OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **4776**

Numerical Methods

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

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1(i)

x	LHS		RHS	
1	1	<	2	(Change of sign implies root.)
2	0.5	>	-1	*(or equivalent)*

[M1A1]

r	0	1	2	3	4	5	6
x_r	1.5	1.333333	0.818182	0.429078	0.355127	0.347961	0.347352

[M1A1]

State or clearly imply convergence outside the interval (1, 2) **[E1]**

(ii) E.g. $x_{r+1} = \sqrt{(3 - 1/x)}$ **[B1]**

r	0	1	2	3	4	5
x_r	1.5	1.527525	1.531452	1.532	1.532077	1.532087

[M1A1]

E.g. $x_{r+1} = 3/x - 1/x^2$ **[B1]**

r	0	1	2	3	4	5
x_r	1.5	1.555556	1.515306	1.544287	1.523326	1.538438

[M1A1]

[TOTAL 8]

2(i)

Forward difference: (0.9996 - 0.9854)/0.2 = 0.071 **[M1A1]**

Central difference: (0.9996 - 0.9508)/0.4 = 0.122 **[M1A1]**

Central difference expected to be more accurate. **[E1]**

(ii)

Forward difference maximum: (0.99965 - 0.98535)/0.2 = 0.0715 **[B1]**

Central difference maximum: (0.99965 - 0.95075)/0.4 = 0.12225 **[B1]**

[TOTAL 7]

3(i)

r is the relative error (in X as an approximation to x) **[E1]**

$X^n = x^n (1 + r)^n$ $(1 + r)^n = 1 + nr$ (provided r is small) **[M1M1A1]**

(ii)

G^2 (= 0.332 929, not required) is about 0.08% smaller than g^2

$\sqrt{G}$ (= 0.795 605, not required) is about 0.02% smaller than $\sqrt{g}$ **[M1A1A1]**

[TOTAL 7]

4(i)

x	sin + tan	$2x$	error	rel error	*accept:*	+ve, +ve
0.2	0.401379	0.4	-0.00138	-0.00344		*-ve, +ve*
0.1	0.200168	0.2	-0.00017	-0.00084		*-ve, -ve*

[M1A1A1A1]

(ii)

$2 \times 0.2^3 / k = 0.00138$ gives $k = 11.59$

$2 \times 0.1^3 / k = 0.00017$ gives $k = 11.76$

Either of these (or other methods) to suggest $k = 12$ **[M1A1]** **[B1]**

[TOTAL 7]

5

Data not equally spaced in x **[E1]**

$f(x) = - 10(x - 3)(x - 6) / (1 - 3)(1 - 6) - 12(x - 1)(x - 6) / (3 - 1)(3 - 6) + 30(x - 1)(x - 3) / (6 - 1)(6 - 3)$

[M1A1A1A1]

$f(x) = - (x^2 - 9x + 18) + 2(x^2 - 7x + 6) + 2(x^2 - 4x + 3)$ **[A1]**

$= 3x^2 - 13x$ **[A1]**

[TOTAL 7]

6(i)

h	M	T	S
0.8	1.547953	1.611209	**1.569038**
0.4	**1.563639**	**1.579581**	**1.568953**
0.2	**1.567619**	**1.571610**	**1.568949**

M: **[M1A1A1]**
T: **[M1]**
S: **[M1]**
[subtotal 7]

(ii) 1.56895 appears justified Comparison of last two S values, e.g.: **[B1]**
last change in S is -0.000004; next change negligible **[E1]**
[subtotal 2]

(iii)

h	M error	T error	
0.8	-0.02100	0.04226	*accept consistent*
0.4	-0.00531	0.01063	*use of other sign*
0.2	-0.00133	0.00266	*convention*

[M1A1A1]

(A) M errors are about half the T errors so M is twice as accurate as T **[E1A1]**
(B) Errors for both T and M reduce by a factor of 4 as h is halved so **[E1]**
the rates of convergence are the same, both second order **[A1A1]**
[subtotal 8]
[TOTAL 17]

7(i) f(0) = 5, f(1) = -2. (Change of sign implies root.) **[M1A1]**

f '(x) = $5x^4$ - 8 hence N-R formula **[M1A1]**

r	0	1	2	3	4
x_r	0.5	0.634146	0.638232	0.638238	0.638238
differences		0.134146	0.004086	5.98E-06	1.29E-11
ratios			0.030457	0.001462	2.17E-06

[M1A1A1]
[A1]
[M1A1]
The ratios of differences are decreasing (fast) so process is faster than first order **[E1]**
[subtotal 11]

(ii)

r	0	1	2	3	4
x_r	1.4	1.5	1.458054	1.462741	1.46312
f(x_r)	-0.82176	0.59375	-0.0747	-0.00559	5.99E-06

[M1A1A1]
root is 1.46 correct to 3 sf **[A1]**

differences	0.1	-0.04195	0.004687	0.000379
ratios		-0.41946	-0.11175	0.080876

[A1]
[M1A1]
The ratios of differences are decreasing (fast) so process is faster than first order **[E1]**
accept 'second order'
[subtotal 8]
[TOTAL 19]

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