OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4758/01

Differential Equations

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Insert for Question 2 (inserted)
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 27 January 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- There is an **insert** for use in Question **2**.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 A particle is attached to a spring and suspended vertically from an oscillating platform. The vertical displacement, y, of the particle from a fixed point at time t is modelled by the differential equation

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0.5\sin t.$$

(i) Find the general solution. **[9]**

Initially the displacement and velocity are both zero.

(ii) Find the solution. **[5]**

(iii) Describe the motion of the particle for large values of t. **[2]**

(iv) Find approximate values of the velocity and displacement at $t = 20\pi$. **[3]**

The motion of the platform is stopped at $t = 20\pi$ and the differential equation modelling the subsequent motion of the particle is

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 0.$$

(v) Write down the general solution. Sketch the solution curve for $t > 20\pi$. **[5]**

2 **There is an insert for use with part (b)(i) of this question.**

(a) The differential equation

$$\frac{dy}{dx} - y\tan x = \tan x$$

is to be solved for $|x| < \frac{1}{2}\pi$.

(i) Find the general solution. **[8]**

(ii) Find the equation of the solution curve that passes through the origin and sketch the curve. **[4]**

(b) The differential equation

$$\frac{dy}{dx} - y^2\tan x = \tan x$$

is to be solved approximately, first by using a tangent field and then by Euler's method.

(i) On the insert is a tangent field for the differential equation. Sketch the solution curves through the origin and through $(0, 1)$. **[4]**

Euler's method is now used, starting at $x = 0$, $y = 1$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$.

(ii) Carry out two steps with a step length of 0.1 to verify that the algorithm gives $x = 0.2$, $y \approx 1.0201$. **[5]**

(iii) Explain why it would be inappropriate to extend this numerical solution as far as $x = 1.6$. **[2]**

(iv) How could the accuracy of the estimate found in part **(b)(ii)** be improved? **[1]**

3 Fig. 3 shows a small ball projected from a point O over horizontal ground. The forces acting on the ball are its weight and air resistance. Its initial horizontal component of velocity is v_1 and its subsequent horizontal velocity $\dot{x}$ is modelled by the differential equation

$$\frac{\mathrm{d}\dot{x}}{\mathrm{d}t} = -k\dot{x},$$

where k is a positive constant.

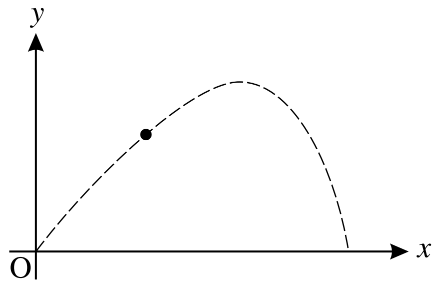

Fig. 3

The units of displacement are metres and the units of time are seconds.

(i) Solve this differential equation to find $\dot{x}$ in terms of t and hence show that the horizontal displacement from O is given by $x = \frac{v_1}{k}\left(1 - \mathrm{e}^{-kt}\right)$. **[8]**

The ball's initial vertical component of velocity is v_2 and its subsequent vertical velocity $\dot{y}$ is modelled by the differential equation

$$\frac{\mathrm{d}\dot{y}}{\mathrm{d}t} = -k\dot{y} - g.$$

(ii) Solve this differential equation to find $\dot{y}$ in terms of t and hence show that the vertical displacement from O is given by $y = \frac{kv_2 + g}{k^2}\left(1 - \mathrm{e}^{-kt}\right) - \frac{g}{k}t$. **[10]**

(iii) Eliminate t between the expressions for x and y to show that $y = \left(\frac{kv_2 + g}{kv_1}\right)x + \frac{g}{k^2}\ln\left(1 - \frac{kx}{v_1}\right)$. **[4]**

(iv) In the case $v_1 = v_2 = 10$, $k = 0.1$, determine whether the ball will pass over a 5 m high wall at a horizontal distance 8 m from O. **[2]**

4 The simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -3x - 4y + 23,$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 2x + y - 7$$

are to be solved.

(i) Show that $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 5x = 5$. **[5]**

(ii) Find the general solution for x. **[7]**

(iii) Find the corresponding general solution for y. **[4]**

When $t = 0$, $x = 8$ and $y = 0$.

(iv) Find the particular solutions for x and y. **[4]**

(v) Show that for sufficiently large t, y is always greater than x. **[4]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

4758/01

Differential Equations

INSERT for Question 2

Wednesday 27 January 2010
Afternoon

Duration: 1 hour 30 minutes

Candidate Forename		Candidate Surname	
Centre Number		Candidate Number	

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- This insert should be used to answer Question **2** part **(b)(i)**.
- Write your answers to Question **2** part **(b)(i)** in the spaces provided in this insert, and **attach it to your Answer Booklet**.

INFORMATION FOR CANDIDATES

- This document consists of **2** pages. Any blank pages are indicated.

Turn over

2 (b) (i)