

Mathematics (MEI)

Advanced GCE

Unit **4763**: Mechanics 3

Mark Scheme for June 2011

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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General marking instructions for MEI Mechanics

Types of marks

- M** A suitable method has been selected and applied in a manner which shows the method is essentially understood. Method marks are not generally lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
- A** Follows **M** mark. Unless modified, it implies that errors already penalized in **earlier** parts of the question will be followed through for **no further loss**. It does **not** imply the following through of errors made in the **current** part of the question. A marks cannot be given unless the associated M mark is earned (including by implication).
- B** Freestanding. Unless modified, it is **cao**.
- E** A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.
- F** Specifies that working should be followed through within a part of a question or emphasizes that follow through is required from an earlier part of a question.

General instructions

1. Mark in red. Correct answers should be ticked, errors which determine marks (usually the first error in each part of a question) should be identified by ringing or by a cross or by underlining or, if an omission, by a caret ^. Do not cross out or obliterate any work. In cases of particular difficulty, brief annotations that explain the marks awarded (or not awarded) are correct or indicate that calculations correctly FT may be helpful should the script be reviewed at a later stage but, in general, comments on the candidate's work or corrections to it should not be written on the script. Each page of the script which includes any part of the candidate's answer must have some indication that it has been seen, e.g. a tick.
2. The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt.
3. Unless otherwise indicated, marks once gained cannot subsequently be lost, (e.g. wrong working following a correct form of answer is ignored). However, this would not apply to a case where a candidate passed through the correct solution as part of a wrong argument, (e.g. when asked for an acceleration, finds the correct vector and then goes on to give just the modulus as the answer).
4. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular M or B mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are run together by the candidate, the earlier marks are implied and full credit must be given.

5. The symbol \surd implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A and B marks are not given for ‘correct’ answers or results obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be agreed at the standardisation meeting.
6. For a partially correct **part** of a question, exhibit the detailed marks, e.g. M1, A0, *in the margin* at the point where the marks have been first earned. Please give sufficient detail to allow your marking to be understood. For a completely correct **part** of a question, only the total mark for that part need be given, *in the margin*. Do not use subtotals (underlined or otherwise). The question total should be ringed and placed in the margin at the end of the question. *This total MUST equal the sum of all the marks in the margin for that question* and should be entered against the question number in the question grid on the front of the script.
7. If a candidate’s answer to a question is in more than one instalment, indicate the carried forward total at the end of one instalment by, for example, ③, and the brought forward total at the start of the next instalment by, for example, ③.
8. The total mark for the paper should be obtained (a) by adding all the unringed marks through the script (checking at the same time that all pages have been marked) and (b) by adding the question marks in the grid in reverse order. The two totals must, of course, tally, and the resulting figure should be written, ringed, on the front of the script.
9. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* get full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.
10. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a single candidate adopts a method which does not correspond to the marking scheme, award marks according to the spirit of the basic scheme; this will usually involve preserving the ratio **M** : **A** + **B** : **F** marks and any emphasis on accuracy specific to that question. If you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.
11. If a GDC is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
12. If in any case the scheme operates with considerable unfairness, mark at discretion but please give a brief reason and initial the mark. *This discretion should be used very rarely, and **only** after consultation with your Team Leader.*
13. If there is any suspicion of cheating or copying, mark according to the scheme and enter the marks on the marksheet as usual. Send the script(s) to your Team Leader, as per OCR instructions. Notes concerning illness etc should be forwarded to OCR, but separately from the scripts or mark sheets. The scripts should be marked exactly as per the scheme, as any special circumstances that may have affected a candidate’s performance are dealt with at a later stage.

14. Examiners are reminded of the *VITAL* importance of checking the accuracy of the addition of marks and of the transcriptions onto the marksheets; in particular that the marks are entered against the right candidates. Do not assume that the scripts are in the same order as the names on the marksheet. As detailed in §8 above, each Examiner must check the paper total, obtaining the same figure twice by different methods. The transcription to the marksheet should also be checked; ideally, the Checker should read out the candidate's name and mark from the marksheet, while the Examiner checks with the front of the script.

