



GCE

## Mathematics

Advanced GCE 4764

Mechanics 4

# Mark Scheme for June 2010

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1(i)

$$(m - |\delta m|)(v + \delta v) + |\delta m|(v - u) - mv = -mg\delta t \quad \text{M1} \quad \text{Impulse} = \text{change in momentum}$$

A1 Accept sign errors in  $\delta m$

$$m\delta v - u|\delta m| - |\delta m|\delta v = -mg\delta t$$

$$m \frac{\delta v}{\delta t} + u \frac{\delta m}{\delta t} + \delta m \frac{\delta v}{\delta t} = -mg \quad \text{M1} \quad \text{Form DE}$$

$$\Rightarrow m \frac{dv}{dt} + u \frac{dm}{dt} = -mg \quad \text{E1} \quad \text{Complete argument (including signs)}$$

4

(ii)

$$\frac{dm}{dt} = -k \Rightarrow m = m_0 - kt \quad \text{M1}$$

$$\text{So } (m_0 - kt) \frac{dv}{dt} - uk = -(m_0 - kt)g \quad \text{A1}$$

$$\frac{dv}{dt} = \frac{uk}{m_0 - kt} - g$$

$$v = \int \left( \frac{uk}{m_0 - kt} - g \right) dt \quad \text{M1} \quad \text{Integrate}$$

$$= -u \ln(m_0 - kt) - gt + c \quad \text{A1}$$

$$t = 0, v = 0 \Rightarrow 0 = -u \ln m_0 + c \quad \text{M1} \quad \text{Use condition}$$

$$v = -u \ln \left( 1 - \frac{k}{m_0} t \right) - gt \quad \text{A1}$$

$$\text{Fuel burnt when } m_0 - kt = 0.25m_0 \quad \text{M1}$$

$$v = -u \ln 0.25 - \frac{0.75m_0 g}{k} \quad \text{A1}$$

8

4764

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June 2010

2(i)	$m \frac{dv}{dt} = -mkv^{\frac{3}{2}}$	M1	N2L
		A1	
	$\int -v^{\frac{3}{2}} dv = \int k dt$	M1	Separate and integrate
	$2v^{-\frac{1}{2}} = kt + c$	A1	
	$t = 0, v = 25 \Rightarrow c = \frac{2}{5}$	M1	Use condition
	$2v^{-\frac{1}{2}} = kt + \frac{2}{5}$	M1	Rearrange
	$v = 4 \left(kt + \frac{2}{5}\right)^{-2}$	E1	
			7
(ii)	$x = \int 4 \left(kt + \frac{2}{5}\right)^{-2} dt$		
	$= -\frac{4}{k} \left(kt + \frac{2}{5}\right)^{-1} + A$	M1	Integrate
	$t = 0, x = 0 \Rightarrow A = \frac{10}{k}$	M1	Use condition
	$x = \frac{1}{k} \left(10 - \frac{4}{kt + \frac{2}{5}}\right)$	A1	
			3
(iii)	The speed decreases, tending to zero	B1	
	The displacement tends to $\frac{10}{k}$	B1	$Cv(10/k)$
			2

4764

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June 2010

3(i)	$V = -mg a \sin \theta + \frac{\lambda}{2(2a)} (3a \sin \theta)^2$	M1	GPE term
		M1	EPE term
		A1	
	$\frac{dV}{d\theta} = -mg a \cos \theta + \frac{\lambda}{4a} \cdot 9a^2 \cdot 2 \sin \theta \cdot \cos \theta$	M1	Differentiate
		A1	
	$= a \cos \theta \left( \frac{9}{2} \lambda \sin \theta - mg \right)$	E1	
			6
(ii)	$\frac{dV}{d\theta} = 0 \Leftrightarrow \cos \theta = 0 \text{ or } \sin \theta = \frac{2mg}{9\lambda}$	M1	Solve $\frac{dV}{d\theta} = 0$
(A)	$\lambda > \frac{2}{9} mg$		
	$\theta = \frac{\pi}{2}$	A1	
	and $\theta = \sin^{-1} \frac{2mg}{9\lambda}$	A1	
	$\frac{d^2V}{d\theta^2} = -a \sin \theta \left( \frac{9}{2} \lambda \sin \theta - mg \right) + a \cos \theta \left( \frac{9}{2} \lambda \cos \theta \right)$	M1	Second derivative (or other valid method)
		A1	Any correct form
	$= a \left( \frac{9}{2} \lambda (1 - 2 \sin^2 \theta) + mg \sin \theta \right)$		
	$V'' \left( \frac{\pi}{2} \right) = a \left( -\frac{9}{2} \lambda + mg \right) < 0$	M1	Substitute $\theta = \frac{\pi}{2}$
	$\Rightarrow$ unstable	A1	Deduce unstable
	$V'' \left( \sin^{-1} \left( \frac{2mg}{9\lambda} \right) \right) = a \left( \frac{9}{2} \lambda \left( 1 - 2 \left( \frac{2mg}{9\lambda} \right)^2 \right) + \frac{2(mg)^2}{9\lambda} \right)$	M1	Substitute other value

