



RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

4754A

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Friday 15 January 2010

Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Find the first three terms in the binomial expansion of $\frac{1+2x}{(1-2x)^2}$ in ascending powers of x . State the set of values of x for which the expansion is valid. [7]

2 Show that $\cot 2\theta = \frac{1 - \tan^2 \theta}{2 \tan \theta}$.

Hence solve the equation

$$\cot 2\theta = 1 + \tan \theta \quad \text{for } 0^\circ < \theta < 360^\circ. \quad [7]$$

3 A curve has parametric equations

$$x = e^{2t}, \quad y = \frac{2t}{1+t}.$$

(i) Find the gradient of the curve at the point where $t = 0$. [6]

(ii) Find y in terms of x . [2]

4 The points A, B and C have coordinates $(1, 3, -2)$, $(-1, 2, -3)$ and $(0, -8, 1)$ respectively.

(i) Find the vectors \overrightarrow{AB} and \overrightarrow{AC} . [2]

(ii) Show that the vector $2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ is perpendicular to the plane ABC. Hence find the equation of the plane ABC. [5]

5 (i) Verify that the lines $\mathbf{r} = \begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ meet at the point $(1, 3, 2)$. [3]

(ii) Find the acute angle between the lines. [4]

Section B (36 marks)

6 In Fig. 6, OAB is a thin bent rod, with $OA = a$ metres, $AB = b$ metres and angle $OAB = 120^\circ$. The bent rod lies in a vertical plane. OA makes an angle θ above the horizontal. The vertical height BD of B above O is h metres. The horizontal through A meets BD at C and the vertical through A meets OD at E.

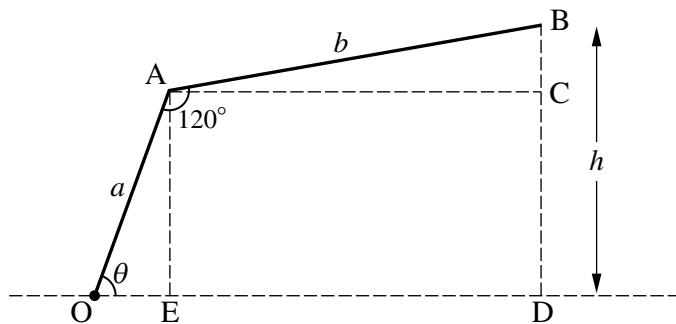


Fig. 6

(i) Find angle BAC in terms of θ . Hence show that

$$h = a \sin \theta + b \sin(\theta - 60^\circ). \quad [3]$$

(ii) Hence show that $h = (a + \frac{1}{2}b) \sin \theta - \frac{\sqrt{3}}{2}b \cos \theta. \quad [3]$

The rod now rotates about O, so that θ varies. You may assume that the formulae for h in parts (i) and (ii) remain valid.

(iii) Show that OB is horizontal when $\tan \theta = \frac{\sqrt{3}b}{2a + b}. \quad [3]$

In the case when $a = 1$ and $b = 2$, $h = 2 \sin \theta - \sqrt{3} \cos \theta$.

(iv) Express $2 \sin \theta - \sqrt{3} \cos \theta$ in the form $R \sin(\theta - \alpha)$. Hence, for this case, write down the maximum value of h and the corresponding value of θ . [7]

[Question 7 is printed overleaf.]

7 Fig. 7 illustrates the growth of a population with time. The proportion of the ultimate (long term) population is denoted by x , and the time in years by t . When $t = 0$, $x = 0.5$, and as t increases, x approaches 1.

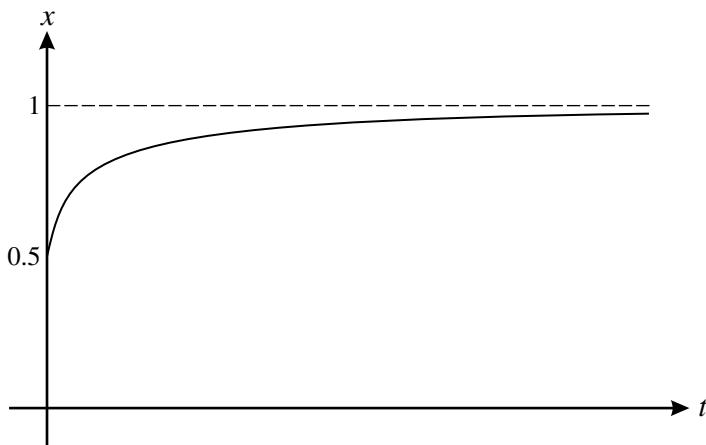


Fig. 7

One model for this situation is given by the differential equation

$$\frac{dx}{dt} = x(1 - x).$$

(i) Verify that $x = \frac{1}{1 + e^{-t}}$ satisfies this differential equation, including the initial condition. [6]

(ii) Find how long it will take, according to this model, for the population to reach three-quarters of its ultimate value. [3]

An alternative model for this situation is given by the differential equation

$$\frac{dx}{dt} = x^2(1 - x),$$

with $x = 0.5$ when $t = 0$ as before.

(iii) Find constants A , B and C such that $\frac{1}{x^2(1 - x)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{1 - x}$. [4]

(iv) Hence show that $t = 2 + \ln\left(\frac{x}{1 - x}\right) - \frac{1}{x}$. [5]

(v) Find how long it will take, according to this model, for the population to reach three-quarters of its ultimate value. [2]



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