OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **4756**

Further Methods for Advanced Mathematics (FP2)

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

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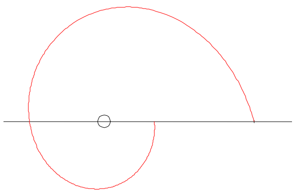
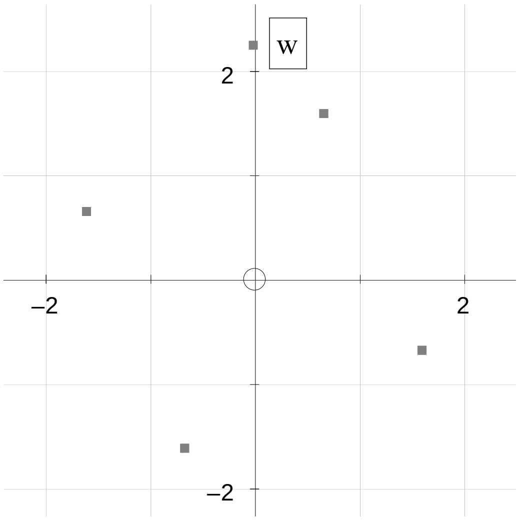
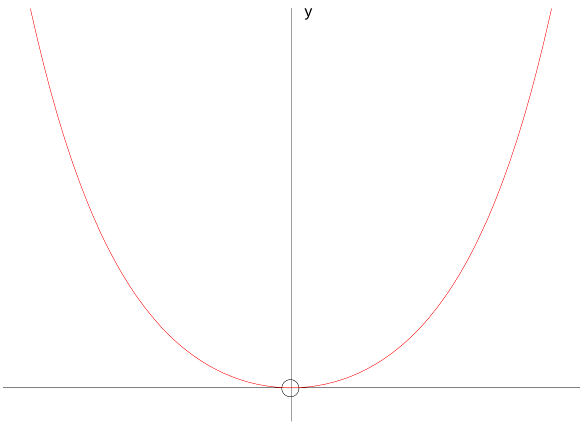
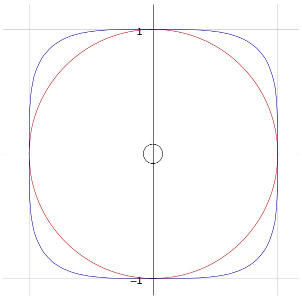
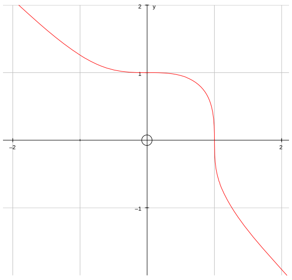
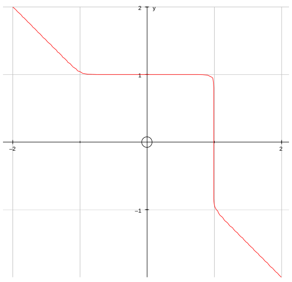
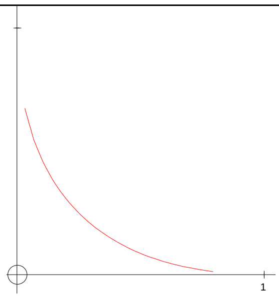
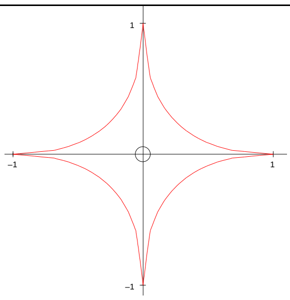
1 (a)(i)	$f(t) = \arcsin t$ $\Rightarrow f'(t) = \dfrac{1}{\sqrt{1-t^2}} = \left(1-t^2\right)^{-\frac{1}{2}}$ $\Rightarrow f''(t) = -\frac{1}{2}\left(1-t^2\right)^{-\frac{3}{2}} \times -2t$ $= \dfrac{t}{\left(1-t^2\right)^{\frac{3}{2}}}$	B1 M1 A1 (ag) **3**	Any form Using Chain Rule
(ii)	$f(x) = \arcsin(x + ½)$ $\Rightarrow f(0) = \arcsin(½) = \dfrac{\pi}{6}$ $f'(0) = \left(1-\left(\frac{1}{2}\right)^2\right)^{-\frac{1}{2}} = \dfrac{2}{\sqrt{3}}$ and $f''(0) = \dfrac{\frac{1}{2}}{\left(1-\left(\frac{1}{2}\right)^2\right)^{\frac{3}{2}}} = \dfrac{4\sqrt{3}}{9}$ $f(x) = f(0) + x f'(0) + \dfrac{x^2}{2} f''(0) + \ldots$ $\Rightarrow$ term in x^2 is $\dfrac{2\sqrt{3}}{9}x^2$	B1 (ag) M1 A1 (ag) M1 A1 **5**	$\dfrac{\pi}{6}$ obtained clearly from $f(0)$ www Clear substitution of $x = 0$ or $t = ½$ Evaluating $f''(0)$ and dividing by 2 Accept $0.385x^2$ or better
(b)	 Area $= \int_0^{\pi} \frac{1}{2} r^2 \, d\theta$ $= \int_0^{\pi} \dfrac{\pi^2 a^2}{2(\pi+\theta)^2} d\theta = \dfrac{\pi^2 a^2}{2} \int_0^{\pi} \dfrac{1}{(\pi+\theta)^2} d\theta$ $= \dfrac{\pi^2 a^2}{2}\left[\dfrac{-1}{\pi+\theta}\right]_0^{\pi}$ $= \dfrac{\pi^2 a^2}{2}\left(\dfrac{-1}{2\pi} + \dfrac{1}{\pi}\right)$ $= \frac{1}{4}\pi a^2$	G1 G1 M1 A1 M1 A1 **6**	Complete spiral with $r(2\pi) < r(0)$ $r(0) = a$, $r(2\pi) = a/3$ indicated or $r(0) > r(\pi/2) > r(\pi) > r(3\pi/2) > r(2\pi)$ Dep. on G1 above Max. G1 if not fully correct Integral expression involving r^2 Correct result of integration with correct limits Substituting limits into an expression of the form $\dfrac{k}{\pi+\theta}$. Dep. on M1 above
(c)	$\int_0^{\frac{3}{2}} \dfrac{1}{9+4x^2} dx = \dfrac{1}{4}\int_0^{\frac{3}{2}} \dfrac{1}{\frac{9}{4}+x^2} dx = \dfrac{1}{4} \times \left[\dfrac{2}{3}\arctan\dfrac{2x}{3}\right]_0^{\frac{3}{2}}$ $= \dfrac{1}{6}\arctan 1$ $= \dfrac{\pi}{24}$	M1 A1A1 M1 A1 **5**	arctan $\dfrac{1}{4} \times \dfrac{2}{3}$ and $\dfrac{2x}{3}$ Substituting limits. Dep. on M1 above Evaluated in terms of π **19**