The Examiner has final responsibility for the accuracy of the mark recorded on the marksheet.

Tariffs and usage for common errors

Note that these general instructions may be varied in individual questions or papers

Wrong value of g used

No penalty. E marks will be lost except when results agree to the accuracy required in the question.

Misread

Lose 1 mark for each consistent misread. FT except when trivialized. E marks lost unless, by chance, the given results are established by equivalent working.

Units

Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metre unless in a particular question all the lengths are km, when this would be assumed to be the unspecified unit.)

Accuracy

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper

Accept any answer that agrees with the correct value to 2 s.f..

FT should be used so that only one mark is lost for each distinct error, except when for errors due to premature approximation which should be penalised only once in the examination.

Rules for crossed out and/or replaced work

If work is crossed out and not replaced, examiners should mark the crossed out work if it is legible.

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If two or more attempts are made at a question, and just one is not crossed out, examiners should ignore the crossed out work and mark the work that is not crossed out.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

Some commonly used abbreviations

AEF	Any Equivalent Form (of answer or result) is equally acceptable
AG	Answer Given on the question paper (so extra care is needed in checking that the detailed working leading to the result is valid)
BOD	Benefit Of Doubt (allowed for work whose validity may not be absolutely plain)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
FT	Follow through
ISW	Ignore Subsequent Working.
MR	Misread
NOS	Used to indicate an allocation of marks that is Not On Scheme
PA	Premature Approximation (resulting in basically correct work that is numerically insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WW	Without Working
WWW	Without Wrong Working

1 (i)	$\frac{dx}{dt} = A\omega \cos \omega t$ $\frac{d^2x}{dt^2} = -A\omega^2 \sin \omega t$ $= -\omega^2 (A \sin \omega t) = -\omega^2 x$ $\left(\frac{dx}{dt}\right)^2 = A^2 \omega^2 \cos^2 \omega t = A^2 \omega^2 (1 - \sin^2 \omega t)$ $= \omega^2 (A^2 - A^2 \sin^2 \omega t) = \omega^2 (A^2 - x^2)$	B1 M1 E1 M1 E1	Obtaining second derivative Using $\cos^2 \omega t = 1 - \sin^2 \omega t$ 5
(ii)	$1.2^2 = \omega^2 (A^2 - 0.7^2)$ $0.75^2 = \omega^2 (A^2 - 2^2)$ $\frac{A^2 - 0.49}{A^2 - 4} = \frac{1.2^2}{0.75^2}$ $A^2 - 0.49 = 2.56(A^2 - 4)$ $9.75 = 1.56A^2$ $A^2 = 6.25$ Amplitude is 2.5 m $1.44 = \omega^2 (2.5^2 - 0.7^2)$ $\omega = 0.5$ Period is $\frac{2\pi}{\omega} = \frac{2\pi}{0.5}$ $= 4\pi = 12.6 \text{ s} \quad (3 \text{ sf})$	M1 A1 M1 E1 M1 A1	Using $v^2 = \omega^2 (A^2 - x^2)$ Two correct equations (M0 if $x = 7.3$ used, etc) Eliminating ω or Eliminating A or Substituting $A = 2.5$ into both equations Correctly shown Using $\frac{2\pi}{\omega}$ 6
(iii)	Maximum speed is $A\omega = 1.25 \text{ m s}^{-1}$	B1	ft only if greater than 1.2 1
(iv)	Magnitude is $0.5^2 \times 1.6$ $= 0.4 \text{ m s}^{-2}$ Direction is upwards	M1 A1 B1	Accept -0.4 B0 for just 'towards centre' 3
(v)	$x = 2.5 \sin(0.5t)$ When $x = -2$, $t = -1.855$ (or 10.71) When $x = 1$, $t = 0.823$ (or 13.39) Time taken is $0.823 - (-1.855)$ $= 2.68 \text{ s} \quad (3 \text{ sf})$	B1 M1 A1	or $x = 2.5 \cos(0.5t)$ or $t = (\pm) 4.996$ or $t = (\pm) 2.319$ Correct strategy for finding time (must use radians) (ft is $1.3388 / \omega$) 3

