



**ADVANCED SUBSIDIARY GCE  
MATHEMATICS (MEI)**

Numerical Methods

**4776/01**



Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

None

**Friday 15 January 2010  
Afternoon**

**Duration: 1 hour 30 minutes**



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of 4 pages. Any blank pages are indicated.

## Section A (36 marks)

1 Show that the equation

$$2^x + \left(\frac{1}{2}\right)^x = 3$$

has a root between  $x = 1.3$  and  $x = 1.5$ . Use the bisection method to find an estimate of this root with a maximum possible error less than 0.02.

Determine how many further iterations would be required to reduce the maximum possible error to less than 0.001. [8]

2 An integral,  $\int_a^b f(x) dx$ , is being evaluated numerically. Some mid-point rule and trapezium rule estimates are shown in the table.

$h$	Mid-point rule	Trapezium rule
1	2.579 768	2.447 490
0.5	2.547 350	

Find the trapezium rule estimate for  $h = 0.5$ .

Find two Simpson's rule estimates and hence state, with a reason, the value of the integral to the accuracy that appears justified. [7]

3 (i) Given that  $f(x) = x^3 - x^2 + 1$ , find  $f(0.5)$ .

Use the formula  $f(x + h) \approx f(x) + h f'(x)$  to show that

$$f(0.5 + h) \approx 0.875 - 0.25h. \quad [3]$$

(ii) Hence determine the approximate range of values of  $x$  for which  $f(x) = 0.875$  correct to 3 decimal places. [4]

4 (i) Show algebraically that

$$(k + 1)^2 + (k - 1)^2 - 2k^2 = 2 \quad (*)$$

for all values of  $k$ . [2]

(ii) Use your calculator to evaluate the left hand side of (\*) for increasingly large values of  $k$  (e.g.  $10^3$ ,  $10^6$ ,  $10^9$ , ...). State briefly two important results in numerical methods that are illustrated by your working. [4]

5 A function  $f(x)$  has the following values correct to 3 decimal places.

$x$	0	1	2	3	4
$f(x)$	1.883	2.342	2.874	3.491	4.206

(i) Show, by means of a difference table, that a cubic polynomial fits these data points closely but not exactly. [4]

(ii) Use Newton's forward difference formula to estimate the value of  $f(1.5)$ . [4]

### Section B (36 marks)

6 (i) The derivative of a function is to be estimated numerically. Show, with the aid of a sketch, that the central difference method will generally be more accurate than the forward difference method. [4]

(ii) The table shows two values of  $\tan x^\circ$  correct to 7 significant figures.

$x$	60	62
$\tan x^\circ$	1.732 051	1.880 726

Use these two values to estimate the derivative of  $\tan x^\circ$  at  $x = 60$ .

Use your calculator to find two further estimates of this derivative, using the forward difference method and taking  $h = 1$  and  $h = 0.5$ . [4]

(iii) Use the central difference method with  $h = 2$ ,  $h = 1$  and  $h = 0.5$  to obtain three estimates of the derivative of  $\tan x^\circ$  at  $x = 60$ . [4]

(iv) Show that the differences between the estimates in part (ii) reduce by a factor of about 0.5 as  $h$  is halved.

By considering the differences between the estimates in part (iii) show that the central difference method seems to converge more rapidly than the forward difference method. [6]

[Question 7 is printed overleaf.]

7 (i) Show, by means of a sketch or otherwise, that the equation

$$x = 3 \sin x, \quad (*)$$

where  $x$  is in radians, has a root,  $\alpha$ , in the interval  $(\frac{1}{2}\pi, \pi)$ . Determine how many other non-zero roots, if any, the equation has. [3]

(ii) Determine whether or not the iteration

$$x_{r+1} = 3 \sin x_r,$$

starting with  $x_0 = 2$ , converges to  $\alpha$ . Illustrate your answer with a staircase or cobweb diagram as appropriate. [7]

(iii) Show that equation (\*) may be rearranged into the form

$$x = \sin x + \frac{2}{3}x.$$

Show that the corresponding iteration, starting with  $x_0 = 2$ , converges rapidly. State to 5 decimal places the value to which the iteration converges. Verify that this value for  $\alpha$  is correct to 5 decimal places. [8]



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