OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754A

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Express $\dfrac{x}{x^2-1}+\dfrac{2}{x+1}$ as a single fraction, simplifying your answer. **[3]**

2 Fig. 2 shows the curve $y=\sqrt{1+x^2}$.

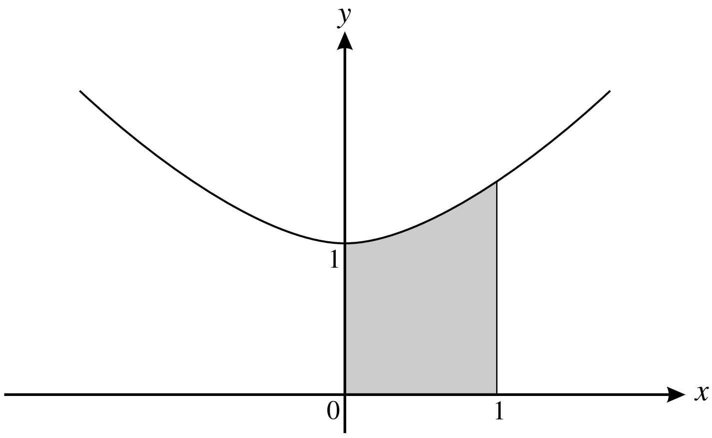

Fig. 2

(i) The following table gives some values of x and y.

x	0	0.25	0.5	0.75	1
y	1	1.0308		1.25	1.4142

Find the missing value of y, giving your answer correct to 4 decimal places.

Hence show that, using the trapezium rule with four strips, the shaded area is approximately 1.151 square units. **[3]**

(ii) Jenny uses a trapezium rule with 8 strips, and obtains a value of 1.158 square units. Explain why she must have made a mistake. **[2]**

(iii) The shaded area is rotated through 360° about the x-axis. Find the exact volume of the solid of revolution formed. **[3]**

3 The parametric equations of a curve are

$$x=\cos 2\theta,\quad y=\sin\theta\cos\theta\quad \text{for } 0\leqslant\theta<\pi.$$

Show that the cartesian equation of the curve is $x^2+4y^2=1$.

Sketch the curve. **[5]**

4 Find the first three terms in the binomial expansion of $\sqrt{4+x}$ in ascending powers of x.

State the set of values of x for which the expansion is valid. **[5]**

5 **(i)** Express $\dfrac{3}{(y-2)(y+1)}$ in partial fractions. **[3]**

(ii) Hence, given that x and y satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2(y-2)(y+1),$$

show that $\dfrac{y-2}{y+1} = Ae^{x^3}$, where A is a constant. **[5]**

6 Solve the equation $\tan(\theta + 45°) = 1 - 2\tan\theta$, for $0° \leqslant \theta \leqslant 90°$. **[7]**

Section B (36 marks)

7 A straight pipeline AB passes through a mountain. With respect to axes Oxyz, with Ox due East, Oy due North and Oz vertically upwards, A has coordinates (–200, 100, 0) and B has coordinates (100, 200, 100), where units are metres.

(i) Verify that $\overrightarrow{AB} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ and find the length of the pipeline. **[3]**

(ii) Write down a vector equation of the line AB, and calculate the angle it makes with the vertical. **[6]**

A thin flat layer of hard rock runs through the mountain. The equation of the plane containing this layer is $x + 2y + 3z = 320$.

(iii) Find the coordinates of the point where the pipeline meets the layer of rock. **[4]**

(iv) By calculating the angle between the line AB and the normal to the plane of the layer, find the angle at which the pipeline cuts through the layer. **[5]**

[Question 8 is printed overleaf.]

8 Part of the track of a roller-coaster is modelled by a curve with the parametric equations

$$x = 2\theta - \sin\theta, \quad y = 4\cos\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

This is shown in Fig. 8. B is a minimum point, and BC is vertical.

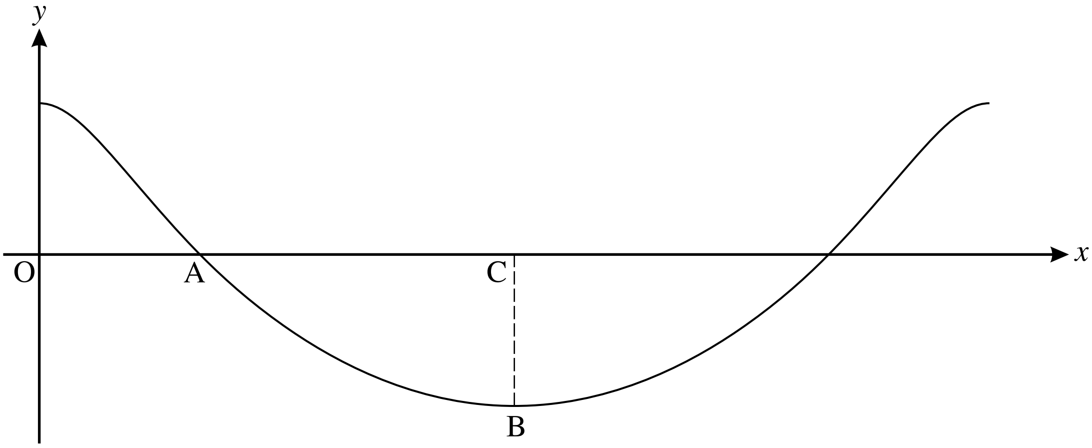

Fig. 8

(i) Find the values of the parameter at A and B.

Hence show that the ratio of the lengths OA and AC is $(\pi - 1) : (\pi + 1)$. **[5]**

(ii) Find $\frac{dy}{dx}$ in terms of θ. Find the gradient of the track at A. **[4]**

(iii) Show that, when the gradient of the track is 1, θ satisfies the equation

$$\cos\theta - 4\sin\theta = 2.$$ **[2]**

(iv) Express $\cos\theta - 4\sin\theta$ in the form $R\cos(\theta + \alpha)$.

Hence solve the equation $\cos\theta - 4\sin\theta = 2$ for $0 \leqslant \theta \leqslant 2\pi$. **[7]**