

Mathematics (MEI)

Advanced GCE 4767

Statistics 2

Mark Scheme for June 2010

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Question 1

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|----------|---|---|--------------------------------|----|----|----|----|----|----|----|-----|---|----|----|----|---|---|----|----|----------|---|---|---|---|---|---|---|---|----------|---|---|---|---|---|---|---|---|-----|---|---|---|----|----|----|---|---|-------|---|---|---|---|---|----|---|---|---|---|
| (i) | <table><tr><td>x</td><td>6</td><td>17</td><td>9</td><td>20</td><td>13</td><td>15</td><td>11</td><td>14</td></tr><tr><td>y</td><td>6</td><td>13</td><td>10</td><td>11</td><td>9</td><td>7</td><td>12</td><td>15</td></tr><tr><td>Rank x</td><td>8</td><td>2</td><td>7</td><td>1</td><td>5</td><td>3</td><td>6</td><td>4</td></tr><tr><td>Rank y</td><td>8</td><td>2</td><td>5</td><td>4</td><td>6</td><td>7</td><td>3</td><td>1</td></tr><tr><td>d</td><td>0</td><td>0</td><td>2</td><td>-3</td><td>-1</td><td>-4</td><td>3</td><td>3</td></tr><tr><td>d^2</td><td>0</td><td>0</td><td>4</td><td>9</td><td>1</td><td>16</td><td>9</td><td>9</td></tr></table> <p>$\Sigma d^2 = 48$</p> $r_s = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)} = 1 - \frac{6 \times 48}{8 \times 63}$ $= 0.429 \text{ (to 3 s.f.) } [allow 0.43 \text{ to 2 s.f.}]$ | x | 6 | 17 | 9 | 20 | 13 | 15 | 11 | 14 | y | 6 | 13 | 10 | 11 | 9 | 7 | 12 | 15 | Rank x | 8 | 2 | 7 | 1 | 5 | 3 | 6 | 4 | Rank y | 8 | 2 | 5 | 4 | 6 | 7 | 3 | 1 | d | 0 | 0 | 2 | -3 | -1 | -4 | 3 | 3 | d^2 | 0 | 0 | 4 | 9 | 1 | 16 | 9 | 9 | <p>M1 for attempt at ranking (allow all ranks reversed)</p> <p>M1 for d^2</p> <p>A1 CAO for Σd^2</p> <p>M1 for method for r_s</p> <p>A1 f.t. for $r_s < 1$ NB No ranking scores zero</p> | 5 |
| x | 6 | 17 | 9 | 20 | 13 | 15 | 11 | 14 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| y | 6 | 13 | 10 | 11 | 9 | 7 | 12 | 15 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Rank x | 8 | 2 | 7 | 1 | 5 | 3 | 6 | 4 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Rank y | 8 | 2 | 5 | 4 | 6 | 7 | 3 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| d | 0 | 0 | 2 | -3 | -1 | -4 | 3 | 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| d^2 | 0 | 0 | 4 | 9 | 1 | 16 | 9 | 9 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (ii) | <p>H_0: no association between X and Y in the population</p> <p>H_1: some positive association between X and Y in the population</p> <p>One tail test critical value at 5% level is 0.6429</p> <p>Since $0.429 < 0.6429$, there is insufficient evidence to reject H_0,</p> <p>i.e. conclude that there is not enough evidence to show positive association between the two judges' scores.</p> | <p>B1 for H_0</p> <p>B1 for H_1</p> <p>B1 for population SOI</p> <p>NB $H_0 H_1$ <u>not</u> ito p</p> <p>B1 for ± 0.6429</p> <p>M1 for sensible comparison with c.v., provided that $r_s < 1$</p> <p>A1 for conclusion in context f.t. their r_s and sensible cv</p> | 3 3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (iii) | <p>A bivariate Normal distribution is required.</p> <p>Scatter diagram.</p> <p>Suitable discussion</p> | <p>B1</p> <p>G1 labelled axes</p> <p>G1 correct points</p> <p>E1</p> <p>E1</p> | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | TOTAL | 16 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

