



RECOGNISING ACHIEVEMENT

## ADVANCED GCE

### MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

**4756**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

- Scientific or graphical calculator

**Friday 11 June 2010**

**Morning**

**Duration:** 1 hour 30 minutes



\* 4 7 5 6 \*

#### INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

#### INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

## Section A (54 marks)

## Answer all the questions

1 (a) (i) Given that  $f(t) = \arcsin t$ , write down an expression for  $f'(t)$  and show that

$$f''(t) = \frac{t}{(1-t^2)^{\frac{3}{2}}}. \quad [3]$$

(ii) Show that the Maclaurin expansion of the function  $\arcsin(x + \frac{1}{2})$  begins

$$\frac{\pi}{6} + \frac{2}{\sqrt{3}}x,$$

and find the term in  $x^2$ .

[5]

(b) Sketch the curve with polar equation  $r = \frac{\pi a}{\pi + \theta}$ , where  $a > 0$ , for  $0 \leq \theta < 2\pi$ .

Find, in terms of  $a$ , the area of the region bounded by the part of the curve for which  $0 \leq \theta \leq \pi$  and the lines  $\theta = 0$  and  $\theta = \pi$ .

[6]

(c) Find the exact value of the integral

$$\int_0^{\frac{3}{2}} \frac{1}{9+4x^2} dx. \quad [5]$$

2 (a) Given that  $z = \cos \theta + j \sin \theta$ , express  $z^n + \frac{1}{z^n}$  and  $z^n - \frac{1}{z^n}$  in simplified trigonometric form.

Hence find the constants  $A, B, C$  in the identity

$$\sin^5 \theta \equiv A \sin \theta + B \sin 3\theta + C \sin 5\theta. \quad [5]$$

(b) (i) Find the 4th roots of  $-9j$  in the form  $re^{j\theta}$ , where  $r > 0$  and  $0 < \theta < 2\pi$ . Illustrate the roots on an Argand diagram.

[6]

(ii) Let the points representing these roots, taken in order of increasing  $\theta$ , be P, Q, R, S. The mid-points of the sides of PQRS represent the 4th roots of a complex number  $w$ . Find the modulus and argument of  $w$ . Mark the point representing  $w$  on your Argand diagram. [5]

3 (a) (i) A  $3 \times 3$  matrix  $\mathbf{M}$  has characteristic equation

$$2\lambda^3 + \lambda^2 - 13\lambda + 6 = 0.$$

Show that  $\lambda = 2$  is an eigenvalue of  $\mathbf{M}$ . Find the other eigenvalues. [4]

(ii) An eigenvector corresponding to  $\lambda = 2$  is  $\begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$ .

Evaluate  $\mathbf{M} \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$  and  $\mathbf{M}^2 \begin{pmatrix} 1 \\ -1 \\ \frac{1}{3} \end{pmatrix}$ .

Solve the equation  $\mathbf{M} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$ . [5]

(iii) Find constants  $A, B, C$  such that

$$\mathbf{M}^4 = A\mathbf{M}^2 + B\mathbf{M} + C\mathbf{I}.$$

[4]

(b) A  $2 \times 2$  matrix  $\mathbf{N}$  has eigenvalues  $-1$  and  $2$ , with eigenvectors  $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$  and  $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$  respectively. Find  $\mathbf{N}$ . [6]

## Section B (18 marks)

### Answer one question

#### Option 1: Hyperbolic functions

4 (i) Prove, using exponential functions, that

$$\sinh 2x = 2 \sinh x \cosh x.$$

Differentiate this result to obtain a formula for  $\cosh 2x$ . [4]

(ii) Sketch the curve with equation  $y = \cosh x - 1$ .

The region bounded by this curve, the  $x$ -axis, and the line  $x = 2$  is rotated through  $2\pi$  radians about the  $x$ -axis. Find, correct to 3 decimal places, the volume generated. (You must show your working; numerical integration by calculator will receive no credit.) [7]

(iii) Show that the curve with equation

$$y = \cosh 2x + \sinh x$$

has exactly one stationary point.

Determine, in exact logarithmic form, the  $x$ -coordinate of the stationary point. [7]

*Option 2: Investigation of curves***This question requires the use of a graphical calculator.****5** In parts (i), (ii), (iii) of this question you are required to investigate curves with the equation

$$x^k + y^k = 1$$

for various positive values of  $k$ .(i) Firstly consider cases in which  $k$  is a positive even integer.

- (A) State the shape of the curve when  $k = 2$ .
- (B) Sketch, on the same axes, the curves for  $k = 2$  and  $k = 4$ .
- (C) Describe the shape that the curve tends to as  $k$  becomes very large.
- (D) State the range of possible values of  $x$  and  $y$ .

**[6]**(ii) Now consider cases in which  $k$  is a positive odd integer.

- (A) Explain why  $x$  and  $y$  may take any value.
- (B) State the shape of the curve when  $k = 1$ .
- (C) Sketch the curve for  $k = 3$ . State the equation of the asymptote of this curve.
- (D) Sketch the shape that the curve tends to as  $k$  becomes very large.

**[6]**(iii) Now let  $k = \frac{1}{2}$ .Sketch the curve, indicating the range of possible values of  $x$  and  $y$ .**[2]**(iv) Now consider the modified equation  $|x|^k + |y|^k = 1$ .

- (A) Sketch the curve for  $k = \frac{1}{2}$ .
- (B) Investigate the shape of the curve for  $k = \frac{1}{n}$  as the positive integer  $n$  becomes very large.

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