OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4753/01**

Methods for Advanced Mathematics (C3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
* 8 page Answer Booklet
* MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes

475301

INSTRUCTIONS TO CANDIDATES

* Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
* Use black ink. Pencil may be used for graphs and diagrams only.
* Read each question carefully and make sure that you know what you have to do before starting your answer.
* Answer **all** the questions.
* Do **not** write in the bar codes.
* You are permitted to use a graphical calculator in this paper.
* Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

* The number of marks is given in brackets **[ ]** at the end of each question or part question.
* You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
* The total number of marks for this paper is **72**.
* This document consists of **4** pages. Any blank pages are indicated.

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Section A (36 marks)

1   Solve the equation $e^{2x} - 5e^x = 0$. **[4]**

2   The temperature T in degrees Celsius of water in a glass t minutes after boiling is modelled by the equation $T = 20 + be^{-kt}$, where b and k are constants. Initially the temperature is $100\,°\text{C}$, and after 5 minutes the temperature is $60\,°\text{C}$.

   (i) Find b and k. **[4]**

   (ii) Find at what time the temperature reaches $50\,°\text{C}$. **[2]**

3   **(i)** Given that $y = \sqrt[3]{1 + 3x^2}$, use the chain rule to find $\dfrac{dy}{dx}$ in terms of x. **[3]**

   (ii) Given that $y^3 = 1 + 3x^2$, use implicit differentiation to find $\dfrac{dy}{dx}$ in terms of x and y. Show that this result is equivalent to the result in part **(i)**. **[4]**

4   Evaluate the following integrals, giving your answers in exact form.

   (i) $\displaystyle\int_0^1 \dfrac{2x}{x^2 + 1}\,dx$. **[3]**

   **(ii)** $\displaystyle\int_0^1 \dfrac{2x}{x + 1}\,dx$. **[5]**

5   The curves in parts **(i)** and **(ii)** have equations of the form $y = a + b\sin cx$, where a, b and c are constants. For each curve, find the values of a, b and c.

   (i)
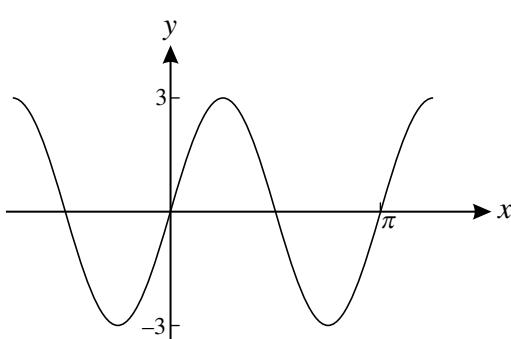
[2]

   (ii)
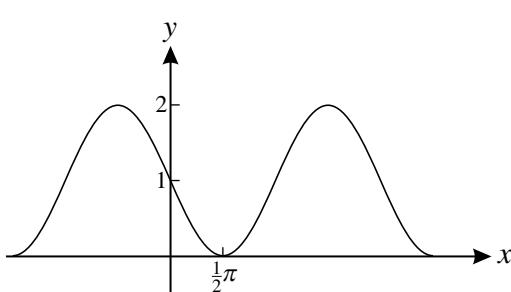
[2]

6   Write down the conditions for f(x) to be an odd function and for g(x) to be an even function.

Hence prove that, if f(x) is odd and g(x) is even, then the composite function gf(x) is even.   **[4]**

7   Given that arcsin x = arccos y, prove that $x^2 + y^2 = 1$. [Hint: let arcsin $x = \theta$.]   **[3]**

Section B (36 marks)

