

**ADVANCED GCE****MATHEMATICS (MEI)**

Applications of Advanced Mathematics (C4) Paper A

**4754A**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

None

**Monday 1 June 2009**  
**Morning**

**Duration:** 1 hour 30 minutes**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

**NOTE**

- This paper will be followed by **Paper B: Comprehension**.

**Section A** (36 marks)

- 1 Express  $4 \cos \theta - \sin \theta$  in the form  $R \cos(\theta + \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ .

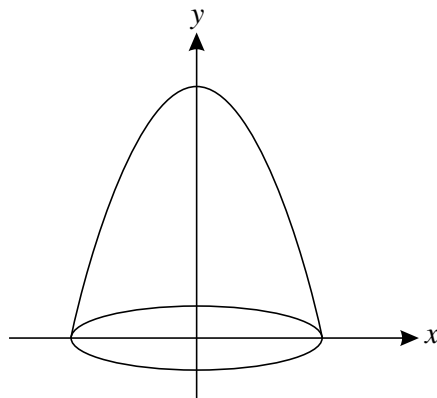
Hence solve the equation  $4 \cos \theta - \sin \theta = 3$ , for  $0 \leq \theta \leq 2\pi$ . [7]

- 2 Using partial fractions, find  $\int \frac{x}{(x+1)(2x+1)} dx$ . [7]

- 3 A curve satisfies the differential equation  $\frac{dy}{dx} = 3x^2y$ , and passes through the point  $(1, 1)$ . Find  $y$  in terms of  $x$ . [4]

- 4 The part of the curve  $y = 4 - x^2$  that is above the  $x$ -axis is rotated about the  $y$ -axis. This is shown in Fig. 4.

Find the volume of revolution produced, giving your answer in terms of  $\pi$ . [5]



**Fig. 4**

- 5 A curve has parametric equations

$$x = at^3, \quad y = \frac{a}{1+t^2},$$

where  $a$  is a constant.

Show that  $\frac{dy}{dx} = \frac{-2}{3t(1+t^2)^2}$ .

Hence find the gradient of the curve at the point  $(a, \frac{1}{2}a)$ . [7]

- 6 Given that  $\operatorname{cosec}^2 \theta - \cot \theta = 3$ , show that  $\cot^2 \theta - \cot \theta - 2 = 0$ .

Hence solve the equation  $\operatorname{cosec}^2 \theta - \cot \theta = 3$  for  $0^\circ \leq \theta \leq 180^\circ$ . [6]

## Section B (36 marks)

- 7 When a light ray passes from air to glass, it is deflected through an angle. The light ray ABC starts at point A (1, 2, 2), and enters a glass object at point B (0, 0, 2). The surface of the glass object is a plane with normal vector  $\mathbf{n}$ . Fig. 7 shows a cross-section of the glass object in the plane of the light ray and  $\mathbf{n}$ .

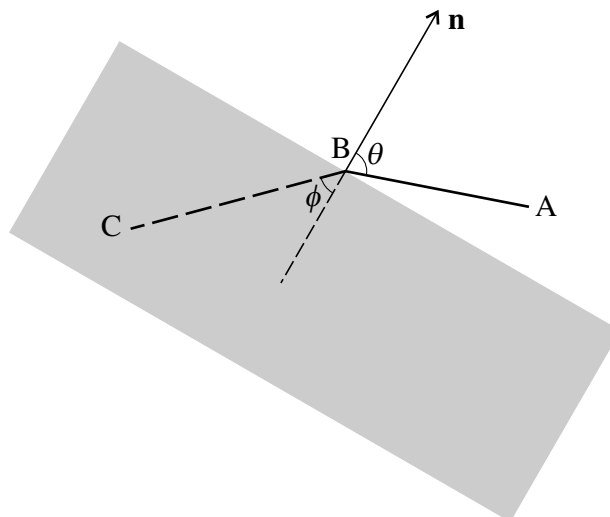


Fig. 7

- (i) Find the vector  $\overrightarrow{AB}$  and a vector equation of the line AB. [2]

The surface of the glass object is a plane with equation  $x + z = 2$ . AB makes an acute angle  $\theta$  with the normal to this plane.

- (ii) Write down the normal vector  $\mathbf{n}$ , and hence calculate  $\theta$ , giving your answer in degrees. [5]

The line BC has vector equation  $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$ . This line makes an acute angle  $\phi$  with the normal to the plane.

- (iii) Show that  $\phi = 45^\circ$ . [3]

- (iv) Snell's Law states that  $\sin \theta = k \sin \phi$ , where  $k$  is a constant called the refractive index. Find  $k$ . [2]

The light ray leaves the glass object through a plane with equation  $x + z = -1$ . Units are centimetres.

- (v) Find the point of intersection of the line BC with the plane  $x + z = -1$ . Hence find the distance the light ray travels through the glass object. [5]

[Question 8 is printed overleaf.]

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- 8 Archimedes, about 2200 years ago, used regular polygons inside and outside circles to obtain approximations for  $\pi$ .

- (i) Fig. 8.1 shows a regular 12-sided polygon inscribed in a circle of radius 1 unit, centre O. AB is one of the sides of the polygon. C is the midpoint of AB. Archimedes used the fact that the circumference of the circle is greater than the perimeter of this polygon.

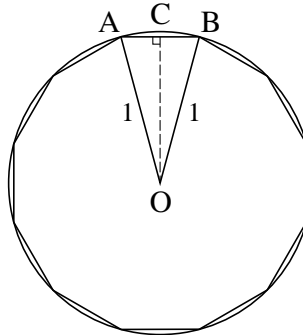


Fig. 8.1

- (A) Show that  $AB = 2 \sin 15^\circ$ . [2]
- (B) Use a double angle formula to express  $\cos 30^\circ$  in terms of  $\sin 15^\circ$ . Using the exact value of  $\cos 30^\circ$ , show that  $\sin 15^\circ = \frac{1}{2}\sqrt{2 - \sqrt{3}}$ . [4]
- (C) Use this result to find an exact expression for the perimeter of the polygon.

Hence show that  $\pi > 6\sqrt{2 - \sqrt{3}}$ . [2]

- (ii) In Fig. 8.2, a regular 12-sided polygon lies outside the circle of radius 1 unit, which touches each side of the polygon. F is the midpoint of DE. Archimedes used the fact that the circumference of the circle is less than the perimeter of this polygon.

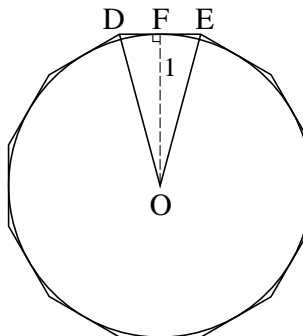


Fig. 8.2

- (A) Show that  $DE = 2 \tan 15^\circ$ . [2]
- (B) Let  $t = \tan 15^\circ$ . Use a double angle formula to express  $\tan 30^\circ$  in terms of  $t$ .  
Hence show that  $t^2 + 2\sqrt{3}t - 1 = 0$ . [3]
- (C) Solve this equation, and hence show that  $\pi < 12(2 - \sqrt{3})$ . [4]

- (iii) Use the results in parts (i)(C) and (ii)(C) to establish upper and lower bounds for the value of  $\pi$ , giving your answers in decimal form. [2]