OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4763

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Friday 19 June 2009
Afternoon

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 A fixed solid sphere has centre O and radius 2.6 m. A particle P of mass 0.65 kg moves on the smooth surface of the sphere.

The particle P is set in motion with horizontal velocity $1.4\ \mathrm{m\,s^{-1}}$ at the highest point of the sphere, and moves in part of a vertical circle. When OP makes an angle θ with the upward vertical, and P is still in contact with the sphere, the speed of P is $v\ \mathrm{m\,s^{-1}}$.

(i) Show that $v^2 = 52.92 - 50.96\cos\theta$. **[3]**

(ii) Find, in terms of θ, the normal reaction acting on P. **[4]**

(iii) Find the speed of P at the instant when it leaves the surface of the sphere. **[4]**

The particle P is now attached to one end of a light inextensible string, and the other end of the string is fixed to a point A, vertically above O, such that AP is tangential to the sphere, as shown in Fig. 1. P moves with constant speed $1.2\ \mathrm{m\,s^{-1}}$ in a **horizontal** circle with radius 2.4 m on the surface of the sphere.

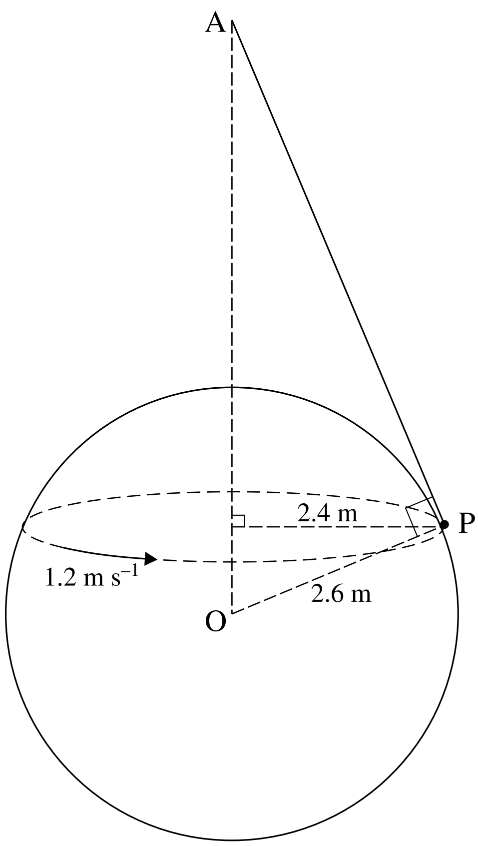

Fig. 1

(iv) Find the tension in the string and the normal reaction acting on P. **[8]**

2 In trials for a vehicle emergency stopping system, a small car of mass 400 kg is propelled towards a buffer. The buffer is modelled as a light spring of stiffness $5000\,\text{N m}^{-1}$. One end of the spring is fixed, and the other end points directly towards the oncoming car. Throughout this question, there is no driving force acting on the car, and there are no resistances to motion apart from those specifically mentioned.

At first, the buffer is mounted on a horizontal surface, and the car has speed $3\,\text{m s}^{-1}$ when it hits the buffer, as shown in Fig. 2.1.

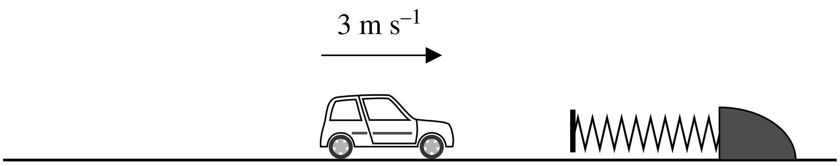

Fig. 2.1

(i) Find the compression of the spring when the car comes (instantaneously) to rest. **[3]**

The buffer is now mounted on a slope making an angle θ with the horizontal, where $\sin\theta = \frac{1}{7}$. The car is released from rest and travels 7.35 m down the slope before hitting the buffer, as shown in Fig. 2.2.

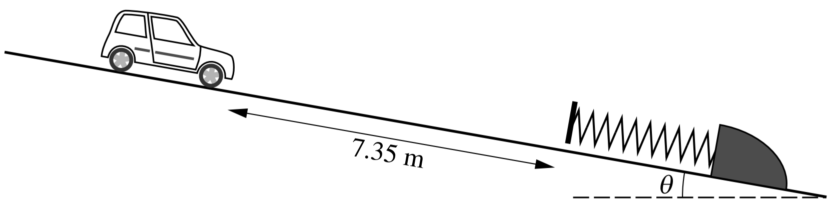

Fig. 2.2

(ii) Verify that the car comes instantaneously to rest when the spring is compressed by 1.4 m. **[4]**

The surface of the slope (including the section under the buffer) is now covered with gravel which exerts a constant resistive force of 7560 N on the car. The car is moving down the slope, and has speed $30\,\text{m s}^{-1}$ when it is 24 m from the buffer, as shown in Fig. 2.3. It comes to rest when the spring has been compressed by x metres.

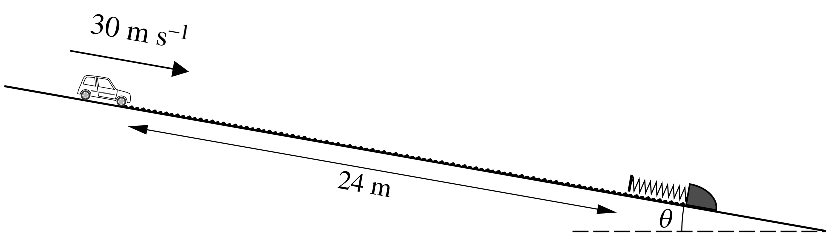

Fig. 2.3

(iii) By considering work and energy, find the value of x. **[10]**

3 **(a)** **(i)** Write down the dimensions of velocity, force and density (which is mass per unit volume). **[3]**

A vehicle moving with velocity v experiences a force F, due to air resistance, given by

$$F = \tfrac{1}{2}C\rho^{\alpha}v^{\beta}A^{\gamma}$$

where ρ is the density of the air, A is the cross-sectional area of the vehicle, and C is a dimensionless quantity called the drag coefficient.

(ii) Use dimensional analysis to find α, β and γ. **[5]**

(b) A light rod is freely pivoted about a fixed point at one end and has a heavy weight attached to its other end. The rod with the weight attached is oscillating in a vertical plane as a simple pendulum with period 4.3 s. The maximum angle which the rod makes with the vertical is 0.08 radians. You may assume that the motion is simple harmonic.

(i) Find the angular speed of the rod when it makes an angle of 0.05 radians with the vertical. **[5]**

(ii) Find the time taken for the pendulum to swing directly from a position where the rod makes an angle of 0.05 radians on one side of the vertical to the position where the rod makes an angle of 0.05 radians on the other side of the vertical. **[5]**

4 **(a)** A uniform lamina occupies the region bounded by the x-axis, the y-axis, the curve $y = e^x$ for $0 \leqslant x \leqslant \ln 3$, and the line $x = \ln 3$. Find, in an exact form, the coordinates of the centre of mass of this lamina. **[9]**

(b) A region is bounded by the x-axis, the curve $y = \dfrac{6}{x^2}$ for $2 \leqslant x \leqslant a$ (where $a > 2$), the line $x = 2$ and the line $x = a$. This region is rotated through 2π radians about the x-axis to form a uniform solid of revolution.

(i) Show that the x-coordinate of the centre of mass of this solid is $\dfrac{3(a^3 - 4a)}{a^3 - 8}$. **[6]**

(ii) Show that, however large the value of a, the centre of mass of this solid is less than 3 units from the origin. **[3]**

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