

Mathematics

Advanced GCE **4764**

Mechanics 4

Mark Scheme for June 2010

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1(i)	$(m - \delta m)(v + \delta v) + \delta m (v - u) - mv = -mg\delta t$	M1	Impulse = change in momentum
		A1	Accept sign errors in δm
	$m\delta v - u \delta m - \delta m \delta v = -mg\delta t$		
	$m\frac{\delta v}{\delta t} + u\frac{\delta m}{\delta t} + \delta m\frac{\delta v}{\delta t} = -mg$	M1	Form DE
	$\Rightarrow m\frac{dv}{dt} + u\frac{dm}{dt} = -mg$	E1	Complete argument (including signs)
			4
(ii)	$\frac{dm}{dt} = -k \Rightarrow m = m_0 - kt$	M1	
	So $(m_0 - kt)\frac{dv}{dt} - uk = -(m_0 - kt)g$	A1	
	$\frac{dv}{dt} = \frac{uk}{m_0 - kt} - g$		
	$v = \int \left(\frac{uk}{m_0 - kt} - g \right) dt$	M1	Integrate
	$= -u \ln(m_0 - kt) - gt + c$	A1	
	$t = 0, v = 0 \Rightarrow 0 = -u \ln m_0 + c$	M1	Use condition
	$v = -u \ln \left(1 - \frac{k}{m_0} t \right) - gt$	A1	
	Fuel burnt when $m_0 - kt = 0.25m_0$	M1	
	$v = -u \ln 0.25 - \frac{0.75m_0 g}{k}$	A1	
			8

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2(i)	$m \frac{dv}{dt} = -mkv^{\frac{3}{2}}$	M1	N2L	
		A1		
	$\int -v^{-\frac{3}{2}} dv = \int k dt$	M1	Separate and integrate	
	$2v^{-\frac{1}{2}} = kt + c$	A1		
	$t = 0, v = 25 \Rightarrow c = \frac{2}{5}$	M1	Use condition	
	$2v^{-\frac{1}{2}} = kt + \frac{2}{5}$	M1	Rearrange	
	$v = 4 \left(kt + \frac{2}{5} \right)^{-2}$	E1		
(ii)	$x = \int 4 \left(kt + \frac{2}{5} \right)^{-2} dt$			7
	$= -\frac{4}{k} \left(kt + \frac{2}{5} \right)^{-1} + A$	M1	Integrate	
	$t = 0, x = 0 \Rightarrow A = \frac{10}{k}$	M1	Use condition	
	$x = \frac{1}{k} \left(10 - \frac{4}{kt + \frac{2}{5}} \right)$	A1		
(iii)	The speed decreases, tending to zero	B1		3
	The displacement tends to $\frac{10}{k}$	B1	Cv (10/k)	

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3(i)	$V = -mga \sin \theta + \frac{\lambda}{2(2a)} (3a \sin \theta)^2$	M1	GPE term
		M1	EPE term
		A1	
	$\frac{dV}{d\theta} = -mga \cos \theta + \frac{\lambda}{4a} \cdot 9a^2 \cdot 2 \sin \theta \cdot \cos \theta$	M1	Differentiate
		A1	
	$= a \cos \theta \left(\frac{9}{2} \lambda \sin \theta - mg \right)$	E1	
			6
(ii)	$\frac{dV}{d\theta} = 0 \Leftrightarrow \cos \theta = 0 \text{ or } \sin \theta = \frac{2mg}{9\lambda}$	M1	Solve $\frac{dV}{d\theta} = 0$
(A)	$\lambda > \frac{2}{9}mg$		
	$\theta = \frac{\pi}{2}$	A1	
	and $\theta = \sin^{-1} \frac{2mg}{9\lambda}$	A1	
	$\frac{d^2V}{d\theta^2} = -a \sin \theta \left(\frac{9}{2} \lambda \sin \theta - mg \right) + a \cos \theta \left(\frac{9}{2} \lambda \cos \theta \right)$	M1	Second derivative (or other valid method)
		A1	Any correct form
	$= a \left(\frac{9}{2} \lambda (1 - 2 \sin^2 \theta) + mg \sin \theta \right)$		
	$V'' \left(\frac{\pi}{2} \right) = a \left(-\frac{9}{2} \lambda + mg \right) < 0$	M1	Substitute $\theta = \frac{\pi}{2}$
	\Rightarrow unstable	A1	Deduce unstable
	$V'' \left(\sin^{-1} \left(\frac{2mg}{9\lambda} \right) \right) = a \left(\frac{9}{2} \lambda \left(1 - 2 \left(\frac{2mg}{9\lambda} \right)^2 \right) + \frac{2(mg)^2}{9\lambda} \right)$	M1	Substitute other value

