



RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

4754A

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Monday 1 June 2009

Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Express $4 \cos \theta - \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $4 \cos \theta - \sin \theta = 3$, for $0 \leq \theta \leq 2\pi$. [7]

2 Using partial fractions, find $\int \frac{x}{(x+1)(2x+1)} dx$. [7]

3 A curve satisfies the differential equation $\frac{dy}{dx} = 3x^2y$, and passes through the point $(1, 1)$. Find y in terms of x . [4]

4 The part of the curve $y = 4 - x^2$ that is above the x -axis is rotated about the y -axis. This is shown in Fig. 4.

Find the volume of revolution produced, giving your answer in terms of π . [5]

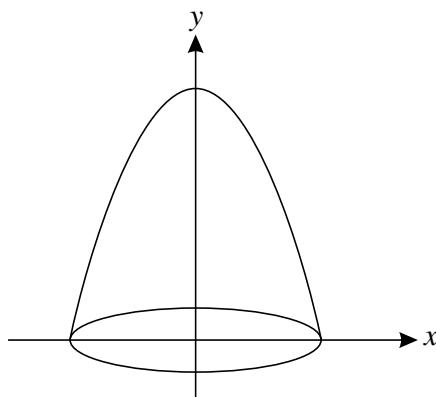


Fig. 4

5 A curve has parametric equations

$$x = at^3, \quad y = \frac{a}{1+t^2},$$

where a is a constant.

Show that $\frac{dy}{dx} = \frac{-2}{3t(1+t^2)^2}$.

Hence find the gradient of the curve at the point $(a, \frac{1}{2}a)$. [7]

6 Given that $\operatorname{cosec}^2 \theta - \cot \theta = 3$, show that $\cot^2 \theta - \cot \theta - 2 = 0$.

Hence solve the equation $\operatorname{cosec}^2 \theta - \cot \theta = 3$ for $0^\circ \leq \theta \leq 180^\circ$. [6]

Section B (36 marks)

7 When a light ray passes from air to glass, it is deflected through an angle. The light ray ABC starts at point A (1, 2, 2), and enters a glass object at point B (0, 0, 2). The surface of the glass object is a plane with normal vector \mathbf{n} . Fig. 7 shows a cross-section of the glass object in the plane of the light ray and \mathbf{n} .

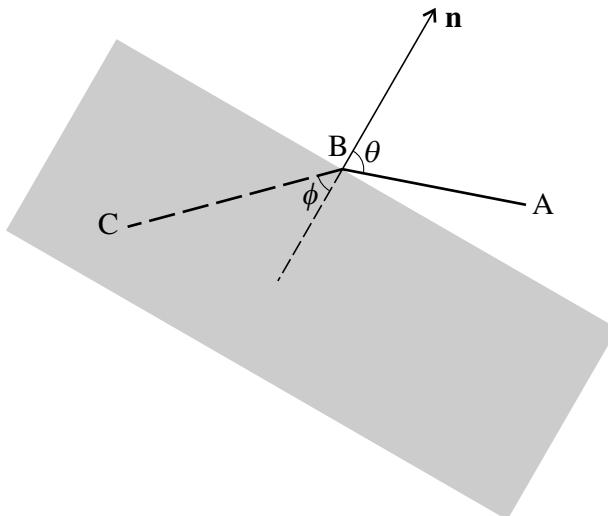


Fig. 7

(i) Find the vector \overrightarrow{AB} and a vector equation of the line AB. [2]

The surface of the glass object is a plane with equation $x + z = 2$. AB makes an acute angle θ with the normal to this plane.

(ii) Write down the normal vector \mathbf{n} , and hence calculate θ , giving your answer in degrees. [5]

The line BC has vector equation $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$. This line makes an acute angle ϕ with the normal to the plane.

(iii) Show that $\phi = 45^\circ$. [3]

(iv) Snell's Law states that $\sin \theta = k \sin \phi$, where k is a constant called the refractive index. Find k . [2]

The light ray leaves the glass object through a plane with equation $x + z = -1$. Units are centimetres.

(v) Find the point of intersection of the line BC with the plane $x + z = -1$. Hence find the distance the light ray travels through the glass object. [5]

[Question 8 is printed overleaf.]

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8 Archimedes, about 2200 years ago, used regular polygons inside and outside circles to obtain approximations for π .

(i) Fig. 8.1 shows a regular 12-sided polygon inscribed in a circle of radius 1 unit, centre O. AB is one of the sides of the polygon. C is the midpoint of AB. Archimedes used the fact that the circumference of the circle is greater than the perimeter of this polygon.

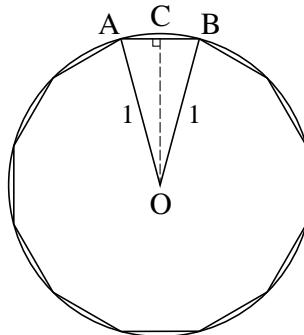


Fig. 8.1

(A) Show that $AB = 2 \sin 15^\circ$. [2]

(B) Use a double angle formula to express $\cos 30^\circ$ in terms of $\sin 15^\circ$. Using the exact value of $\cos 30^\circ$, show that $\sin 15^\circ = \frac{1}{2}\sqrt{2 - \sqrt{3}}$. [4]

(C) Use this result to find an exact expression for the perimeter of the polygon.

Hence show that $\pi > 6\sqrt{2 - \sqrt{3}}$. [2]

(ii) In Fig. 8.2, a regular 12-sided polygon lies outside the circle of radius 1 unit, which touches each side of the polygon. F is the midpoint of DE. Archimedes used the fact that the circumference of the circle is less than the perimeter of this polygon.

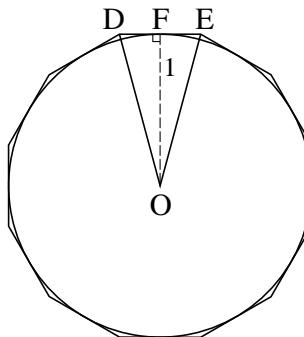


Fig. 8.2

(A) Show that $DE = 2 \tan 15^\circ$. [2]

(B) Let $t = \tan 15^\circ$. Use a double angle formula to express $\tan 30^\circ$ in terms of t .
Hence show that $t^2 + 2\sqrt{3}t - 1 = 0$. [3]

(C) Solve this equation, and hence show that $\pi < 12(2 - \sqrt{3})$. [4]

(iii) Use the results in parts (i)(C) and (ii)(C) to establish upper and lower bounds for the value of π , giving your answers in decimal form. [2]