OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **4777**

Numerical Computation

Mark Scheme for June 2010

Oxford Cambridge and RSA Examinations

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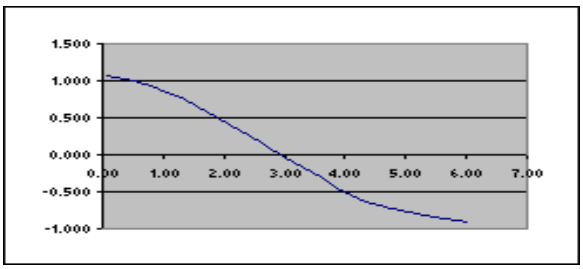
1 (i) The data are not evenly spaced so (ordinary) differences will not work **[E1]**
Lagrange's method is not well suited to increasing the degree of the **[E1]**
approximating polynomial because it requires complete recalculation **[E1]**
[subtotal 3]

(ii)

x	f
0.09	1.076
0.93	0.897
1.91	0.498
4.10	-0.544
4.91	-0.740
6.04	-0.900



[G2]
[subtotal 2]

(iii)

x	f	1DD	2DD	3DD	4DD	5DD
1.91	0.498					
4.10	-0.544	-0.4758				
4.91	-0.740	-0.24198	0.077941			
0.93	0.897	-0.41131	0.053417	0.025025		
0.09	1.076	-0.2131	-0.04112	0.023576	0.000796	
6.04	-0.900	-0.3321	-0.02329	0.015782	-0.00402	-0.00117

re-order: **[M1A1]**
table: **[M1A1]**

f(3)				
=	0.498			
+	-0.51862	-0.021	linear	**[M1A1]**
+	-0.09345	-0.114	quadratic	**[M1A1]**
+	0.057309	-0.057	cubic	**[M1A1]**
+	0.003774	-0.053	quartic	**[M1A1]**

f(3) approximately zero, but difficult to say whether -0.05 or -0.06, -0.1 or 0.0. **[E1E1]**
[subtotal 14]

(iv)

x	f	1DD	2DD	3DD	4DD	5DD
1.91	0.498					
4.10	-0.544	-0.4758				
4.91	-0.740	-0.24198	0.077941			
0.93	0.897	-0.41131	0.053417	0.025025		
0.09	1.076	-0.2131	-0.04112	0.023576	0.000796	
6.04	-0.900	-0.3321	-0.02329	0.015782	-0.00402	-0.00117

	user-specified x:	2.89	0.498			*adjust SS to allow*
			-0.46628	0.032		*user-specified x:* **[M1A1]**
			-0.09242	-0.061		
			0.056679	-0.004		*trial and error:* **[M1A1]**
			0.003738	0.000		*answer:* **[A1]**

[subtotal 5]
[TOTAL 24]

2 (i) $T_n - I = A_2h^2 + A_4h^4 + A_6h^6 + \ldots$

$T_{2n} - I = A_2(h/2)^2 + A_4(h/2)^4 + A_6(h/2)^6 + \ldots$ **[M1A1]**

$4(T_{2n} - I) - (T_n - I) = b_4h^4 + b_6h^6 + \ldots$ **[M1]**

$4T_{2n} - T_n - 3I = b_4h^4 + b_6h^6 + \ldots$ **[A1]**

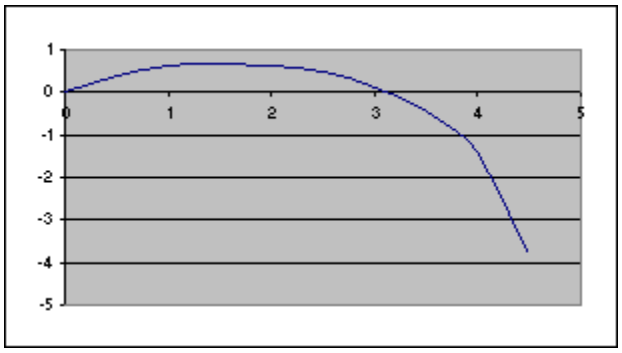
$(4T_{2n} - T_n)/3 - I = B_4h^4 + B_6h^6 + \ldots$ **[A1]**

($T_n^* = (4T_{2n} - T_n)/3$ has error of order h^4 as given)

$T_n^{**} = (16T_{2n}^* - T_n^*)/15$ has error of order h^6 **[B1]**

[subtotal 6]

(ii)



[G2]

[subtotal 2]

(iii)

x	f(x)	T	T*	T**	T***	(T****)
0	0					
3.141593	2.22E-16	3.49E-16				
1.570796	0.693147	1.088793	1.451724			
0.785398	0.5348					
2.356194	0.5348	1.384458	1.483014	1.485099		
0.392699	0.324026					
1.178097	0.654344					
1.963495	0.654344					
2.748894	0.324026	1.460639	1.486033	1.486234	1.486252	
0.19635	0.178222					
0.589049	0.441842					
0.981748	0.605119					
1.374447	0.683493					
1.767146	0.683493					
2.159845	0.605119					
2.552544	0.441842					
2.945243	0.178222	1.479855	1.48626	1.486275	1.486276	1.486276

f: **[A1]**

T: **[M1A2]**

T:* **[M1A1]**

*T**:* **[M1A1]**

*T**** **[M1A1]**

answer: **[A1]**

[subtotal 11]