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$= \frac{9}{2} \lambda a \left(1 - \left(\frac{2mg}{9\lambda} \right)^2 \right)$			
$\lambda > \frac{2}{9} mg \Rightarrow \left(\frac{2mg}{9\lambda} \right)^2 < 1 \Rightarrow V'' > 0$	M1	Consider second derivative	
$\Rightarrow \text{stable}$	A1	Complete argument	
			10
(B) $\lambda < \frac{2}{9} mg \Rightarrow$		M1	Consider solutions
$\theta = \frac{\pi}{2}$ only		A1	
$V''\left(\frac{\pi}{2}\right) = a \left(-\frac{9}{2} \lambda + mg \right) > 0$		M1	Consider second derivative
$\Rightarrow \text{stable}$		A1	Complete argument
			4
(C) $\lambda = \frac{2}{9} mg$ gives $\theta = \frac{1}{2} \pi$ only (from both factors)		M1	Consider solutions
		A1	
$V''\left(\frac{\pi}{2}\right) = 0$			
$V'\left(\frac{\pi}{2} - \epsilon\right) = (+)(-) = (-)$			
$V'\left(\frac{\pi}{2} + \epsilon\right) = (-)(-) = (+)$		M1	Valid method
Hence stable		A1	Complete argument
			4

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4(i)	Mass of slice $\approx \rho \pi y^2 \delta x$	M1	
	So $I_{\text{slice}} \approx \frac{1}{2} (\rho \pi y^2 \delta x) y^2$	M1	
	$= \frac{1}{32} \rho \pi x^4 \delta x$	A1	
	So $I_{\text{cone}} \approx \int_0^{2a} \frac{1}{32} \rho \pi x^4 dx$	M1	
	$= \left[\frac{1}{160} \rho \pi x^5 \right]_0^{2a}$	A1	ft
	$= \frac{1}{5} \pi \rho a^5$	A1	
	$\rho = \frac{M}{\frac{2}{3} \pi a^3}$	M1	
	$\Rightarrow I_{\text{cone}} = \frac{3}{10} M a^2$	E1	
			8
(ii)	Mass of small cone $= \left(\frac{1}{2}\right)^3 M = \frac{1}{8} M$		
	Mass of frustum $= \frac{7}{8} M$	B1	
	$I_{\text{large cone}} = I_{\text{small cone}} + I$	M1	
	$\frac{3}{10} M a^2 = \frac{3}{10} \left(\frac{1}{8} M\right) \left(\frac{1}{2} a\right)^2 + I$	M1	Moment of inertia of small cone
	$\Rightarrow I = \frac{93}{320} M a^2$		
	$\frac{7}{8} M = 2.8, a = 0.1 \Rightarrow I = 0.0093$	E1	
			4

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(iii)	$C = I\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{0.05}{0.0093}$	M1				
		A1				
	$t = \frac{10}{\ddot{\theta}} = 1.86$	M1				
		A1				4

(iv)	Centre of mass:					
	$\frac{7}{8}M\bar{x} + \frac{1}{8}M \cdot \frac{3a}{4} = M \cdot \frac{3a}{2}$	M1				
		A1				
	$OG = \bar{x} = \frac{45a}{28} = \frac{4.5}{28} \approx 0.1607$	A1	Any distance which locates G			
	i.e. G is $\frac{1.7}{28} \approx 0.0607$ m from the small circular face					3

(v)	$0.1J = I(10 - 5)$	M1	Moment of impulse = ang. momentum			
	$J = 0.465$	A1				
	Radius at G is $\frac{1}{2}\bar{x}$	B1				
	$\left(\frac{4.5}{56}\right)J = I(5 - \omega)$	M1	Moment of impulse = ang. momentum			
	$\Rightarrow \omega = \frac{55}{56} \approx 0.98$	A1				5

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