2(a)(i)	$0.6 + 0.2 \times 9.8 = 0.2 \times \frac{u^2}{3.2}$ <p>Speed is 6.4 m s^{-1}</p>	M1 A1 A1 3	For acceleration $\frac{u^2}{3.2}$
(ii)	<p>(A)</p> $\frac{1}{2} m(v^2 - u^2) = m \times 9.8(3.2 + 3.2 \cos 60^\circ)$ $v^2 = 135.04$ <p>Radial component is $\frac{v^2}{3.2} = 42.2 \text{ m s}^{-2}$</p> <p>Tangential component is $g \sin 60^\circ$ $= 8.49 \text{ m s}^{-2} \quad (3 \text{ sf})$</p>	M1 A1 A1 M1 A1 5	<p>Equation involving KE and PE</p> <p>(ft is $29.4 + \frac{u^2}{3.2}$)</p> <p>M1A0 for $g \cos 60^\circ$ M0 for $mg \sin 60^\circ$ <i>If radial and tangential components are interchanged, withhold first A1</i></p>
	<p>(B)</p> $T - mg \cos 60^\circ = ma$ $T - 0.2 \times 9.8 \cos 60^\circ = 0.2 \times 42.2$ <p>Tension is 9.42 N</p>	M1 A1 A1 cao 3	<p>Radial equation (three terms) (Allow M1 for $T - mg = ma$) <i>This M1 can be awarded in (A)</i></p> <p>ft dependent on M1 for energy in (A)</p> <p>SC If 60° with upward vertical, (A) M1A0A0 M1A1 (8.49) (B) M1A1A1 (3.54)</p>
(b)(i)	$T \cos 36^\circ + 0.75 \sin 36^\circ = 0.2 \times 9.8$ <p>Tension is $1.88 \text{ N} \quad (3 \text{ sf})$</p>	M1 A1 2	<p>Resolving vertically (three terms) Allow sin/cos confusion, but both T and R must be resolved</p>
(ii)	<p>Angular speed $\omega = \frac{2\pi}{1.8} \quad (= 3.491)$</p> $T \sin 36^\circ - 0.75 \cos 36^\circ = 0.2 r \left(\frac{2\pi}{1.8} \right)^2$ $r = 0.204$ <p>Length of string is $\frac{r}{\sin 36^\circ}$ $= 0.347 \text{ m} \quad (3 \text{ sf})$</p>	B1 M1 A1 M1 A1 cao 5	<p>Or $v = \frac{2\pi r}{1.8}$</p> <p>Horiz eqn involving $r\omega^2$ or v^2/r Equation for r (or l)</p> <p><i>Dependent on previous M1</i></p>

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3 (i)	Elastic energy is $\frac{1}{2} \times \frac{573.3}{3.9} \times 0.9^2$ $= 59.535 \text{ J}$	M1 A1 2	Allow one error (Allow 60 A0 for 59)
(ii)	Length of string at bottom is $2\sqrt{1.8^2 + 2.4^2} (= 6)$ $\frac{1}{2} \times \frac{573.3}{3.9} \times (2.1^2 - 0.9^2) = m \times 9.8 \times 1.8$ $324.135 - 59.535 = 17.64m$ Mass is 15 kg	M1 M1 B1B1 E1 5	Finding length of string (or half-string) Equation involving EE and PE For change in EE and change in PE
(iii)	Length of string is $2\sqrt{1.0^2 + 2.4^2} = 5.2$ Tension $T = \frac{573.3}{3.9} \times 1.3 (= 191.1)$ $2T \sin \alpha - mg = 2 \times 191.1 \times \frac{1.0}{2.6} - 15 \times 9.8$ $= 147 - 147$ $= 0$, hence it is in equilibrium	M1 A1 M1 E1 4	Finding tension (via Hooke's law) Finding vertical component of tension Give A1 for $T = 191.1$ obtained from resolving vertically SC If 573.3 is used as stiffness: (i) M1A0 (ii) M1M1B0B1E0 (iii) M1A1 (745.29) M1E0
(iv)	$[8\pi^2 h^3] = L^3$, $[8h^3 - ad^2] = L^3$ So $\frac{8\pi^2 h^3}{8h^3 - ad^2}$ is dimensionless	E1 1	Condone ' $L^3 / L^3 = 0$, dimensionless' But E0 for $\frac{L^3}{L^3 - L^3} = \frac{L^3}{0}$
(v)	$T = M^\alpha L^\beta (MLT^{-2})^\gamma$ $\gamma = -\frac{1}{2}$ $\alpha + \gamma = 0$, so $\alpha = \frac{1}{2}$ $\beta + \gamma = 0$, so $\beta = \frac{1}{2}$	B1 B1 B1 B1 4	For $[\lambda] = MLT^{-2}$ If γ is wrong but non-zero, give B1 ft for $\alpha = \beta = -\gamma$
(vi)	$a = 3.9$, $\lambda = 573.3$, $d = 4.8$, $h = 2.6$, $m = 15$ Period is $\sqrt{\frac{8\pi^2 h^3}{8h^3 - ad^2}} m^{1/2} a^{1/2} \lambda^{-1/2} = 1.67 \text{ s}$ (3 sf)	M1 A1 cao 2	Using formula with numerical α , β , γ (must use the complete formula)