Question 2

| | | | |
|--------------|---|--|---------------------------------|
| (i) | Counts have a uniform average rate of occurrence | E1 | 2 |
| | All counts are independent | E1 | |
| (ii) | Variance = 3.4 | B1 | 1 |
| (iii) | <p>(A) Either $P(X=3) = 0.5584 - 0.3397 = 0.2187$</p> <p>Or $P(X=3) = e^{-3.4} \frac{3.4^3}{3!} = 0.2186$</p> <p>(B) Using tables: $P(X \geq 3) = 1 - P(X \leq 2)$</p> <p>$= 1 - 0.3397$</p> <p>$= 0.6603$</p> | <p>M1 for use of tables or calculation</p> <p>A1</p> <p>M1 for $1 - P(X \leq 2)$</p> <p>M1 correct use of Poisson tables</p> <p>A1</p> | <p>2</p> <p>3</p> |
| (iv) | <p>$\lambda = 12 \times 3.4 = 40.8$</p> <p>$P(X=40) = e^{-40.8} \frac{40.8^{40}}{40!} = 0.0625$</p> | <p>B1 for mean</p> <p>M1 for calculation</p> <p>A1</p> | 3 |
| (v) | <p>Mean no. per hour = $12 \times 3.4 = 40.8$</p> <p>Using Normal approx. to the Poisson,</p> <p>$X \sim N(40.8, 40.8)$</p> <p>$P(X \geq 40) = P\left(Z > \frac{39.5 - 40.8}{\sqrt{40.8}}\right)$</p> <p>$= P(Z > -0.2035) = \Phi(0.2035)$</p> <p>$= 0.5806$</p> | <p>B1 for Normal approx.</p> <p>B1 for correct parameters (SOI)</p> <p>B1 for correct continuity corr.</p> <p>M1 for probability using correct tail</p> <p>A1 CAO (3 s.f.)</p> | 5 |
| (vi) | <p>Overall mean = 4.8</p> <p>$P(X \geq 8) = 1 - P(X \leq 7)$</p> <p>$= 1 - 0.8867 = 0.1133$</p> | <p>B1 for 4.8</p> <p>M1</p> <p>A1</p> | 3 |
| | | TOTAL | 19 |

Question 3

| | | | |
|-------|---|--|---------------------------------|
| (i) | <p>(A) $P(X < 65) =$ $P\left(Z < \frac{65-63}{5.2}\right)$ $= P(Z < 0.3846)$ $= \Phi(0.3846) = 0.6497$</p> <p>(B) $P(60 < X < 65) = P\left(\frac{60-63}{5.2} < Z < \frac{65-63}{5.2}\right)$ $= P(-0.5769 < Z < 0.3846)$ $= \Phi(0.3846) - (1 - \Phi(0.5769))$ $= 0.6497 - (1 - 0.7181)$ $= 0.3678$</p> | <p>M1 for standardizing</p> <p>M1 for structure A1 CAO (min 3 s.f.), NB When a candidate's answers suggest that (s)he appears to have neglected to use the difference column of the Normal distribution tables penalise the first occurrence only</p> <p>M1 for standardizing both M1 for correct structure</p> <p>A1 CAO 3s.f.</p> | <p>3</p> <p>3</p> |
| (ii) | <p>$P(\text{All 5 between 60 and 65})$ $= 0.3678^5 = 0.00673$</p> | <p>M1 A1 FT (min 2sf)</p> | <p>2</p> |
| (iii) | <p>From tables $\Phi^{-1}(0.95) = 1.645$</p> <p>$\frac{k-63}{5.2} = -1.645$</p> <p>$x = 63 - 5.2 \times 1.645 = 54.45$ mins</p> | <p>B1 for ± 1.645 seen M1 for correct equation in k</p> <p>A1 CAO</p> | <p>3</p> |
| (iv) | <p>$H_0: \mu = 63$ minutes; $H_1: \mu < 63$ minutes. Where μ denotes the population mean time on the new course.</p> <p>Test statistic $= \frac{61.7-63}{5.2/\sqrt{15}} = \frac{-1.3}{1.3426}$ $= -0.968$</p> <p>5% level 1 tailed critical value of $z = 1.645$ $-0.968 > -1.645$ so not significant. There is not sufficient evidence to reject H_0</p> <p>There is insufficient evidence to conclude that the new course results in lower times.</p> | <p>B1 for use of 63 B1 for both correct B1 for definition of μ</p> <p>M1 must include $\sqrt{15}$</p> <p>A1</p> <p>B1 for ± 1.645 M1 for sensible comparison leading to a conclusion</p> <p>A1 FT for correct conclusion in words in context</p> | <p>3</p> <p>5</p> |
| | | | 19 |