2 (a)	$z^n + \frac{1}{z^n} = 2\cos n\theta$, $z^n - \frac{1}{z^n} = 2j\sin n\theta$	B1	Both
	$\left(z - \frac{1}{z}\right)^5 = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$	M1	Expanding $\left(z - \frac{1}{z}\right)^5$
	$= z^5 - \frac{1}{z^5} - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$		
	$\Rightarrow$ $32j\sin^5\theta = 2j\sin 5\theta - 10j\sin 3\theta + 20j\sin\theta$	M1 A1	Introducing sines (and possibly cosines) of multiple angles RHS
	$\Rightarrow$ $\sin^5\theta = \frac{1}{16}\sin 5\theta - \frac{5}{16}\sin 3\theta + \frac{5}{8}\sin\theta$	A1ft	Division by 32(j)
	$A = \frac{5}{8}$, $B = -\frac{5}{16}$, $C = \frac{1}{16}$	5	
(b)(i)	4^{th} roots of $-9j = 9e^{\frac{3}{2}\pi j}$ are $re^{j\theta}$ where		
	$r = \sqrt{3}$	B1	Accept $9^{\frac{1}{4}}$
	$\theta = \frac{3\pi}{8}$	B1	
	$\Rightarrow$ $\theta = \frac{3\pi}{8} + \frac{2k\pi}{4}$	M1	Implied by at least two correct (ft) further values
	$\Rightarrow$ $\theta = \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$	A1	Or stating $k = (0), 1, 2, 3$ Allow arguments in range $-\pi \le \theta \le \pi$
	 	M1 A1 6	Points at vertices of a square centre O or 3 correct points (ft) or 1 point in each quadrant
(ii)	Mid-point of SP has argument $\frac{\pi}{8}$	B1	
	and modulus of $\sqrt{\frac{3}{2}}$	B1	
	Argument of $w = 4 \times \frac{\pi}{8} = \frac{\pi}{2}$		
	and modulus $= \left(\sqrt{\frac{3}{2}}\right)^4 = \frac{9}{4}$	M1 A1 G1 5	Multiplying argument by 4 and modulus raised to power of 4 Both correct w plotted on imag. axis above level of P 16

