



RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

4754A

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

- Scientific or graphical calculator

Wednesday 9 June 2010

Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Express $\frac{x}{x^2 - 1} + \frac{2}{x + 1}$ as a single fraction, simplifying your answer. [3]

2 Fig. 2 shows the curve $y = \sqrt{1 + x^2}$.

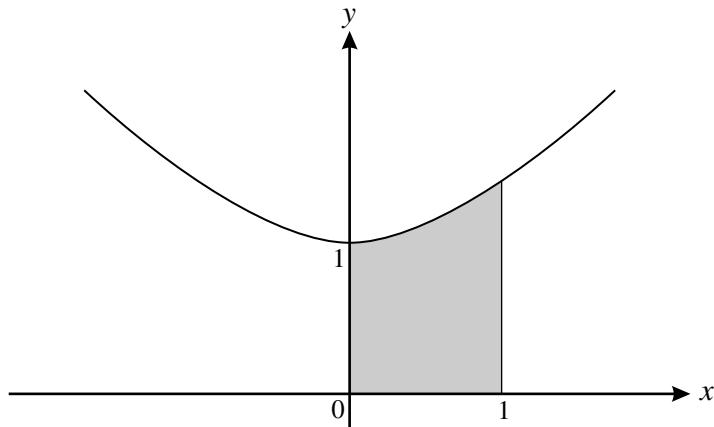


Fig. 2

(i) The following table gives some values of x and y .

x	0	0.25	0.5	0.75	1
y	1	1.0308		1.25	1.4142

Find the missing value of y , giving your answer correct to 4 decimal places.

Hence show that, using the trapezium rule with four strips, the shaded area is approximately 1.151 square units. [3]

(ii) Jenny uses a trapezium rule with 8 strips, and obtains a value of 1.158 square units. Explain why she must have made a mistake. [2]

(iii) The shaded area is rotated through 360° about the x -axis. Find the exact volume of the solid of revolution formed. [3]

3 The parametric equations of a curve are

$$x = \cos 2\theta, \quad y = \sin \theta \cos \theta \quad \text{for } 0 \leq \theta < \pi.$$

Show that the cartesian equation of the curve is $x^2 + 4y^2 = 1$.

Sketch the curve. [5]

4 Find the first three terms in the binomial expansion of $\sqrt{4+x}$ in ascending powers of x .

State the set of values of x for which the expansion is valid. [5]

5 (i) Express $\frac{3}{(y-2)(y+1)}$ in partial fractions. [3]

(ii) Hence, given that x and y satisfy the differential equation

$$\frac{dy}{dx} = x^2(y-2)(y+1),$$

show that $\frac{y-2}{y+1} = Ae^{x^3}$, where A is a constant. [5]

6 Solve the equation $\tan(\theta + 45^\circ) = 1 - 2 \tan \theta$, for $0^\circ \leq \theta \leq 90^\circ$. [7]

Section B (36 marks)

7 A straight pipeline AB passes through a mountain. With respect to axes Oxyz, with Ox due East, Oy due North and Oz vertically upwards, A has coordinates $(-200, 100, 0)$ and B has coordinates $(100, 200, 100)$, where units are metres.

(i) Verify that $\overrightarrow{AB} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ and find the length of the pipeline. [3]

(ii) Write down a vector equation of the line AB, and calculate the angle it makes with the vertical. [6]

A thin flat layer of hard rock runs through the mountain. The equation of the plane containing this layer is $x + 2y + 3z = 320$.

(iii) Find the coordinates of the point where the pipeline meets the layer of rock. [4]

(iv) By calculating the angle between the line AB and the normal to the plane of the layer, find the angle at which the pipeline cuts through the layer. [5]

[Question 8 is printed overleaf.]

8 Part of the track of a roller-coaster is modelled by a curve with the parametric equations

$$x = 2\theta - \sin \theta, \quad y = 4 \cos \theta \quad \text{for } 0 \leq \theta \leq 2\pi.$$

This is shown in Fig. 8. B is a minimum point, and BC is vertical.

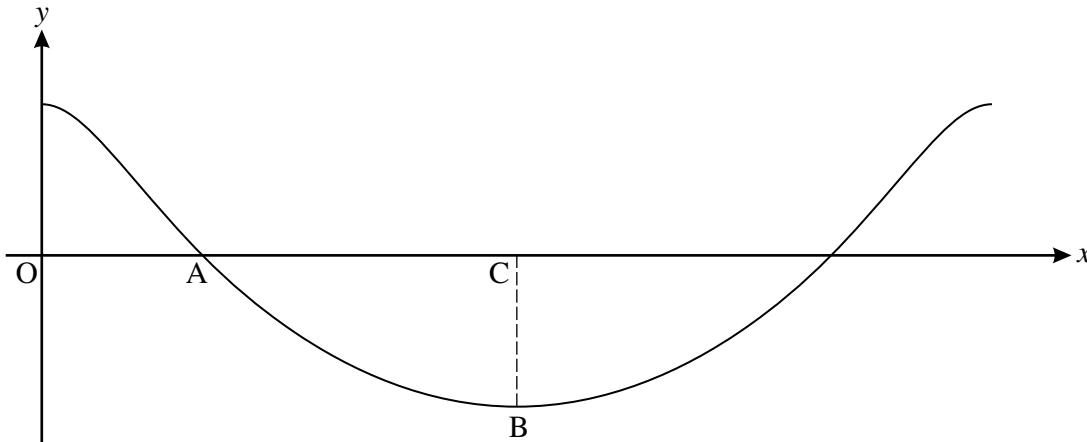


Fig. 8

(i) Find the values of the parameter at A and B.

Hence show that the ratio of the lengths OA and AC is $(\pi - 1) : (\pi + 1)$. [5]

(ii) Find $\frac{dy}{dx}$ in terms of θ . Find the gradient of the track at A. [4]

(iii) Show that, when the gradient of the track is 1, θ satisfies the equation

$$\cos \theta - 4 \sin \theta = 2.$$

[2]

(iv) Express $\cos \theta - 4 \sin \theta$ in the form $R \cos(\theta + \alpha)$.

Hence solve the equation $\cos \theta - 4 \sin \theta = 2$ for $0 \leq \theta \leq 2\pi$. [7]

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