OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4762

Mechanics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Thursday 11 June 2009
Morning

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 **(a)** Two small objects, P of mass m kg and Q of mass km kg, slide on a smooth horizontal plane. Initially, P and Q are moving in the same straight line towards one another, each with speed $u\,\text{m s}^{-1}$.

After a direct collision with P, the direction of motion of Q is reversed and it now has a speed of $\frac{1}{3}u\,\text{m s}^{-1}$. The velocity of P is now $v\,\text{m s}^{-1}$, where the positive direction is the original direction of motion of P.

(i) Draw a diagram showing the velocities of P and Q before and after the impact. **[1]**

(ii) By considering the linear momentum of the objects before and after the collision, show that $v = \left(1 - \frac{4}{3}k\right)u$. **[3]**

(iii) Hence find the condition on k for the direction of motion of P to be reversed. **[2]**

The coefficient of restitution in the collision is 0.5.

(iv) Show that $v = -\frac{2}{3}u$ and calculate the value of k. **[5]**

(b) Particle A has a mass of 5 kg and velocity $\begin{pmatrix} 3 \\ 2 \end{pmatrix}\,\text{m s}^{-1}$. Particle B has mass 3 kg and is initially at rest. A force $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ N acts for 9 seconds on B and subsequently (in the absence of the force), A and B collide and stick together to form an object C that moves off with a velocity $\mathbf{V}\,\text{m s}^{-1}$.

(i) Show that $\mathbf{V} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. **[4]**

The object C now collides with a smooth barrier which lies in the direction $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. The coefficient of restitution in the collision is 0.5.

(ii) Calculate the velocity of C after the impact. **[3]**

2 **(a)** A small block of mass 25 kg is on a long, horizontal table. Each side of the block is connected to a small sphere by means of a light inextensible string passing over a smooth pulley. Fig. 2 shows this situation. Sphere A has mass 5 kg and sphere B has mass 20 kg. Each of the spheres hangs freely.

Fig. 2

Initially the block moves on a smooth part of the table. With the block at a point O, the system is released from rest with both strings taut.

(i) (*A*) Is mechanical energy conserved in the subsequent motion? Give a brief reason for your answer. **[1]**

(*B*) Why is no work done by the block against the reaction of the table on it? **[1]**

The block reaches a speed of 1.5 m s^{-1} at point P.

(ii) Use an energy method to calculate the distance OP. **[5]**

The block continues moving beyond P, at which point the table becomes rough. After travelling two metres beyond P, the block passes through point Q. The block does 180 J of work against resistances to its motion from P to Q.

(iii) Use an energy method to calculate the speed of the block at Q. **[5]**

(b) A tree trunk of mass 450 kg is being pulled up a slope inclined at 20° to the horizontal.

Calculate the power required to pull the trunk at a steady speed of 2.5 m s^{-1} against a frictional force of 2000 N. **[5]**

3 A non-uniform beam AB has weight 85 N. The length of the beam is 5 m and its centre of mass is 3 m from A. In this question all the forces act in the same vertical plane.

Fig. 3.1 shows the beam in horizontal equilibrium, supported at its ends.

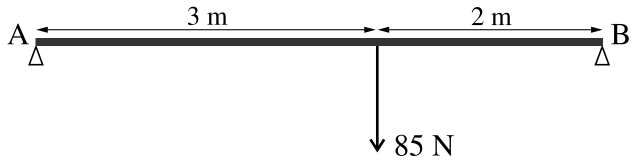

Fig. 3.1

(i) Calculate the reactions of the supports on the beam. **[4]**

Using a smooth hinge, the beam is now attached at A to a vertical wall. The beam is held in equilibrium at an angle α to the horizontal by means of a horizontal force of magnitude 27.2 N acting at B. This situation is shown in Fig. 3.2.