3 (a)(i)	$2\lambda^3+\lambda^2-13\lambda+6=0 \Rightarrow (\lambda-2)(2\lambda^2+5\lambda-3)=0$ $\Rightarrow \lambda=2$ or $2\lambda^2+5\lambda-3=0$ $\Rightarrow (2\lambda-1)(\lambda+3)=0$ $\Rightarrow \lambda=\frac{1}{2}, \lambda=-3$	B1 M1 A1A1 **4**	Substituting $\lambda=2$ or factorising Obtaining and solving a quadratic
(ii)	$\mathbf{M}\begin{pmatrix}3\\-3\\1\end{pmatrix}=2\begin{pmatrix}3\\-3\\1\end{pmatrix}=\begin{pmatrix}6\\-6\\2\end{pmatrix}$ $\mathbf{M}^2\mathbf{v}=2^2\mathbf{v}=4\begin{pmatrix}1\\-1\\\frac{1}{3}\end{pmatrix}=\begin{pmatrix}4\\-4\\\frac{4}{3}\end{pmatrix}$ $\mathbf{M}\begin{pmatrix}\frac{3}{2}\\-\frac{3}{2}\\\frac{1}{2}\end{pmatrix}=2\begin{pmatrix}\frac{3}{2}\\-\frac{3}{2}\\\frac{1}{2}\end{pmatrix}=\begin{pmatrix}3\\-3\\1\end{pmatrix}$ $\Rightarrow x=\frac{3}{2}, y=-\frac{3}{2}, z=\frac{1}{2}$	B1 B2 M1 A1 **5**	 Give B1 for one component with the wrong sign Recognising that the solution is a multiple of the given RHS Correct multiple
(iii)	$2\lambda^3+\lambda^2-13\lambda+6=0$ $\Rightarrow 2\mathbf{M}^3+\mathbf{M}^2-13\mathbf{M}+6\mathbf{I}=\mathbf{0}$ $\Rightarrow \mathbf{M}^3=-\frac{1}{2}\mathbf{M}^2+\frac{13}{2}\mathbf{M}-3\mathbf{I}$ $\Rightarrow \mathbf{M}^4=-\frac{1}{2}\mathbf{M}^3+\frac{13}{2}\mathbf{M}^2-3\mathbf{M}$ $\Rightarrow \mathbf{M}^4=-\frac{1}{2}(-\frac{1}{2}\mathbf{M}^2+\frac{13}{2}\mathbf{M}-3\mathbf{I})+\frac{13}{2}\mathbf{M}^2-3\mathbf{M}$ $\Rightarrow \mathbf{M}^4=\frac{27}{4}\mathbf{M}^2-\frac{25}{4}\mathbf{M}+\frac{3}{2}\mathbf{I}$ $A=\frac{27}{4}, B=-\frac{25}{4}, C=\frac{3}{2}$	 M1 M1 M1 A1 **4**	 Using Cayley-Hamilton Theorem Multiplying by **M** Substituting for $\mathbf{M}^3$
(b)	$\mathbf{N}=\mathbf{PDP}^{-1}$ where $\mathbf{D}=\begin{pmatrix}-1&0\\0&2\end{pmatrix}$ and $\mathbf{P}=\begin{pmatrix}1&-1\\2&1\end{pmatrix}$ $\Rightarrow \mathbf{P}^{-1}=\frac{1}{3}\begin{pmatrix}1&1\\-2&1\end{pmatrix}$ $\Rightarrow \mathbf{N}=\frac{1}{3}\begin{pmatrix}1&-1\\2&1\end{pmatrix}\begin{pmatrix}-1&0\\0&2\end{pmatrix}\begin{pmatrix}1&1\\-2&1\end{pmatrix}$ $=\frac{1}{3}\begin{pmatrix}-1&-2\\-2&2\end{pmatrix}\begin{pmatrix}1&1\\-2&1\end{pmatrix}$ $=\frac{1}{3}\begin{pmatrix}3&-3\\-6&0\end{pmatrix}=\begin{pmatrix}1&-1\\-2&0\end{pmatrix}$	B1 B1 B1 B1ft M1 A1	Order must be correct For B1B1, order must be consistent Ft their **P** Attempting matrix product
	OR Let $\mathbf{N}=\begin{pmatrix}a&c\\b&d\end{pmatrix}$ $\begin{pmatrix}a&c\\b&d\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}=-1\begin{pmatrix}1\\2\end{pmatrix}$ B1 $\begin{pmatrix}a&c\\b&d\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix}=2\begin{pmatrix}-1\\1\end{pmatrix}$ B1 $\Rightarrow a+2c=-1, -a+c=-2$ B1 $b+2d=-2, -b+d=2$ B1 $\Rightarrow a=1, c=-1; b=-2, d=0$ M1A1	 **6**	Or $\begin{pmatrix}a+1&c\\b&d+1\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ Or $\begin{pmatrix}a-2&c\\b&d-2\end{pmatrix}\begin{pmatrix}-1\\1\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ Solving both pairs of equations **19**

