



GCE

Mathematics (MEI)

Advanced GCE 4767

Statistics 2

Mark Scheme for June 2010

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Question 1

(i)	<table border="1"> <tr><td>x</td><td>6</td><td>17</td><td>9</td><td>20</td><td>13</td><td>15</td><td>11</td><td>14</td></tr> <tr><td>y</td><td>6</td><td>13</td><td>10</td><td>11</td><td>9</td><td>7</td><td>12</td><td>15</td></tr> <tr><td>Rank x</td><td>8</td><td>2</td><td>7</td><td>1</td><td>5</td><td>3</td><td>6</td><td>4</td></tr> <tr><td>Rank y</td><td>8</td><td>2</td><td>5</td><td>4</td><td>6</td><td>7</td><td>3</td><td>1</td></tr> <tr><td>d</td><td>0</td><td>0</td><td>2</td><td>-3</td><td>-1</td><td>-4</td><td>3</td><td>3</td></tr> <tr><td>d^2</td><td>0</td><td>0</td><td>4</td><td>9</td><td>1</td><td>16</td><td>9</td><td>9</td></tr> </table> <p>$\Sigma d^2 = 48$</p> $r_s = 1 - \frac{6 \Sigma d^2}{n(n^2 - 1)} = 1 - \frac{6 \times 48}{8 \times 63}$ $= 0.429 \text{ (to 3 s.f.)} \quad [\text{allow 0.43 to 2 s.f.}]$	x	6	17	9	20	13	15	11	14	y	6	13	10	11	9	7	12	15	Rank x	8	2	7	1	5	3	6	4	Rank y	8	2	5	4	6	7	3	1	d	0	0	2	-3	-1	-4	3	3	d^2	0	0	4	9	1	16	9	9	M1 for attempt at ranking (allow all ranks reversed) M1 for d^2 A1 CAO for Σd^2 M1 for method for r_s A1 f.t. for $ r_s < 1$ NB No ranking scores zero	5
x	6	17	9	20	13	15	11	14																																																	
y	6	13	10	11	9	7	12	15																																																	
Rank x	8	2	7	1	5	3	6	4																																																	
Rank y	8	2	5	4	6	7	3	1																																																	
d	0	0	2	-3	-1	-4	3	3																																																	
d^2	0	0	4	9	1	16	9	9																																																	
(ii)	<p>H_0: no association between X and Y in the population</p> <p>H_1: some positive association between X and Y in the population</p> <p>One tail test critical value at 5% level is 0.6429</p> <p>Since $0.429 < 0.6429$, there is insufficient evidence to reject H_0,</p> <p>i.e. conclude that there is not enough evidence to show positive association between the two judges' scores.</p>	B1 for H_0 B1 for H_1 B1 for population SOI NB $H_0 H_1$ <u>not</u> ito ρ B1 for ± 0.6429 M1 for sensible comparison with c.v., provided that $ r_s < 1$ A1 for conclusion in context f.t. their r_s and sensible cv	3																																																						
(iii)	<p>A bivariate Normal distribution is required.</p> <p>Scatter diagram.</p> <p>Suitable discussion</p>	B1 G1 labelled axes G1 correct points E1 E1	5																																																						
		TOTAL	16																																																						

Question 2

(i)	Counts have a uniform average rate of occurrence All counts are independent	E1 E1	2
(ii)	Variance = 3.4	B1	1
(iii)	(A) Either $P(X=3) = 0.5584 - 0.3397 = 0.2187$ Or $P(X=3) = e^{-3.4} \frac{3.4^3}{3!} = 0.2186$ (B) Using tables: $P(X \geq 3) = 1 - P(X \leq 2)$ $= 1 - 0.3397$ $= 0.6603$	M1 for use of tables or calculation A1 M1 for $1 - P(X \leq 2)$ M1 correct use of Poisson tables A1	2 3
(iv)	$\lambda = 12 \times 3.4 = 40.8$ $P(X=40) = e^{-40.8} \frac{40.8^{40}}{40!} = 0.0625$	B1 for mean M1 for calculation A1	3
(v)	Mean no. per hour = $12 \times 3.4 = 40.8$ Using Normal approx. to the Poisson, $X \sim N(40.8, 40.8)$ $P(X \geq 40) = P\left(Z > \frac{39.5 - 40.8}{\sqrt{40.8}}\right)$ $= P(Z > -0.2035) = \Phi(0.2035)$ $= 0.5806$	B1 for Normal approx. B1 for correct parameters (SOI) B1 for correct continuity corr. M1 for probability using correct tail A1 CAO (3 s.f.)	5
(vi)	Overall mean = 4.8 $P(X \geq 8) = 1 - P(X \leq 7)$ $= 1 - 0.8867 = 0.1133$	B1 for 4.8 M1 A1	3
		TOTAL	19

