.667) for comparison with 200.) c.a.o. but ft from here in any case if wrong. Use of 200 – $\bar{x}$ scores M1A0, but ft. No ft from here if wrong. P(t > 2.081) = 0.0336. No ft from here if wrong. ft only c's test statistic. ft only c's test statistic.	6
				18

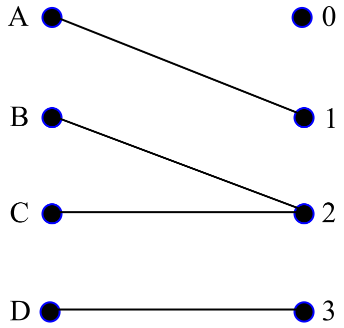
Q3				
(i)	In this situation a paired test is appropriate because there are clearly differences between specimens … … which the pairing eliminates.	 E1 E1		2
(ii)	H$_0$: $\mu_D = 0$ H$_1$: $\mu_D > 0$	B1	Both. Accept alternatives e.g. $\mu_D < 0$ for H$_1$, or $\mu_A - \mu_B$ etc provided adequately defined. Hypotheses in words only must include "population".	
	Where μ_D is the (population) mean reduction in hormone concentration.	B1	For adequate verbal definition. Allow absence of "population" if correct notation μ is used, but do NOT allow "$\bar{X} = \ldots$" or similar unless $\bar{X}$ is clearly and explicitly stated to be a <u>population</u> mean.	
	Must assume • Sample is random • Normality of differences	 B1 B1		4
(iii)	<u>MUST</u> be PAIRED COMPARISON t test. Differences (reductions) (before – after) are		Allow "after – before" if consistent with alternatives above.	
	–0.75 2.71 2.59 6.07 0.71 –1.85 –0.98 3.56 1.77 2.95 1.59 4.17 0.38 0.88 0.95			
	$\bar{x} = 1.65$ $s_{n-1} = 2.100(3)$ $(s_{n-1}^{\ 2} = 4.4112)$	B1	Do not allow $s_n = 2.0291$ $(s_n^{\ 2} = 4.1171)$	
	Test statistic is $\dfrac{1.65 - 0}{\frac{2.100}{\sqrt{15}}}$	M1	Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 0 + (c's 2.624) × $\frac{2.100}{\sqrt{15}}$ (= 1.423) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ – (c's 2.624) × $\frac{2.100}{\sqrt{15}}$ (= 0.227) for comparison with 0.)	
	= 3.043.	A1	c.a.o. but ft from here in any case if wrong. Use of $0 - \bar{x}$ scores M1A0, but ft.	
	Refer to t_{14}.	M1	No ft from here if wrong. P(t > 3.043) = 0.00438.	
	Single-tailed 1% point is 2.624.	A1	No ft from here if wrong.	
	Significant.	A1	ft only c's test statistic.	
	Seems mean concentration of hormone has fallen.	A1	ft only c's test statistic.	7
(iv)	CI is 1.65 ±	M1	ft c's $\bar{x}$ ±.	
	$k \times \dfrac{2.100}{\sqrt{15}}$	M1	ft c's $s_{\tilde{n}1}$.	
	= (0.4869, 2.8131)	A1	A correct equation in k using either end of the interval or the width of the interval.	
	∴ $k = 2.145$	A1	Allow ft c's $\bar{x}$ and $s_{\tilde{n}1}$.	
	By reference to t_{14} tables this is a 95% CI.	A1	c.a.o.	5
				18