Question 4

| (i) | <p>H_0: no association between category of runner and type of running; H_1: some association between category of runner and type of running;</p> <table border="1" data-bbox="172 470 877 622"> <thead> <tr> <th>EXPECTED</th><th>Junior</th><th>Senior</th><th>Veteran</th></tr> </thead> <tbody> <tr> <td>Track</td><td>5.13</td><td>7.84</td><td>6.03</td></tr> <tr> <td>Road</td><td>6.48</td><td>9.90</td><td>7.62</td></tr> <tr> <td>Both</td><td>5.40</td><td>8.25</td><td>6.35</td></tr> </tbody> </table> <table border="1" data-bbox="172 694 877 846"> <thead> <tr> <th>CONTRIBUTN</th><th>Junior</th><th>Senior</th><th>Veteran</th></tr> </thead> <tbody> <tr> <td>Track</td><td>2.9257</td><td>0.0032</td><td>2.6949</td></tr> <tr> <td>Road</td><td>0.9468</td><td>0.3663</td><td>2.5190</td></tr> <tr> <td>Both</td><td>0.3615</td><td>0.3694</td><td>0.0192</td></tr> </tbody> </table> <p>$\chi^2 = 10.21$</p> <p>Refer to χ^2_4</p> <p>Critical value at 5% level = 9.488</p> <p>Result is significant</p> <p>There is evidence to suggest that there is some association between category of runner and type of running. NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1 or final E1</p> | EXPECTED | Junior | Senior | Veteran | Track | 5.13 | 7.84 | 6.03 | Road | 6.48 | 9.90 | 7.62 | Both | 5.40 | 8.25 | 6.35 | CONTRIBUTN | Junior | Senior | Veteran | Track | 2.9257 | 0.0032 | 2.6949 | Road | 0.9468 | 0.3663 | 2.5190 | Both | 0.3615 | 0.3694 | 0.0192 | <p>B1</p> <p>M1 A2 for expected values (to 2 dp) (allow A1 for at least one row or column correct)</p> <p>M1 for valid attempt at $(O-E)^2/E$ A1 for all correct NB These M1/A1 marks cannot be implied by a correct final value of χ^2</p> <p>M1 for summation A1 for χ^2</p> <p>B1 for 4 deg of f B1 CAO for cv</p> <p>B1 FT their 'sensible' χ^2</p> <p>E1 must be consistent with their χ^2</p> | <p>1</p> <p>7</p> <p>4</p> |
|------------|--|--|-----------|--------|---------|-------|------|------|------|------|------|------|------|------|------|------|------|------------|--------|--------|---------|-------|--------|--------|--------|------|--------|--------|--------|------|--------|--------|--------|---|----------------------------|
| EXPECTED | Junior | Senior | Veteran | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Track | 5.13 | 7.84 | 6.03 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Road | 6.48 | 9.90 | 7.62 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Both | 5.40 | 8.25 | 6.35 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CONTRIBUTN | Junior | Senior | Veteran | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Track | 2.9257 | 0.0032 | 2.6949 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Road | 0.9468 | 0.3663 | 2.5190 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Both | 0.3615 | 0.3694 | 0.0192 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (ii) | <ul style="list-style-type: none"> • Juniors appear be track runners more often than expected and road less often than expected. • Seniors tend to be as expected in all three categories of running. • Veterans tend to be road runners more than expected and track runners less than expected. | <p>E1 E1</p> <p>E1 E1</p> <p>E1 E1</p> | <p>6</p> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | | TOTAL | 18 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

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