



ADVANCED SUBSIDIARY GCE
MATHEMATICS (MEI)
 Numerical Methods

4776/01

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Friday 15 January 2010
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

- 1** Show that the equation

$$2^x + \left(\frac{1}{2}\right)^x = 3$$

has a root between $x = 1.3$ and $x = 1.5$. Use the bisection method to find an estimate of this root with a maximum possible error less than 0.02.

Determine how many further iterations would be required to reduce the maximum possible error to less than 0.001. [8]

- 2** An integral, $\int_a^b f(x) dx$, is being evaluated numerically. Some mid-point rule and trapezium rule estimates are shown in the table.

| h | Mid-point rule | Trapezium rule |
|-----|----------------|----------------|
| 1 | 2.579 768 | 2.447 490 |
| 0.5 | 2.547 350 | |

Find the trapezium rule estimate for $h = 0.5$.

Find two Simpson's rule estimates and hence state, with a reason, the value of the integral to the accuracy that appears justified. [7]

- 3** (i) Given that $f(x) = x^3 - x^2 + 1$, find $f(0.5)$.

Use the formula $f(x + h) \approx f(x) + h f'(x)$ to show that

$$f(0.5 + h) \approx 0.875 - 0.25h. \quad [3]$$

- (ii) Hence determine the approximate range of values of x for which $f(x) = 0.875$ correct to 3 decimal places. [4]

- 4** (i) Show algebraically that

$$(k + 1)^2 + (k - 1)^2 - 2k^2 = 2 \quad (*)$$

for all values of k . [2]

- (ii) Use your calculator to evaluate the left hand side of (*) for increasingly large values of k (e.g. 10^3 , 10^6 , 10^9 , ...). State briefly two important results in numerical methods that are illustrated by your working. [4]

- 5 A function $f(x)$ has the following values correct to 3 decimal places.

| | | | | | |
|--------|-------|-------|-------|-------|-------|
| x | 0 | 1 | 2 | 3 | 4 |
| $f(x)$ | 1.883 | 2.342 | 2.874 | 3.491 | 4.206 |

- (i) Show, by means of a difference table, that a cubic polynomial fits these data points closely but not exactly. [4]
- (ii) Use Newton's forward difference formula to estimate the value of $f(1.5)$. [4]

Section B (36 marks)

- 6 (i) The derivative of a function is to be estimated numerically. Show, with the aid of a sketch, that the central difference method will generally be more accurate than the forward difference method. [4]
- (ii) The table shows two values of $\tan x^\circ$ correct to 7 significant figures.

| | | |
|----------------|-----------|-----------|
| x | 60 | 62 |
| $\tan x^\circ$ | 1.732 051 | 1.880 726 |

Use these two values to estimate the derivative of $\tan x^\circ$ at $x = 60$.

Use your calculator to find two further estimates of this derivative, using the forward difference method and taking $h = 1$ and $h = 0.5$. [4]

- (iii) Use the central difference method with $h = 2$, $h = 1$ and $h = 0.5$ to obtain three estimates of the derivative of $\tan x^\circ$ at $x = 60$. [4]
- (iv) Show that the differences between the estimates in part (ii) reduce by a factor of about 0.5 as h is halved.

By considering the differences between the estimates in part (iii) show that the central difference method seems to converge more rapidly than the forward difference method. [6]

[Question 7 is printed overleaf.]

- 7 (i) Show, by means of a sketch or otherwise, that the equation

$$x = 3 \sin x, \quad (*)$$

where x is in radians, has a root, α , in the interval $(\frac{1}{2}\pi, \pi)$. Determine how many other non-zero roots, if any, the equation has. [3]

- (ii) Determine whether or not the iteration

$$x_{r+1} = 3 \sin x_r,$$

starting with $x_0 = 2$, converges to α . Illustrate your answer with a staircase or cobweb diagram as appropriate. [7]

- (iii) Show that equation (*) may be rearranged into the form

$$x = \sin x + \frac{2}{3}x.$$

Show that the corresponding iteration, starting with $x_0 = 2$, converges rapidly. State to 5 decimal places the value to which the iteration converges. Verify that this value for α is correct to 5 decimal places. [8]

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