<table>
<tr><td>Q4</td><td></td><td></td><td></td><td></td></tr>
<tr><td>(i)</td><td>Sampling which selects from those that are (easily) available.
Circumstances may mean that it is the only economically viable method available.
Likely to be neither random nor representative.</td><td>E1
E1
E1</td><td></td><td>3</td></tr>
<tr><td>(ii)</td><td>$p + pq + pq^2 + pq^3 + pq^4 + pq^5 + q^6$
$= \frac{p(1-q^6)}{1-q} + q^6 = \frac{p(1-q^6)}{p} + q^6$
$= 1 - q^6 + q^6 = 1$</td><td>M1
A1</td><td>Use of GP formula to sum probabilities,
or expand in terms of p or in terms of q.
Algebra shown convincingly.
Beware answer given.</td><td>2</td></tr>
<tr><td>(iii)</td><td colspan="3">With $p = 0.25$
<table>
<tr><td>Probability</td><td>0.25</td><td>0.1875</td><td>0.140625</td><td>0.105469</td><td>0.079102</td><td>0.059326</td><td>0.177979</td></tr>
<tr><td>Expected fr</td><td>25.00</td><td>18.75</td><td>14.0625</td><td>10.5469</td><td>7.9102</td><td>5.9326</td><td>17.7979</td></tr>
</table></td><td></td></tr>
<tr><td></td><td></td><td>M1
M1
A1</td><td>Probabilities correct to 3 dp or better.
× 100 for expected frequencies.
All correct and sum to 100.</td><td></td></tr>
<tr><td></td><td>$X^2 = 0.04 + 0.0033 + 0.6136 + 0.5706 + 1.2069 + 0.7204 + 7.8206$
$= 10.97(54)$</td><td>M1
A1</td><td>c.a.o.</td><td></td></tr>
<tr><td></td><td>(If e.g. only 2dp used for expected f's then
$X^2 = 0.04 + 0.0033 + 0.6148 + 0.5690 + 1.2071 + 0.7226 + 7.8225$
$= 10.97(93)$)</td><td></td><td></td><td></td></tr>
<tr><td></td><td>Refer to χ_6^2.</td><td>M1</td><td>Allow correct df (= cells – 1) from wrongly grouped table and ft.
Otherwise, no ft if wrong.
$P(X^2 > 10.975) = 0.0891$.</td><td></td></tr>
<tr><td></td><td>Upper 10% point is 10.64.
Significant.
Suggests model with $p = 0.25$ does not fit.</td><td>A1
A1
A1</td><td>No ft from here if wrong.
ft only c's test statistic.
ft only c's test statistic.</td><td>9</td></tr>
<tr><td>(iv)</td><td>Now with $X^2 = 9.124$
Refer to χ_5^2.</td><td>M1</td><td>Allow correct df (= cells – 2) from wrongly grouped table and ft.
Otherwise, no ft if wrong.
$P(X^2 > 9.124) = 0.1042$.</td><td></td></tr>
<tr><td></td><td>Upper 10% point is 9.236.
Not significant. (Suggests new model does fit.)
Improvement to the model is due to estimation of p from the data.</td><td>A1
A1
E1</td><td>No ft from here if wrong.
Correct conclusion.
Comment about the effect of estimated p, consistent with conclusion in part (iii).</td><td>4</td></tr>
<tr><td></td><td></td><td></td><td></td><td>18</td></tr>
</table>

4771 Decision Mathematics 1

1.

(i)		M1 bipartite A1 one arc from each letter A1 David A1 rest



(ii)	Can't both have someone shaking hands with everyone and someone not shaking hands at all.	B1 $0 \Rightarrow \sim 3$ B1 $3 \Rightarrow \sim 0$
(iii)	n arcs leaving By (ii) only n–1 destinations	B1 B1

2.

(i)

