OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4756

Further Methods for Advanced Mathematics (FP2)

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 10 January 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (54 marks)

Answer all the questions

1 **(a)** A curve has polar equation $r = 2(\cos\theta + \sin\theta)$ for $-\frac{1}{4}\pi \leqslant \theta \leqslant \frac{3}{4}\pi$.

(i) Show that a cartesian equation of the curve is $x^2 + y^2 = 2x + 2y$. Hence or otherwise sketch the curve. **[5]**

(ii) Find, by integration, the area of the region bounded by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{2}\pi$. Give your answer in terms of π. **[7]**

(b) **(i)** Given that $f(x) = \arctan\left(\frac{1}{2}x\right)$, find $f'(x)$. **[2]**

(ii) Expand $f'(x)$ in ascending powers of x as far as the term in x^4.

Hence obtain an expression for $f(x)$ in ascending powers of x as far as the term in x^5. **[5]**

2 **(a)** **(i)** Given that $z = \cos\theta + \mathrm{j}\sin\theta$, express $z^n + z^{-n}$ and $z^n - z^{-n}$ in simplified trigonometrical form. **[2]**

(ii) By considering $\left(z + z^{-1}\right)^6$, show that

$$\cos^6\theta = \tfrac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10).$$ **[3]**

(iii) Obtain an expression for $\cos^6\theta - \sin^6\theta$ in terms of $\cos 2\theta$ and $\cos 6\theta$. **[5]**

(b) The complex number w is $8\mathrm{e}^{\mathrm{j}\pi/3}$. You are given that z_1 is a square root of w and that z_2 is a cube root of w. The points representing z_1 and z_2 in the Argand diagram both lie in the third quadrant.

(i) Find z_1 and z_2 in the form $r\mathrm{e}^{\mathrm{j}\theta}$. Draw an Argand diagram showing w, z_1 and z_2. **[6]**

(ii) Find the product $z_1 z_2$, and determine the quadrant of the Argand diagram in which it lies. **[3]**

3 **(i)** Show that the characteristic equation of the matrix

$$\mathbf{M} = \begin{pmatrix} 1 & -4 & 5 \\ 2 & 3 & -2 \\ -1 & 4 & 1 \end{pmatrix}$$

is $\lambda^3 - 5\lambda^2 + 28\lambda - 66 = 0$. **[4]**

(ii) Show that $\lambda = 3$ is an eigenvalue of $\mathbf{M}$, and determine whether or not $\mathbf{M}$ has any other real eigenvalues. **[4]**

(iii) Find an eigenvector, $\mathbf{v}$, of unit length corresponding to $\lambda = 3$.

State the magnitude of the vector $\mathbf{M}^n\mathbf{v}$, where n is an integer. **[5]**

(iv) Using the Cayley-Hamilton theorem, obtain an equation for $\mathbf{M}^{-1}$ in terms of $\mathbf{M}^2$, $\mathbf{M}$ and $\mathbf{I}$. **[3]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Solve the equation

$$\sinh t + 7\cosh t = 8,$$

expressing your answer in exact logarithmic form. **[6]**

A curve has equation $y = \cosh 2x + 7\sinh 2x$.

(ii) Using part **(i)**, or otherwise, find, in an exact form, the coordinates of the points on the curve at which the gradient is 16.

Show that there is no point on the curve at which the gradient is zero.

Sketch the curve. **[8]**

(iii) Find, in an exact form, the positive value of a for which the area of the region between the curve, the x-axis, the y-axis and the line $x = a$ is $\frac{1}{2}$. **[4]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 A curve has parametric equations

$$x = t + a\sin t, \qquad y = 1 - a\cos t,$$

where a is a positive constant.

(i) Draw, on separate diagrams, sketches of the curve for $-2\pi < t < 2\pi$ in the cases $a = 1$, $a = 2$ and $a = 0.5$.

By investigating other cases, state the value(s) of a for which the curve has

(*A*) loops,

(*B*) cusps. **[7]**

(ii) Suppose that the point P (x, y) lies on the curve. Show that the point P′ $(-x, y)$ also lies on the curve. What does this indicate about the symmetry of the curve? **[3]**

(iii) Find an expression in terms of a and t for the gradient of the curve. Hence find, in terms of a, the coordinates of the turning points on the curve for $-2\pi < t < 2\pi$ and $a \neq 1$. **[5]**

(iv) In the case $a = \frac{1}{2}\pi$, show that $t = \frac{1}{2}\pi$ and $t = \frac{3}{2}\pi$ give the same point. Find the angle at which the curve crosses itself at this point. **[3]**