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June 2010

$$= \frac{9}{2} \lambda a \left( 1 - \left( \frac{2mg}{9\lambda} \right)^2 \right)$$

$$\lambda > \frac{2}{9} mg \Rightarrow \left( \frac{2mg}{9\lambda} \right)^2 < 1 \Rightarrow V'' > 0$$

M1 Consider second derivative

 $\Rightarrow$  stable

A1 Complete argument

10

$$(B) \quad \lambda < \frac{2}{9} mg \Rightarrow$$

M1 Consider solutions

$$\theta = \frac{\pi}{2} \text{ only}$$

A1

$$V'' \left( \frac{\pi}{2} \right) = a \left( -\frac{9}{2} \lambda + mg \right) > 0$$

M1 Consider second derivative

 $\Rightarrow$  stable

A1 Complete argument

4

$$(C) \quad \lambda = \frac{2}{9} mg \text{ gives } \theta = \frac{1}{2} \pi \text{ only (from both factors)}$$

M1 Consider solutions

A1

$$V'' \left( \frac{\pi}{2} \right) = 0$$

$$V' \left( \frac{\pi}{2} - \epsilon \right) = (+)(-) = (-)$$

$$V' \left( \frac{\pi}{2} + \epsilon \right) = (-)(+) = (+)$$

M1 Valid method

Hence stable

A1 Complete argument

4

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June 2010

4(i) Mass of slice  $\approx \rho \pi y^2 \delta x$  M1

$$\text{So } I_{\text{slice}} \approx \frac{1}{2} (\rho \pi y^2 \delta x) y^2 \text{ M1}$$

$$= \frac{1}{32} \rho \pi x^4 \delta x \text{ A1}$$

$$\text{So } I_{\text{cone}} \approx \int_0^{2a} \frac{1}{32} \rho \pi x^4 dx \text{ M1}$$

$$= \left[ \frac{1}{160} \rho \pi x^5 \right]_0^{2a} \text{ A1 ft}$$

$$= \frac{1}{5} \pi \rho a^5 \text{ A1}$$

$$\rho = \frac{M}{\frac{2}{3} \pi a^3} \text{ M1}$$

$$\Rightarrow I_{\text{cone}} = \frac{3}{10} Ma^2 \text{ E1}$$

8

(ii) Mass of small cone  $= \left(\frac{1}{2}\right)^3 M = \frac{1}{8} M$

$$\text{Mass of frustum} = \frac{7}{8} M \text{ B1}$$

$$I_{\text{large cone}} = I_{\text{small cone}} + I \text{ M1}$$

$$\frac{3}{10} Ma^2 = \frac{3}{10} \left(\frac{1}{8} M\right) \left(\frac{1}{2} a\right)^2 + I \text{ M1 Moment of inertia of small cone}$$

$$\Rightarrow I = \frac{93}{320} Ma^2$$

$$\frac{7}{8} M = 2.8, a = 0.1 \Rightarrow I = 0.0093 \text{ E1}$$

4

4764

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June 2010

(iii)  $C = I\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{0.05}{0.0093}$  M1

A1

$t = \frac{10}{\ddot{\theta}} = 1.86$  M1

A1

4

(iv) Centre of mass:

$\frac{7}{8}M\bar{x} + \frac{1}{8}M \cdot \frac{3a}{4} = M \cdot \frac{3a}{2}$  M1

A1

$OG = \bar{x} = \frac{45a}{28} = \frac{4.5}{28} \approx 0.1607$  A1 Any distance which locates G

i.e. G is  $\frac{1.7}{28} \approx 0.0607$  m from the small circular face

3

(v)  $0.1J = I(10 - 5)$  M1 Moment of impulse = ang. momentum

$J = 0.465$  A1

Radius at G is  $\frac{1}{2}\bar{x}$  B1

$\left(\frac{4.5}{56}\right)J = I(5 - \omega)$  M1 Moment of impulse = ang. momentum

$\Rightarrow \omega = \frac{55}{56} \approx 0.98$  A1

5

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