8   Fig. 8 shows part of the curve $y = x \cos 3x$.

The curve crosses the x-axis at O, P and Q.
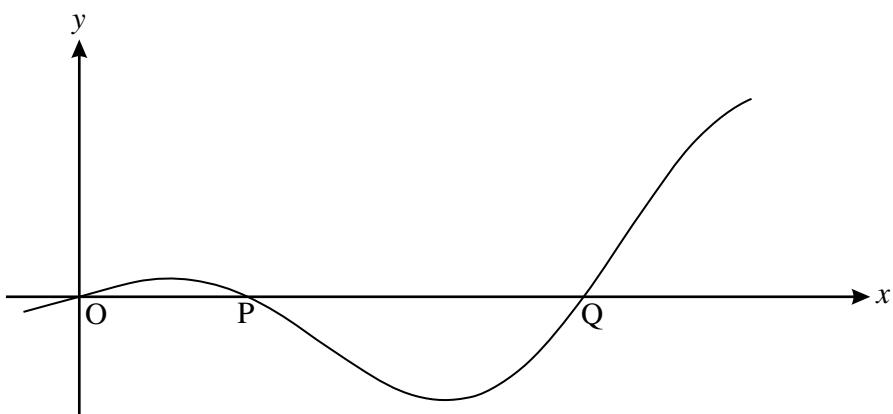
Fig. 8

   (i) Find the exact coordinates of P and Q.   **[4]**

  (ii) Find the exact gradient of the curve at the point P.

Show also that the turning points of the curve occur when $x \tan 3x = \frac{1}{3}$.   **[7]**

 (iii) Find the area of the region enclosed by the curve and the x-axis between O and P, giving your answer in exact form.   **[6]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{2x^2 - 1}{x^2 + 1}$ for the domain $0 \leqslant x \leqslant 2$.
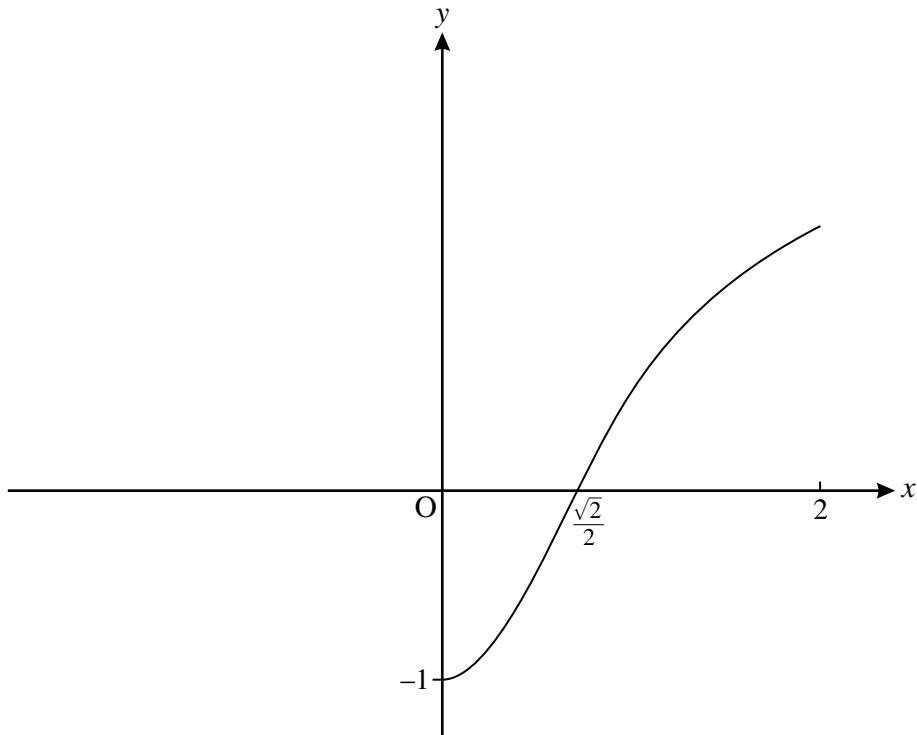
Fig. 9

(i) Show that $\mathrm{f}'(x) = \dfrac{6x}{(x^2 + 1)^2}$, and hence that $\mathrm{f}(x)$ is an increasing function for $x > 0$. **[5]**

(ii) Find the range of $\mathrm{f}(x)$. **[2]**

(iii) **Given** that $\mathrm{f}''(x) = \dfrac{6 - 18x^2}{(x^2 + 1)^3}$, find the maximum value of $\mathrm{f}'(x)$. **[4]**

The function $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$.

(iv) Write down the domain and range of $\mathrm{g}(x)$. Add a sketch of the curve $y = \mathrm{g}(x)$ to a copy of Fig. 9. **[4]**

(v) Show that $\mathrm{g}(x) = \sqrt{\dfrac{x + 1}{2 - x}}$. **[4]**