n	i	j	k
5	1	3	3
	2	2	8
	3	1	13
	4	0	16

B1
B1
B1
B1

k = 16 B1

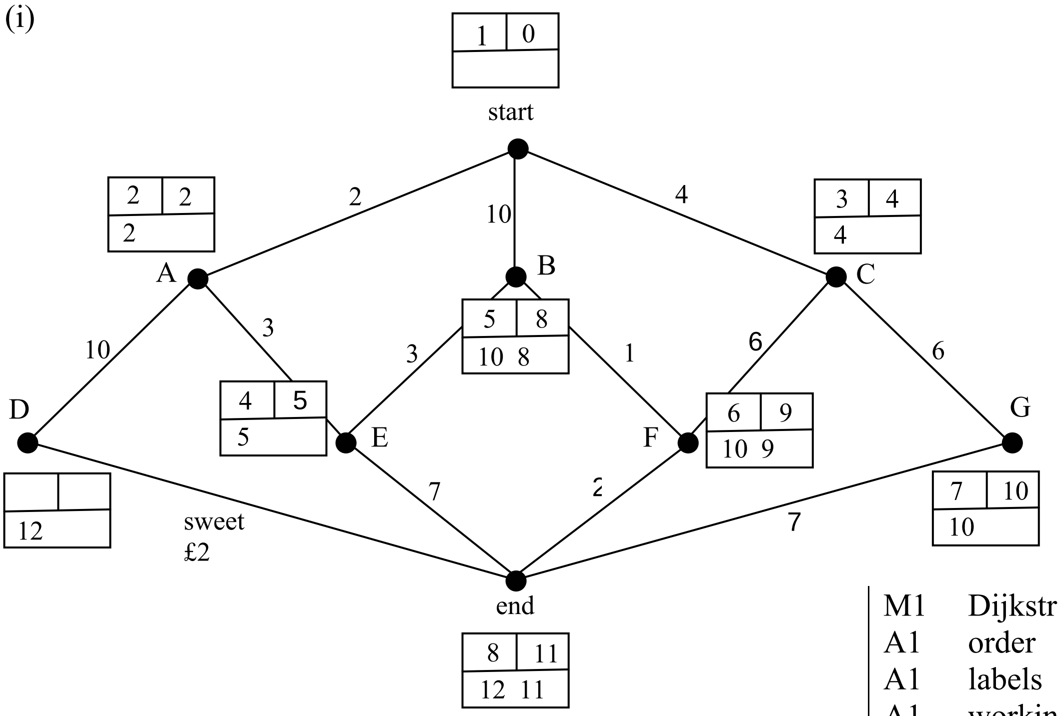
(ii) $f(5) = 125/6 – 35/6 + 1 = 90/6 + 1 = 16$ M1 substituting

(Need to see 125 or $20.8\dot{3}$ for A1) A1

(iii) cubic complexity B1

3.

(i)



Cheapest: £11
[start (£2 starter)] → A (£3 main) → E (£3 main) → B (£1 main) → F (£2 sweet) → [end]

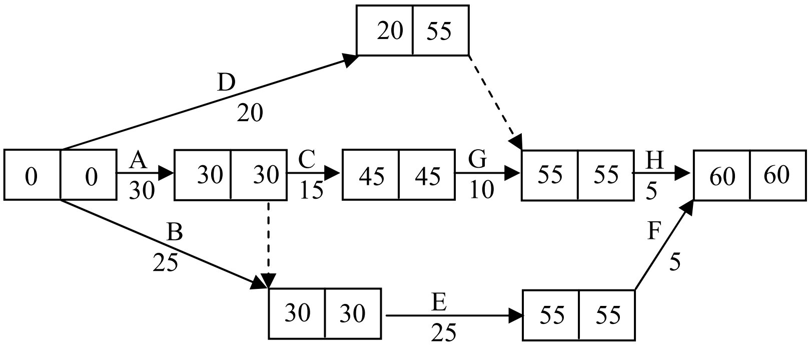
(ii) repeated mains !
directed network

M1	Dijkstra
A1	order
A1	labels
A1	working values
B1	£11
B1	route
B1	
B1	

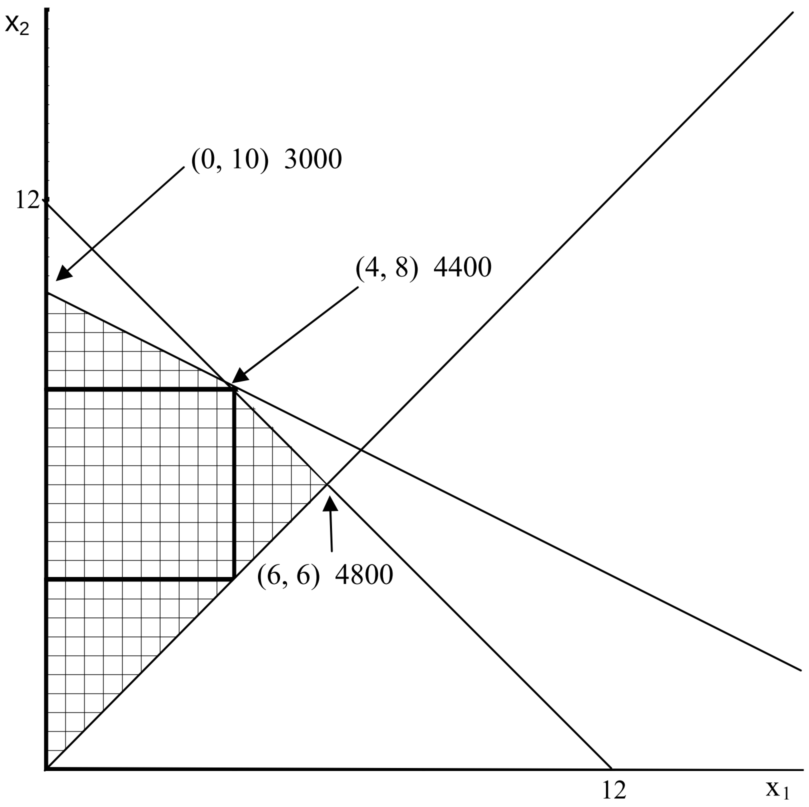
4.