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4 (i)	<p>Area is $\int_0^3 (x^2 + 5) dx$</p> $= \left[\frac{1}{3}x^3 + 5x \right]_0^3 \quad (= 24)$ <p>$\int xy dx = \int_0^3 (x^3 + 5x) dx$</p> $= \left[\frac{1}{4}x^4 + \frac{5}{2}x^2 \right]_0^3 \quad (= \frac{171}{4})$ <p>$\bar{x} = \frac{42.75}{24} = \frac{57}{32} = 1.78125$</p> <p>$\int \frac{1}{2}y^2 dx = \int_0^3 \frac{1}{2}(x^4 + 10x^2 + 25) dx$</p> $= \left[\frac{1}{10}x^5 + \frac{5}{3}x^3 + \frac{25}{2}x \right]_0^3 \quad (= 106.8)$ <p>$\bar{y} = \frac{106.8}{24} = \frac{89}{20} = 4.45$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A2</p> <p>A1</p> <p>9</p>	<p>For $\int (x^2 + 5) dx$</p> <p>For $\frac{1}{3}x^3 + 5x$</p> <p>For $\int xy dx$</p> <p>For $\frac{1}{4}x^4 + \frac{5}{2}x^2$</p> <p>For $\int y^2 dx$</p> <p>For $\frac{1}{10}x^5 + \frac{5}{3}x^3 + \frac{25}{2}x$</p> <p>Give A1 for two terms correct</p>
(ii)	<p>Volume is $\int \pi x^2 dy = \int_5^{14} \pi(y - 5) dy$</p> $= \pi \left[\frac{1}{2}y^2 - 5y \right]_5^{14} \quad (= 40.5\pi)$ <p>$\int \pi x^2 y dy = \int_5^{14} \pi(y^2 - 5y) dy$</p> $= \pi \left[\frac{1}{3}y^3 - \frac{5}{2}y^2 \right]_5^{14} \quad (= 445.5\pi)$ <p>$\bar{y} = \frac{445.5\pi}{40.5\pi}$</p> <p>$= 11$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>6</p>	<p>For $\int (y - 5) dy$</p> <p>For $\left[\frac{1}{2}y^2 - 5y \right]_5^{14}$</p> <p>For $\int x^2 y dy$</p> <p>For $\frac{1}{3}y^3 - \frac{5}{2}y^2$</p> <p><i>Dependent on previous M1M1</i></p>
(iii)	<p>Volume of whole cylinder is $\pi \times 3^2 \times 14 = 126\pi$</p> <p>$126\pi \times 7 = 40.5\pi \times 11 + (126\pi - 40.5\pi) \times \bar{y}_A$</p> $\bar{y}_A = \frac{126\pi \times 7 - 40.5\pi \times 11}{126\pi - 40.5\pi}$ $= \frac{97}{19} = 5.105 \quad (4 \text{ sf})$	<p>M1</p> <p>A1</p> <p>A1 cao</p> <p>3</p>	<p>Using formula for composite body</p>

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