4 (i)	$2\sinh x\cosh x$ $= 2\times\frac{e^x+e^{-x}}{2}\times\frac{e^x-e^{-x}}{2}$ $=\frac{e^{2x}-e^{-2x}}{2}$ $=\sinh 2x$ Differentiating, $2\cosh 2x = 2\cosh^2 x + 2\sinh^2 x$ $\Rightarrow\quad \cosh 2x = \cosh^2 x + \sinh^2 x$	M1 A1 (ag) B1 B1 **4**	Using exponential definitions and multiplying or factorising One side correct Correct completion
(ii)	 Volume $=\pi\int_0^2(\cosh x-1)^2\,dx$ $=\pi\int_0^2\cosh^2 x-2\cosh x+1\,dx$ $=\pi\int_0^2\tfrac{1}{2}\cosh 2x-2\cosh x+\tfrac{3}{2}\,dx$ $=\pi\left[\tfrac{1}{4}\sinh 2x-2\sinh x+\tfrac{3}{2}x\right]_0^2$ $=\pi\left[\tfrac{1}{4}\sinh 4-2\sinh 2+3\right]$ $= 8.070$	G1 M1 A1 M1 A2 A1 **7**	Correct shape and through origin $\int(\cosh x-1)^2\,dx$ A correct expanded integral expression including limits 0, 2 (may be implied by later work) Attempting to obtain an integrable form Dep. on M1 above Give A1 for two terms correct 3 d.p. required. Condone 8.07
(iii)	$y = \cosh 2x + \sinh x$ $\Rightarrow\quad \frac{dy}{dx} = 2\sinh 2x + \cosh x$ At S.P. $2\sinh 2x + \cosh x = 0$ $\Rightarrow\quad 4\sinh x\cosh x + \cosh x = 0$ $\Rightarrow\quad \cosh x(4\sinh x + 1) = 0$ $\Rightarrow\quad \cosh x = 0$ (rejected) $\Rightarrow\quad \sinh x = -\tfrac{1}{4}$ $\Rightarrow\quad x = \ln\left(-\frac{1}{4}+\frac{\sqrt{17}}{4}\right)$	B1 M1 M1 A1 A1 M1 A1 **7**	Any correct form Setting derivative equal to zero and using identity Solving $\frac{dy}{dx}$=0 to obtain value of sinh x Repudiating $\cosh x = 0$ Using log form of arsinh, or setting up and solving quadratic in e^x A0 if extra "roots" quoted **18**

5(i)(A)	Circle	B1	
(B)		G1 G1	Sketch of circle, centre (0, 0) Sketch of "squarer" circle on same axes
(C)	Square	B1	
(D)	$-1 \le x \le 1$ $-1 \le y \le 1$	B1 B1 **6**	Give B1B0 for not all non-strict or unclear
(ii)(A)	Odd roots exist for all real numbers	B1	Any equivalent explanation
(B)	Line	B1	Sketch insufficient
(C)		G1	
	Asymptote: $x + y = 0$	B1	
(D)		G1 G1 **6**	Line $x + y = 0$ outside unit square Lines $y = 1$ and $x = 1$ on unit square
(iii)		G1	G0 if curve beyond (1, 0) or (0, 1)
	$0 \le x, y \le 1$	B1 **2**	Accept strict, or indication on graph
(iv)(A)		G2ft	Give G1 for a partial attempt. Ft from (iii) on shape
(B)	Limit is a "plus sign" where $x \to 0$ for $-1 \le y \le 1$ and vice versa	B1 B1 **4**	**18**

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

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