OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) **4758/01**

Differential Equations

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 26 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(a)** The displacement, x m, of a particle at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 4e^t.$$

(i) Find the general solution. **[9]**

The particle is initially at rest at the origin.

(ii) Find the particular solution. **[4]**

(b) The differential equation

$$\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

is to be solved.

(i) Show that 1 is a root of the auxiliary equation and find the other two roots. Hence find the general solution. **[5]**

When $x = 0$, $y = 1$ and $\frac{dy}{dx} = -4$. As $x \to \infty$, $y \to 0$.

(ii) Find the particular solution subject to these conditions. **[4]**

(iii) Find the value of x for which $y = 0$. **[2]**

2 **(a)** The differential equation

$$\frac{dy}{dx} + 2xy = e^{-x^2} \sin x$$

is to be solved subject to the condition $x = 0$, $y = 1$.

(i) Find the particular solution for y in terms of x. **[9]**

(ii) Show that $y > 0$ for all x and that y has a stationary point when $x = 0$. State the limiting value of y as $|x| \to \infty$. Hence draw a simple sketch graph of the solution, given that the stationary point at $x = 0$ is a maximum. **[6]**

(b) The differential equation

$$\frac{dy}{dx} + 2xy = 1$$

is to be solved numerically subject to the condition $x = 0$, $y = 1$.

(i) Use Euler's method with a step length of 0.1 to estimate y when $x = 0.2$. The algorithm is given by $x_{r+1} = x_r + h$, $y_{r+1} = y_r + hy'_r$. **[4]**

(ii) Use the integrating factor method and the approximation $\int_0^{0.2} e^{x^2}\, dx \approx 0.2027$ to estimate y when $x = 0.2$. **[5]**

3 The differential equation

$$\frac{dy}{dx} + ky = \cos 3x,$$

where k is a constant, is to be solved.

(i) Find the complementary function. Hence find the general solution for y in terms of x and k. **[8]**

(ii) Find the particular solution subject to the condition that $\frac{dy}{dx} = 1$ when $x = 0$. **[4]**

Now consider the differential equation

$$\frac{dy}{dx} + ky = 2e^{-kx}.$$

(iii) Find the general solution. **[6]**

Now consider the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 4e^{2x}.$$

(iv) Using your answer to part **(iii)**, or otherwise, solve this differential equation subject to the conditions that $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$. **[6]**

4 The populations of foxes, x, and rabbits, y, on an island at time t are modelled by the simultaneous differential equations

$$\frac{dx}{dt} = 0.1x + 0.1y,$$

$$\frac{dy}{dt} = -0.2x + 0.3y.$$

(i) Show that $\frac{d^2x}{dt^2} - 0.4\frac{dx}{dt} + 0.05x = 0.$ **[5]**

(ii) Find the general solution for x. **[4]**

(iii) Find the corresponding general solution for y. **[4]**

Initially there are x_0 foxes and y_0 rabbits.

(iv) Find the particular solutions. **[4]**

(v) In the case $y_0 = 10x_0$, find the time at which the model predicts the rabbits will die out. Determine whether the model predicts the foxes die out before the rabbits. **[7]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.