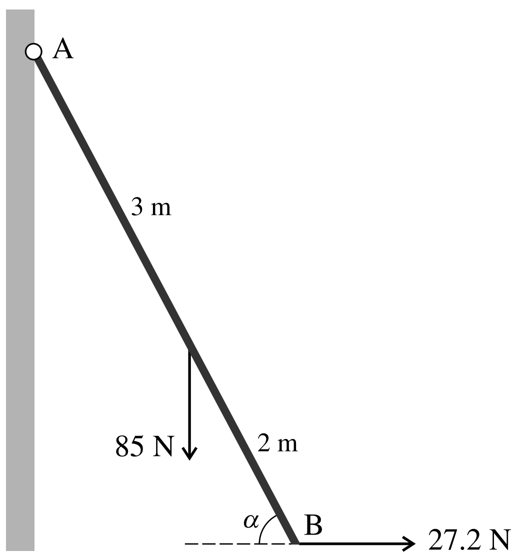

Fig. 3.2

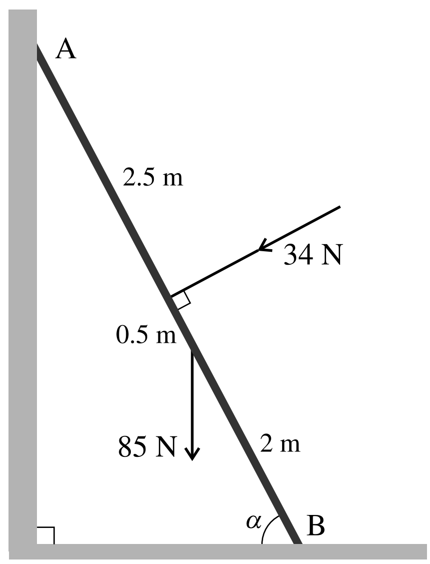

Fig. 3.3

(ii) Show that $\tan\alpha = \frac{15}{8}$. **[4]**

The hinge and 27.2 N force are removed. The beam now rests with B on a rough horizontal floor and A on a smooth vertical wall, as shown in Fig. 3.3. It is at the same angle α to the horizontal. There is now a force of 34 N acting at right angles to the beam at its centre in the direction shown. The beam is in equilibrium and on the point of slipping.

(iii) Draw a diagram showing the forces acting on the beam.

Show that the frictional force acting on the beam is 7.4 N.

Calculate the value of the coefficient of friction between the beam and the floor. **[10]**

4 In this question you may use the following facts: as illustrated in Fig. 4.1, the centre of mass, G, of a uniform thin open hemispherical shell is at the mid-point of OA on its axis of symmetry; the surface area of this shell is $2\pi r^2$, where r is the distance OA.

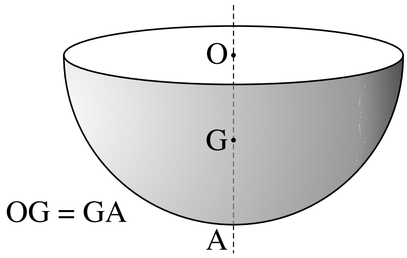

Fig. 4.1

A perspective view and a cross-section of a dog bowl are shown in Fig. 4.2. The bowl is made throughout from thin uniform material. An open hemispherical shell of radius 8 cm is fitted inside an open circular cylinder of radius 8 cm so that they have a common axis of symmetry and the rim of the hemisphere is at one end of the cylinder. The height of the cylinder is k cm. The point O is on the axis of symmetry and at the end of the cylinder.

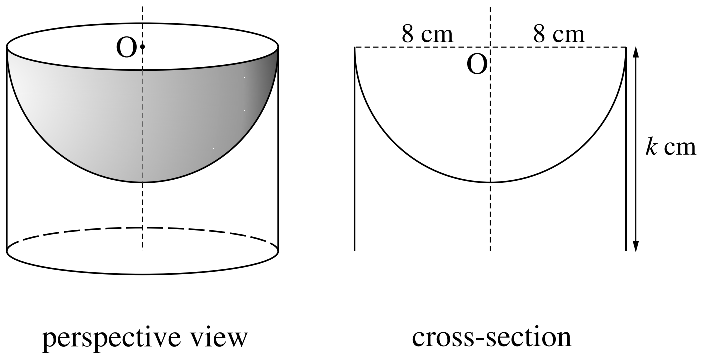

Fig. 4.2

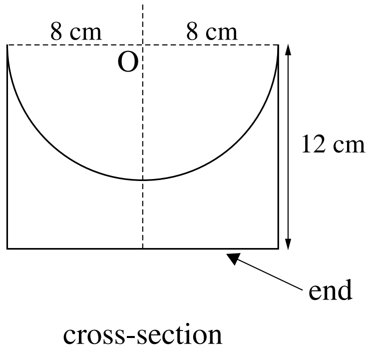

Fig. 4.3

(i) Show that the centre of mass of the bowl is a distance $\dfrac{64 + k^2}{16 + 2k}$ cm from O. **[6]**

A version of the bowl for the 'senior dog' has $k = 12$ and an end to the cylinder, as shown in Fig. 4.3. The end is made from the same material as the original bowl.

(ii) Show that the centre of mass of this bowl is a distance $6\frac{1}{3}$ cm from O. **[5]**

This bowl is placed on a rough slope inclined at θ to the horizontal.

(iii) Assume that the bowl is prevented from sliding and is on the point of toppling.

Draw a diagram indicating the position of the centre of mass of the bowl with relevant lengths marked.

Calculate the value of θ. **[5]**

(iv) If the bowl is not prevented from sliding, determine whether it will slide when placed on the slope when there is a coefficient of friction between the bowl and the slope of 1.5. **[3]**

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