Question 3

(i)	<p>(A) $P(X < 65) =$ $P\left(Z < \frac{65-63}{5.2}\right)$ $= P(Z < 0.3846)$ $= \Phi(0.3846) = 0.6497$</p> <p>(B) $P(60 < X < 65) = P\left(\frac{60-63}{5.2} < Z < \frac{65-63}{5.2}\right)$ $= P(-0.5769 < Z < 0.3846)$ $= \Phi(0.3846) - (1 - \Phi(0.5769))$ $= 0.6497 - (1 - 0.7181)$ $= 0.3678$</p>	<p>M1 for standardizing</p> <p>M1 for structure</p> <p>A1 CAO (min 3 s.f.), NB When a candidate's answers suggest that (s)he appears to have neglected to use the difference column of the Normal distribution tables penalise the first occurrence only</p> <p>M1 for standardizing both</p> <p>M1 for correct structure</p> <p>A1 CAO 3s.f.</p>	3
(ii)	<p>$P(\text{All 5 between 60 and 65})$ $= 0.3678^5 = 0.00673$</p>	<p>M1 A1 FT (min 2sf)</p>	2
(iii)	<p>From tables $\Phi^{-1}(0.95) = 1.645$</p> $\frac{k-63}{5.2} = -1.645$ $x = 63 - 5.2 \times 1.645 = 54.45 \text{ mins}$	<p>B1 for ± 1.645 seen</p> <p>M1 for correct equation in k</p> <p>A1 CAO</p>	3
(iv)	<p>$H_0: \mu = 63$ minutes; $H_1: \mu < 63$ minutes. Where μ denotes the population mean time on the new course.</p> <p>Test statistic $= \frac{61.7 - 63}{5.2 / \sqrt{15}} = \frac{-1.3}{1.3426} = -0.968$</p> <p>5% level 1 tailed critical value of $z = 1.645$ $-0.968 > -1.645$ so not significant. There is not sufficient evidence to reject H_0</p> <p>There is insufficient evidence to conclude that the new course results in lower times.</p>	<p>B1 for use of 63</p> <p>B1 for both correct</p> <p>B1 for definition of μ</p> <p>M1 must include $\sqrt{15}$</p> <p>A1</p> <p>B1 for ± 1.645</p> <p>M1 for sensible comparison leading to a conclusion</p> <p>A1 FT for correct conclusion in words in context</p>	5
			19

Question 4

(i)	H_0 : no association between category of runner and type of running; H_1 : some association between category of runner and type of running;	B1	1																
	<table border="1" data-bbox="173 469 863 624"> <thead> <tr> <th>EXPECTED</th><th>Junior</th><th>Senior</th><th>Veteran</th></tr> </thead> <tbody> <tr> <td>Track</td><td>5.13</td><td>7.84</td><td>6.03</td></tr> <tr> <td>Road</td><td>6.48</td><td>9.90</td><td>7.62</td></tr> <tr> <td>Both</td><td>5.40</td><td>8.25</td><td>6.35</td></tr> </tbody> </table>	EXPECTED	Junior	Senior	Veteran	Track	5.13	7.84	6.03	Road	6.48	9.90	7.62	Both	5.40	8.25	6.35	M1 A2 for expected values (to 2 dp) (allow A1 for at least one row or column correct)	
EXPECTED	Junior	Senior	Veteran																
Track	5.13	7.84	6.03																
Road	6.48	9.90	7.62																
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	<table border="1" data-bbox="173 691 863 846"> <thead> <tr> <th>CONTRIBUTN</th><th>Junior</th><th>Senior</th><th>Veteran</th></tr> </thead> <tbody> <tr> <td>Track</td><td>2.9257</td><td>0.0032</td><td>2.6949</td></tr> <tr> <td>Road</td><td>0.9468</td><td>0.3663</td><td>2.5190</td></tr> <tr> <td>Both</td><td>0.3615</td><td>0.3694</td><td>0.0192</td></tr> </tbody> </table>	CONTRIBUTN	Junior	Senior	Veteran	Track	2.9257	0.0032	2.6949	Road	0.9468	0.3663	2.5190	Both	0.3615	0.3694	0.0192	M1 for valid attempt at $(O-E)^2/E$ A1 for all correct NB These M1A1 marks cannot be implied by a correct final value of X^2	
CONTRIBUTN	Junior	Senior	Veteran																
Track	2.9257	0.0032	2.6949																
Road	0.9468	0.3663	2.5190																
Both	0.3615	0.3694	0.0192																
	$X^2 = 10.21$ Refer to X^2 Critical value at 5% level = 9.488 Result is significant There is evidence to suggest that there is some association between category of runner and type of running. NB if H_0 H_1 reversed, or 'correlation' mentioned, do not award first B1 or final E1	M1 for summation A1 for X^2 B1 for 4 deg of f B1 CAO for cv B1 FT their 'sensible' X^2 E1 must be consistent with their X^2	7																
(ii)	<ul style="list-style-type: none"> Juniors appear be track runners more often than expected and road less often than expected. Seniors tend to be as expected in all three categories of running. Veterans tend to be road runners more than expected and track runners less than expected. 	E1 E1 E1 E1 E1 E1	4																
		TOTAL	18																

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