(i)	e.g.	00–47→90 48–79→80 80–95→40 96, 97,98, 99 ignore		M1 some rejected A3 correct proportions (–1 each error) A1 efficient
(ii)	smaller proportion rejected			B1
(iii)	e.g.	90, 90, 90, 80	350	M1 A1 A1√
(iv)	e.g.	90, 80, 90, 80	340	
		80, 90, 80, 80	330	M1
		90, 40, 80, 90	300	A3 (–1 each error) √
		40, 90, 90, 90	310	
		90, 90, 90, 90	360	
		80, 80, 40, 90	290	
		80, 80, 80, 90	330	
		90, 80, 90, 90	350	
		90, 40, 40, 80	250	
	prob (load>325) = 0.6			M1 A1
(v)	e.g. family groups			B1

5.

(i)&(ii) e.g. 	M1 sca (activity on arc) A1 single start & end A1 dummy A1 rest M1 forward pass A1 M1 backward pass A1
time – 60 minutes	B1 √
critical – A; C; E; F; G; H	B1 cao
(iii) A and B at £300	B1 2 out of A, B, E B1 A B1 B B1 300 from A and B
A; C; G; H	B1
B; E; F	B1

6.

(i) x_i represents the number of tonnes produced in month i $x_2 \le x_3$ $x_1 + x_2 \le 12$	M1 A1 B1 B1	quantities tonnes
(ii) Substitute $x_3 = 20 - x_1 - x_2$ $x_2 \le x_3 \to x_1 + 2x_2 \le 20$ Min $2000x_1 + 2200x_2 + 2500x_3 \to$ Max $500x_1 + 300x_2$	M1 A1 A1	
(iii) 	M1 A3 A1 M1 A1	sca lines shading >1 evaluated point or profit line (6, 6) or 4800
Production plan: 6 tonnes in month 1 6 tonnes in month 2 8 tonnes in month 3	M1	√ all 3
Cost = £45200	A1	cao

4776 Numerical Methods

1(i)	x	y	1st diff	2nd diff		
	-3	-16				
	-1	-2	14			
	1	4	6	-8		[M1A1]
	3	2	-2	-8	2nd difference constant so quadratic fits	[E1]

(ii) $f(x) = -16 + 14(x + 3)/2 - 8(x + 3)(x + 1)/8$ **[M1A1A1A1]**

$= -16 + 7x + 21 - x^2 - 4x - 3$

$= 2 + 3x - x^2$ **[A1]**

[TOTAL 8]

2(i) Convincing algebra to demonstrate result **[M1A1]**

(ii)(A) Direct subtraction: 0.0022 **[B1]**

(B) Using (*): 1/(223.6090+223.6068) = 0.002236057 **[M1A1]**

Second value has many more significant figures ("more accurate") -- may be implied **[E1]**

Subtraction of nearly equal quantities loses precision **[E1]**

[TOTAL 7]

3(i)	x	f(x)				
	0	1				
	0.8	0.819951	T1 =	0.72798		[M1]
	0.4	0.994867	M1 =	0.795893		[M1]
			hence S1 =	0.773256		[M1]
					all values	[A1]

(ii) T2 = 0.761937 **[B1]**

M2 = 0.784069 so S2 = 0.776692 **[M1A1]**

S2 will be much more accurate than S1 so 0.78 or 0.777 would be justified **[A1]**

[TOTAL 8]

4(i)	x	cosx	$1 - 0.5x^2$	error	rel error		
	0.3	0.955336	0.955	-0.000336	-0.000352	*condone signs here but require correct sign for k*	[M1A1A1A1]

(ii) want $k\ 0.3^4 = 0.000336$ **[M1]**

gives k = 0.041542 (0.0415, 0.042, 1/24) **[A1]**

[TOTAL 6]

