



**ADVANCED GCE**  
**MATHEMATICS (MEI)**  
 Differential Equations

**4758/01**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- Insert for Question 2 (inserted)
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

None

**Wednesday 27 January 2010**  
**Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- There is an **insert** for use in Question 2.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 A particle is attached to a spring and suspended vertically from an oscillating platform. The vertical displacement,  $y$ , of the particle from a fixed point at time  $t$  is modelled by the differential equation

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0.5 \sin t.$$

- (i) Find the general solution. [9]

Initially the displacement and velocity are both zero.

- (ii) Find the solution. [5]

- (iii) Describe the motion of the particle for large values of  $t$ . [2]

- (iv) Find approximate values of the velocity and displacement at  $t = 20\pi$ . [3]

The motion of the platform is stopped at  $t = 20\pi$  and the differential equation modelling the subsequent motion of the particle is

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0.$$

- (v) Write down the general solution. Sketch the solution curve for  $t > 20\pi$ . [5]

2 There is an insert for use with part (b)(i) of this question.

- (a) The differential equation

$$\frac{dy}{dx} - y \tan x = \tan x$$

is to be solved for  $|x| < \frac{1}{2}\pi$ .

- (i) Find the general solution. [8]

- (ii) Find the equation of the solution curve that passes through the origin and sketch the curve. [4]

- (b) The differential equation

$$\frac{dy}{dx} - y^2 \tan x = \tan x$$

is to be solved approximately, first by using a tangent field and then by Euler's method.

- (i) On the insert is a tangent field for the differential equation. Sketch the solution curves through the origin and through  $(0, 1)$ . [4]

Euler's method is now used, starting at  $x = 0$ ,  $y = 1$ . The algorithm is given by  $x_{r+1} = x_r + h$ ,  $y_{r+1} = y_r + hy'_r$ .

- (ii) Carry out two steps with a step length of 0.1 to verify that the algorithm gives  $x = 0.2$ ,  $y \approx 1.0201$ . [5]

- (iii) Explain why it would be inappropriate to extend this numerical solution as far as  $x = 1.6$ . [2]

- (iv) How could the accuracy of the estimate found in part (b)(ii) be improved? [1]

3

- 3 Fig. 3 shows a small ball projected from a point O over horizontal ground. The forces acting on the ball are its weight and air resistance. Its initial horizontal component of velocity is  $v_1$  and its subsequent horizontal velocity  $\dot{x}$  is modelled by the differential equation

$$\frac{d\dot{x}}{dt} = -k\dot{x},$$

where  $k$  is a positive constant.

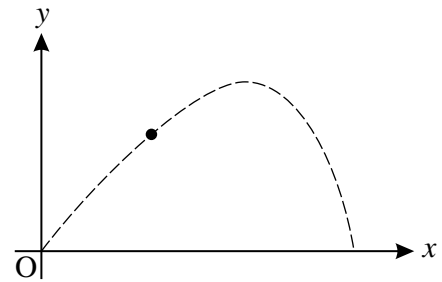


Fig. 3

The units of displacement are metres and the units of time are seconds.

- (i) Solve this differential equation to find  $\dot{x}$  in terms of  $t$  and hence show that the horizontal displacement from O is given by  $x = \frac{v_1}{k}(1 - e^{-kt})$ . [8]

The ball's initial vertical component of velocity is  $v_2$  and its subsequent vertical velocity  $\dot{y}$  is modelled by the differential equation

$$\frac{d\dot{y}}{dt} = -k\dot{y} - g.$$

- (ii) Solve this differential equation to find  $\dot{y}$  in terms of  $t$  and hence show that the vertical displacement from O is given by  $y = \frac{kv_2 + g}{k^2}(1 - e^{-kt}) - \frac{g}{k}t$ . [10]

- (iii) Eliminate  $t$  between the expressions for  $x$  and  $y$  to show that  $y = \left(\frac{kv_2 + g}{kv_1}\right)x + \frac{g}{k^2} \ln\left(1 - \frac{kx}{v_1}\right)$ . [4]

- (iv) In the case  $v_1 = v_2 = 10$ ,  $k = 0.1$ , determine whether the ball will pass over a 5 m high wall at a horizontal distance 8 m from O. [2]

- 4 The simultaneous differential equations

$$\frac{dx}{dt} = -3x - 4y + 23,$$

$$\frac{dy}{dt} = 2x + y - 7$$

are to be solved.

