



GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit 4776: Numerical Methods

Mark Scheme for January 2011

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Question		Answer	Marks	Guidance												
1	(i)	$ \begin{array}{ccccc} x & \text{LHS} & & \text{RHS} \\ 1 & 2 & > & 1.557408 \\ 1.2 & 2.2 & < & 2.572152 \end{array} $	M1 A1 [2]	no explicit explanation required												
	(ii)	$ \begin{array}{cccccc} r & 0 & 1 & 2 & 3 & 4 \\ x_r & 1.1 & 0.96476 & 0.442927 & -0.52564 & -1.58007 \end{array} $	M1 A1 [2]	$r = 3$ required												
	(iii)	<p>e.g. re-arrange to $x = \arctan(1 + x)$</p> $ \begin{array}{ccccccccc} r & 0 & 1 & 2 & 3 & 4 & 5 \\ x_r & 1.1 & 1.126377 & 1.131203 & 1.132076 & 1.132233 & 1.132261 & 1.132 \end{array} $	B1 M1 A1 A1 [4]													
2		<table border="1"> <thead> <tr> <th>h</th> <th>M</th> <th>T</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>1.987467</td> <td>1.354440</td> </tr> <tr> <td>1</td> <td>1.830595</td> <td>1.670954</td> </tr> <tr> <td>0.5</td> <td></td> <td>1.750774</td> </tr> </tbody> </table> <p>Simpson's rule $(2M + T) / 3$ 1.776458 1.777381</p> <p>Reference to justification/accuracy : 1.777 or 1.78</p>	h	M	T	2	1.987467	1.354440	1	1.830595	1.670954	0.5		1.750774	T: M1A1A1 S: M1A1A1 E1 A1 [8]	Lose 1 for any additional 'answer'(s) but do not penalise extrapolation
h	M	T														
2	1.987467	1.354440														
1	1.830595	1.670954														
0.5		1.750774														
3	(i)	$ \begin{array}{ll} h = 1 & g'(0) = (2.0100 - 1.4509)/1 = 0.5591 \\ h = 0.5 & g'(0) = (1.6799 - 1.4509)/0.5 = 0.458 \end{array} $ <p>Estimate with smaller h (0.458) likely to be more accurate: smaller h is more accurate (provided there is no great loss of significant figures)</p>	B1 B1 B1 E1 [4]													
	(ii)	$ \begin{array}{l} h = 0.5 \ g'(0.5) = (2.0100 - 1.4509)/1 = 0.5591 \\ \text{This estimate, central diff, likely to be more accurate than either of the forward diffs} \end{array} $	M1 E1 [2]													

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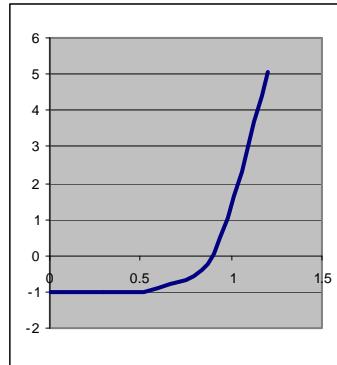
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Question		Answer	Marks	Guidance
4	(i)	Max poss loss: 365 (or 366) times 0.01 pence: = 3.65 (or 3.66) pence Arises if each daily amount would round up but gets chopped down Average loss 1.825 (or 1.83) pence, because average is half of max.	B1 E1 B1 E1 [4]	
	(ii)	£150 000 divided by 1.825 pence: about 8.2 million (8 million) accounts	M1 A1 [2]	
5		$ \begin{array}{cccccc} x & P(x) & \Delta P(x) & \Delta^2 P(x) & \Delta^3 P(x) & \\ -1 & -11 & & & & \text{(i) bold:} \\ 1 & -10 & 1 & & & \text{Diff table} \\ 3 & 3 & 13 & 12 & & \text{3rd diffs constant} \\ 5 & 44 & 41 & 28 & 16 & \text{so cubic} \\ 7 & 129 & 85 & 44 & 16 & \text{(ii) italic:} \\ 9 & 274 & 145 & 60 & 16 & \text{working forwards} \\ 11 & 495 & 221 & 76 & 16 & \text{working backwards} \end{array} $	M1 A1 E1 B1 M1 A1 M1 A1 [4] + [4]	
6	(i)	$ \begin{array}{cccccccc} x & f & g & h & \text{abs err } g & \text{rel err } g & \text{abs err } h & \text{rel err } h \\ 0.2 & 0.013351 & 0.013333 & 0.013423 & 0.0000179 & -0.0013424 & 0.0000716 & 0.0053600 \\ 0.1 & 0.003334 & 0.003333 & 0.003339 & 0.0000011 & -0.0003339 & 0.0000045 & 0.0013350 \end{array} $ A1 A1 A1 A1 A1 A1 A1	abs M1 rel M1 [9]	f, g, h values may be implied
	(ii)	Errors in g and h are of opposite sign; g is about 4 times as accurate as h. $ \begin{array}{ccccc} x & f & (4g+h)/5 & \text{abs err} & \text{rel err} \\ 0.2 & 0.013351 & 0.013351 & -2.5E-08 & -1.9E-06 \\ 0.1 & 0.003334 & 0.003334 & -4E-10 & -1.2E-07 \end{array} $ A1 A1 A1	E1 E1 M1 [6]	
	(iii)	$x / \sin x \approx 1.000\ 000\ 002 \approx 1$ $g(10^{-4}) = 3.33 \times 10^{-9}$ Subtraction of nearly equal quantities	B1 B1 E1 [3]	

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Question		Answer	Marks	Guidance																
7	(i)	<p>$f(0) = -1$ $f(1) = 1$ (hence root) $f'(x) = 7x^6 + 5x^4$ which is zero only at $x = 0$. Convincing argument that this is not a turning point No turning points implies no other roots.</p> 	<p>B1 M1 A1 B1 E1 G2</p>																	
	(ii)	<p>NR iteration: $x_{r+1} = x_r - (x_r^7 + x_r^5 - 1) / (7x_r^6 + 5x_r^4)$</p> <table style="margin-left: 100px;"> <tr> <td>r</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>x_r</td> <td>0.6</td> <td>1.51756</td> <td>1.289164</td> </tr> </table> <p>On graph: tangent at 0.6, intersection at 1.5, ordinate & tangent, intersection at 1.3</p>	r	0	1	2	x_r	0.6	1.51756	1.289164	<p>B1 A1 A1 G4 [7]</p>									
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