5	r	0	1	2	
	x_r	3	3	3	
	x_r	2.99	2.9701	2.911194	[M1A1A1]
	x_r	3.01	3.0301	3.091206	

Derivative is 2x - 3. Evaluates to 3 at x = 3 **[M1A1]**

3 is clearly a root, but the iteration does not converge **[E1]**

Need $-1 < g'(x) < 1$ at root for convergence **[E1]**

[TOTAL 7]

6(i) Demonstrate change of sign (f(a), f(b) below) and hence existence of root **[B1]**

a	b	f(a)	f(b)	x	mpe	f(x)	
0.2	0.3	-0.06429	0.021031	0.25	0.05	-0.01827	**[M1]**
0.25	0.3	-0.01827	0.021031	0.275	0.025	0.002134	**[M1]**
0.25	**0.275**			**0.2625**	**0.0125**	-0.00787	**[A1A1A1]**

[subtotal 6]

(ii)

r	x_r	f_r	
0	0.2	-0.06429	
1	0.3	0.021031	
2	0.275352	0.00241	**[M1A1]**
3	0.272161	-0.0001	**[M1A1]**

accept 0.27 or 0.272 as secure **[A1]**

secant method much faster **[E1]**

[subtotal 6]

(iii)

r	x_r	e_r	e_{r+1}/e_r^2		
0	1.4	0.101496		*e col:*	**[M1A1]**
1	1.314351	0.015847	1.538329	*e/e^2 col:*	**[M1A1]**
2	1.298887	0.000383	1.525122		
3	1.298504	= root			

equal values show 2nd order convergence **[E1]**

second order convergence: each error is proportional to the square of the previous error **[E1]**

[subtotal 6]

[TOTAL 18]

7(i) fwd diff:

h	0.4	0.2	0.1		
f '(0)	0.444758	0.473525	0.48711		**[M1A1A1]**
diffs		0.028768	0.013585	approx halved	**[B1]**

[subtotal 4]

(ii) cent diff:

h	0.4	0.2	0.1		
f '(0)	0.491631	0.498315	0.50008		**[M1A1A1]**
diffs		0.006684	0.001765	reduction greater than for forward difference	**[B1]**

[subtotal 4]

(iii) $(D_2 - d) = 0.5\ (D_1 - d)$ convincing algebra to $d = 2D_2 - D_1$ **[M1A1]**

$(D_2 - d) = 0.25\ (D_1 - d)$ convincing algebra to $d = (4D_2 - D_1)/3$ **[M1A1A1]**

[subtotal 5]

(iv) fwd diff: 2(0.48711) - 0.473525 = 0.500695 **[M1A1]**

cent diff: (4(0.50008) - 0.498315) / 3 = 0.500668 **[M1A1]**

0.5007 seems secure **[E1]**

[subtotal 5]

[TOTAL 18]

Grade Thresholds

Advanced GCE (Subject) (Aggregation Code(s))
January 2009 Examination Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	61	53	45	37	30	0
4752	Raw	72	60	53	46	40	34	0
4753/01	Raw	72	61	54	47	40	32	0
4753/02	Raw	18	15	13	11	9	8	0
4754	Raw	90	75	66	57	49	41	0
4755	Raw	72	57	49	41	33	26	0
4756	Raw	72	53	47	42	37	32	0
4758/01	Raw	72	61	53	45	37	29	0
4758/02	Raw	18	15	13	11	9	8	0
4761	Raw	72	58	50	42	34	27	0
4762	Raw	72	57	49	41	33	26	0
4763	Raw	72	53	46	39	32	25	0
4766/G241	Raw	72	57	48	40	32	24	0
4767	Raw	72	60	52	45	38	31	0
4768	Raw	72	53	46	39	33	27	0
4771	Raw	72	57	51	45	39	33	0
4776/01	Raw	72	56	49	43	37	30	0
4776/02	Raw	18	14	12	10	8	7	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3895-3898	300	240	210	180	150	120	0
7895-7898	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3895	18.3	43.5	65.4	83.8	96.0	100.0	640
3896	39.2	58.8	78.4	86.3	96.1	100.0	94
3897	100.0	100.0	100.0	100.0	100.0	100.0	1
7895	22.2	57.6	81.7	93.0	98.1	100.0	186
7896	18.8	56.3	87.5	87.5	93.8	100.0	16

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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