- (i) Show that  $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 5$ . [5]

- (ii) Find the general solution for  $x$ . [7]

- (iii) Find the corresponding general solution for  $y$ . [4]

When  $t = 0$ ,  $x = 8$  and  $y = 0$ .

- (iv) Find the particular solutions for  $x$  and  $y$ . [4]

- (v) Show that for sufficiently large  $t$ ,  $y$  is always greater than  $x$ . [4]

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.**



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**MATHEMATICS (MEI)**

Differential Equations  
 INSERT for Question 2

**4758/01**

**Wednesday 27 January 2010**  
**Afternoon**

**Duration:** 1 hour 30 minutes



Candidate Forename						Candidate Surname					
Centre Number						Candidate Number					

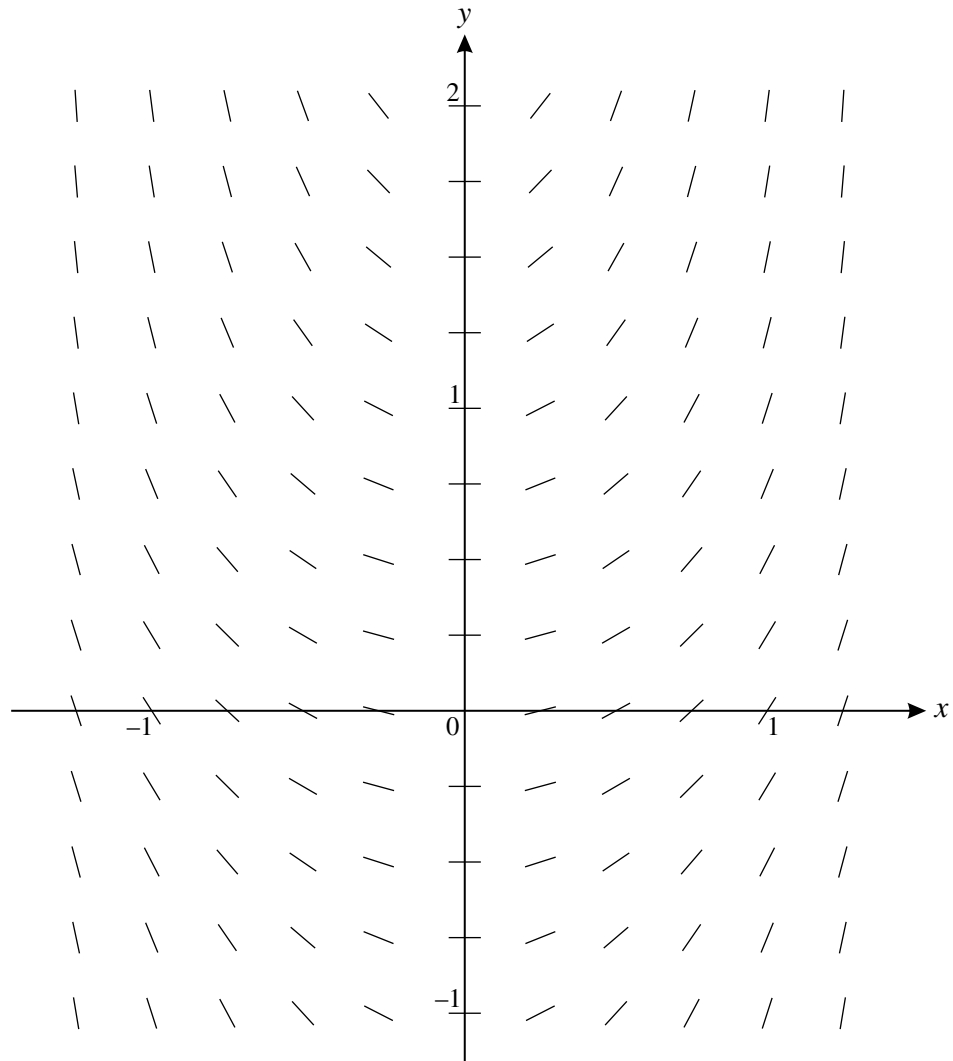
**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- This insert should be used to answer Question 2 part **(b)(i)**.
- Write your answers to Question 2 part **(b)(i)** in the spaces provided in this insert, and **attach it to your Answer Booklet**.

**INFORMATION FOR CANDIDATES**

- This document consists of **2** pages. Any blank pages are indicated.

2 (b) (i)

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