(iv) Spreadsheet as above, but seen to work for user-specified c in place of 3.141593 **[M2]**

Sequence of values representing trial and error towards solution:

c	4	4.5	4.4	4.45	4.44	4.442
I	0.977343	-0.20713	0.133659	-0.02687	0.006681	0.00003

[M1A1]

Answer 4.442 to 3 decimal places **[A1]**

[subtotal 5]

[TOTAL 24]

3 (i) Modified Euler method

h	x	y	k1	k2	new y
0.1	1	1	0.141421	0.150185	1.145803
	1.1	1.145803	0.150346	0.159856	1.300904
	1.2	1.300904	0.160034	0.170271	1.466056
	1.3	1.466056	0.170466	0.181415	1.641997
	1.4	1.641997	0.181626	0.193273	1.829446
	1.5	1.829446	0.193499	0.205833	2.029112
	1.6	2.029112	0.206072	0.219085	2.24169
	1.7	2.24169	0.219337	0.23302	2.467869
	1.8	2.467869	0.233284	0.247633	2.708328
	1.9	2.708328	0.247908	0.262916	2.963739
	2	2.963739			

setup: **[M2]**

first run: **[A2]**

h	α	diffs	ratio of diffs
0.1	2.963739		
0.05	2.964219	0.000480	
0.025	2.964341	0.000122	0.254789
0.0125	2.964372	0.000031	0.252418
0.00625	2.964380	0.000008	0.251215

further runs: **[A1A1A1]**

differences: **[M1]**

ratios: **[M1A1]**

Correct to 4 dp, α = 2.9644 **[A1]**

Ratio of differences indicates 2nd order convergence **[E1]**

[subtotal 12]

(ii) Predictor corrector method

h	x	y	y pred	y corr1	y corr2	y corr3
0.1	1	1	1.141421	1.145803	1.145884	1.145885
	1.1	1.145885	1.296234	1.300989	1.301078	1.30108
	1.2	1.30108	1.46112	1.466239	1.466336	1.466338
	1.3	1.466338	1.636815	1.64229	1.642395	1.642397
	1.4	1.642397	1.824039	1.829862	1.829975	1.829978
	1.5	1.829978	2.023497	2.029664	2.029784	2.029786
	1.6	2.029786	2.235885	2.242392	2.242518	2.24252
	1.7	2.24252	2.461889	2.468732	2.468864	2.468866
	1.8	2.468866	2.702189	2.709364	2.709501	2.709504
	1.9	2.709504	2.957457	2.964961	2.965104	2.965107
	2	2.965107				

setup: **[M2]**

first run: **[A2]**

h	α	diffs	ratio of diffs
0.1	2.965107		
0.05	2.964564	-0.000543	
0.025	2.964428	-0.000136	0.250154
0.0125	2.964394	-0.000034	0.250039
0.00625	2.964385	-0.000008	0.25001

these --> may appear in (iii)

further runs: **[A1A1A1]**

differences and ratios: **[M1]**

[subtotal 8]

(iii) The rate of convergence (see ratio of differences) is the same for both methods. **[E1]**

Magnitude of errors about the same for a given h **[E1]**

More programming required for predictor-corrector **[E1]**

Modified Euler (at least in this case) is preferable **[E1]**

[subtotal 4]

[TOTAL 24]

4 (i)

7.1	6	5	4	1	x1 =	0.320827	*Gauss elim:*
6	5.1	4	3	1			**[M2A2]**
5	4	3.1	2	1			*pivoting:*
4	3	2	1.1	1			**[M1A2]**
	0.029577	-0.22535	-0.38028	0.15493			
	-0.22535	-0.42113	-0.8169	0.295775			*back subn:*
	-0.38028	-0.8169	-1.15352	0.43662	x2 =	0.103317	**[M1A2]**
		-0.28889	-0.47	0.188889	x3 =	-0.11419	
		0.062963	-0.13333	0.037037			*solutions:*
			-0.23577	0.078205	x4 =	-0.3317	**[A2]**

product of pivots: -0.18390 magnitude of determinant: 0.18390 **[M1A1]**

[subtotal 14]

(ii) α = 0.01 β = 0.01

7.01	6	5	4	1.01	x1 =	0.599796
6	5.01	4	3	1		
5	4	3.01	2	1		
4	3	2	1.01	1		
	-0.12552	-0.2796	-0.42368	0.135521		
	-0.2796	-0.55633	-0.85307	0.279601		
	-0.42368	-0.85307	-1.27245	0.42368	x2 =	-0.2999
		-0.02687	-0.0467	0.01	x3 =	-0.1996
		0.006633	-0.01333	0		
			-0.02486	0.002469	x4 =	-0.09929

product of pivots: -0.00198 magnitude of determinant: 0.001984 **[M1A1]**

α = 0.01	(A) β = 0	(B)β = 0.1	
x1	0.302	0.600	*solutions:*
x2	0.100	-0.300	**[M1A1]**
x3	-0.101	-0.200	**[M1A1]**
x4	-0.303	-0.099	

Very large changes in the solution for small change in one coefficient. **[E1E1]**

The determinant is very small in relation to the magnitude of the coefficients. **[E1E1]**

[subtotal 10]

[TOTAL 24]

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