



**ADVANCED GCE**

**MATHEMATICS (MEI)**

Methods for Advanced Mathematics (C3)

**4753/01**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

- Scientific or graphical calculator

**Friday 11 June 2010**

**Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

## Section A (36 marks)

- 1 Evaluate  $\int_0^{\frac{1}{6}\pi} \cos 3x \, dx$ . [3]
- 2 Given that  $f(x) = |x|$  and  $g(x) = x + 1$ , sketch the graphs of the composite functions  $y = fg(x)$  and  $y = gf(x)$ , indicating clearly which is which. [4]
- 3 (i) Differentiate  $\sqrt{1 + 3x^2}$ . [3]
- (ii) Hence show that the derivative of  $x\sqrt{1 + 3x^2}$  is  $\frac{1 + 6x^2}{\sqrt{1 + 3x^2}}$ . [4]
- 4 A piston can slide inside a tube which is closed at one end and encloses a quantity of gas (see Fig. 4).

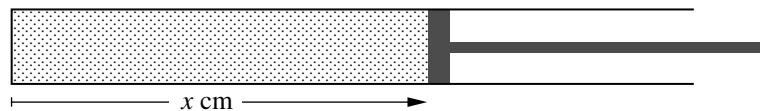


Fig. 4

The pressure of the gas in atmospheric units is given by  $p = \frac{100}{x}$ , where  $x$  cm is the distance of the piston from the closed end. At a certain moment,  $x = 50$ , and the piston is being pulled away from the closed end at 10 cm per minute. At what rate is the pressure changing at that time? [6]

- 5 Given that  $y^3 = xy - x^2$ , show that  $\frac{dy}{dx} = \frac{y - 2x}{3y^2 - x}$ . [7]
- Hence show that the curve  $y^3 = xy - x^2$  has a stationary point when  $x = \frac{1}{8}$ .

- 6 The function  $f(x)$  is defined by

$$f(x) = 1 + 2 \sin 3x, \quad -\frac{\pi}{6} \leq x \leq \frac{\pi}{6}.$$

You are given that this function has an inverse,  $f^{-1}(x)$ .

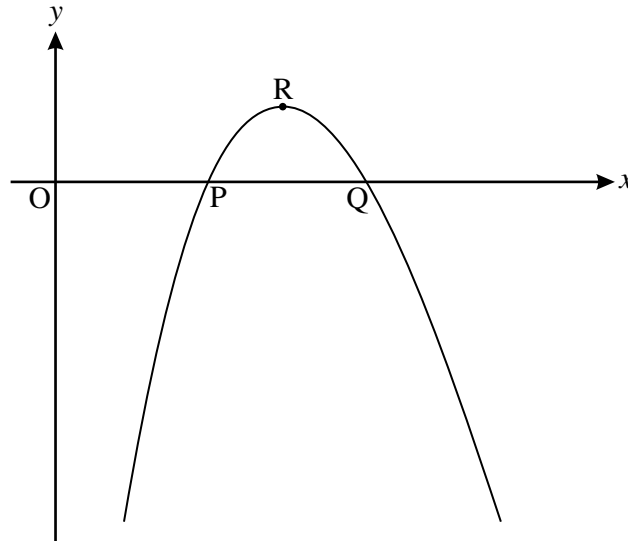
Find  $f^{-1}(x)$  and its domain. [6]

- 7 State whether the following statements are true or false; if false, provide a counter-example. [3]
- (i) If  $a$  is rational and  $b$  is rational, then  $a + b$  is rational.
- (ii) If  $a$  is rational and  $b$  is irrational, then  $a + b$  is irrational.
- (iii) If  $a$  is irrational and  $b$  is irrational, then  $a + b$  is irrational.

**Section B** (36 marks)

- 8 Fig. 8 shows the curve  $y = 3 \ln x + x - x^2$ .

The curve crosses the  $x$ -axis at P and Q, and has a turning point at R. The  $x$ -coordinate of Q is approximately 2.05.



**Fig. 8**

- (i) Verify that the coordinates of P are (1, 0). [1]

- (ii) Find the coordinates of R, giving the  $y$ -coordinate correct to 3 significant figures.

Find  $\frac{d^2y}{dx^2}$ , and use this to verify that R is a maximum point. [9]

- (iii) Find  $\int \ln x \, dx$ .

Hence calculate the area of the region enclosed by the curve and the  $x$ -axis between P and Q, giving your answer to 2 significant figures. [7]

[Question 9 is printed overleaf.]

4

- 9 Fig. 9 shows the curve  $y = f(x)$ , where  $f(x) = \frac{e^{2x}}{1 + e^{2x}}$ . The curve crosses the  $y$ -axis at P.

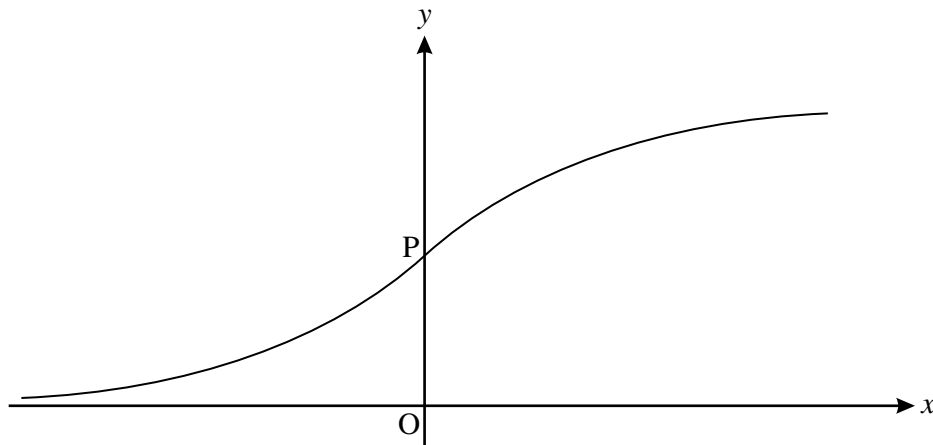


Fig. 9

- (i) Find the coordinates of P. [1]

- (ii) Find  $\frac{dy}{dx}$ , simplifying your answer.

Hence calculate the gradient of the curve at P. [4]

- (iii) Show that the area of the region enclosed by  $y = f(x)$ , the  $x$ -axis, the  $y$ -axis and the line  $x = 1$  is  $\frac{1}{2} \ln\left(\frac{1 + e^2}{2}\right)$ . [5]

The function  $g(x)$  is defined by  $g(x) = \frac{1}{2} \left( \frac{e^x - e^{-x}}{e^x + e^{-x}} \right)$ .

- (iv) Prove algebraically that  $g(x)$  is an odd function.

Interpret this result graphically. [3]

- (v) (A) Show that  $g(x) + \frac{1}{2} = f(x)$ .

(B) Describe the transformation which maps the curve  $y = g(x)$  onto the curve  $y = f(x)$ .

- (C) What can you conclude about the symmetry of the curve  $y = f(x)$ ? [6]

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