



GCE

Mathematics (MEI)

Advanced GCE

Unit 4756: Further Methods for Advanced Mathematics

Mark Scheme for June 2011

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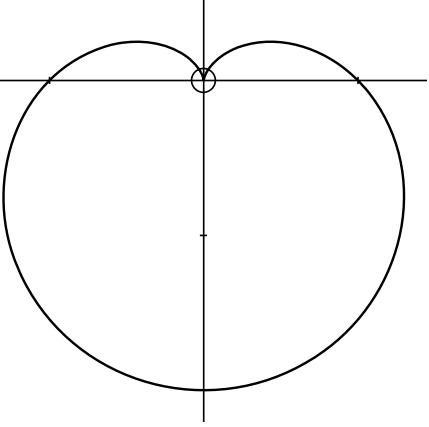
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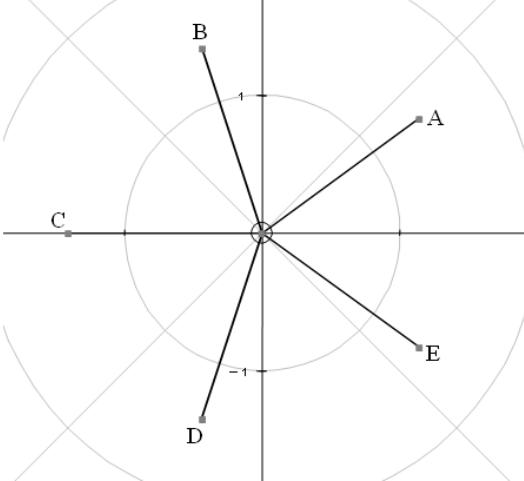
4756 (FP2) Further Methods for Advanced Mathematics

1 (a)(i)		G1 G1	Correct general shape including symmetry in vertical axis Correct form at O and no extra sections. Dependent on first G1 For an otherwise correct curve with a sharp point at the bottom, award G1G0
(ii)	$\begin{aligned} \text{Area} &= \frac{1}{2} a^2 \int_0^{2\pi} (1 - \sin \theta)^2 d\theta \\ &= \frac{1}{2} a^2 \int_0^{2\pi} (1 - 2\sin \theta + \sin^2 \theta) d\theta \\ &= \frac{1}{2} a^2 \int_0^{2\pi} \left(\frac{3}{2} - 2\sin \theta - \frac{1}{2} \cos 2\theta \right) d\theta \\ &= \frac{1}{2} a^2 \left[\frac{3}{2} \theta + 2\cos \theta - \frac{1}{4} \sin 2\theta \right]_0^{2\pi} \\ &= \frac{3}{2} \pi a^2 \end{aligned}$	M1 M1 A1 M1 A2 A1	2 Integral expression involving $(1 - \sin \theta)^2$ Expanding Correct integral expression, incl. limits (which may be implied by later work) Using $\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$ Correct result of integration. Give A1 for one error Dependent on previous A2
(b)(i)	$\begin{aligned} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1+4x^2} dx &= \frac{1}{4} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\frac{1}{4}+x^2} dx = \frac{1}{4} \left[2 \arctan 2x \right]_{-\frac{1}{2}}^{\frac{1}{2}} \\ &= \frac{1}{2} \left(\frac{\pi}{4} - \left(-\frac{\pi}{4} \right) \right) \\ &= \frac{\pi}{4} \end{aligned}$	M1 A1 A1	7 arctan alone, or any tan substitution $\frac{1}{4} \times 2$ and $2x$ Evaluated in terms of π
(ii)	$\begin{aligned} x &= \frac{1}{2} \tan \theta \\ \Rightarrow dx &= \frac{1}{2} \sec^2 \theta d\theta \\ \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{\left(\sec^2 \theta \right)^{\frac{3}{2}}} \times \frac{\sec^2 \theta}{2} d\theta \\ &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2} \cos \theta d\theta \\ &= \left[\frac{1}{2} \sin \theta \right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \\ &= \frac{1}{2} \left(\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}} \right) \right) \\ &= \frac{1}{\sqrt{2}} \end{aligned}$	M1 A1A1 M1 A1ft A1	3 Any tan substitution $\frac{1}{\left(\sec^2 \theta \right)^{\frac{3}{2}}}, \frac{\sec^2 \theta}{2}$ Integrating $a \cos b\theta$ and using consistent limits. Dependent on M1 above $\frac{a}{b} \sin b\theta$

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<p>2 (a) $\cos 5\theta + j \sin 5\theta = (\cos \theta + j \sin \theta)^5$ $= c^5 + 5c^4js - 10c^3s^2 - 10c^2js^3 + 5cs^4 + js^5$</p> $\Rightarrow \cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$ $\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$ $\Rightarrow \tan 5\theta = \frac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4}$ $= \frac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4}$ $= \frac{t(t^4 - 10t^2 + 5)}{5t^4 - 10t^2 + 1}$	M1 M1 A1 A1 M1 A1 (ag)	Expanding Separating real and imaginary parts. Dependent on first M1 Alternative: $16c^5 - 20c^3 + 5c$ Alternative: $16s^5 - 20s^3 + 5s$ Using $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and simplifying
6		
(b)(i) $\arg(-4\sqrt{2}) = \pi$ \Rightarrow fifth roots have $r = \sqrt{2}$ and $\theta = \frac{\pi}{5}$ $\Rightarrow z = \sqrt{2}e^{\frac{1}{5}j\pi}, \sqrt{2}e^{\frac{3}{5}j\pi}, \sqrt{2}e^{j\pi}, \sqrt{2}e^{\frac{7}{5}j\pi}, \sqrt{2}e^{\frac{9}{5}j\pi}$	B1 B1 M1 A1	No credit for arguments in degrees Adding (or subtracting) $\frac{2\pi}{5}$ All correct. Allow $-\pi \leq \theta < \pi$
4		
(ii) 	G1 G1	Points at vertices of "regular" pentagon, with one on negative real axis Points correctly labelled
2		
(iii) $\arg(w) = \frac{1}{2} \left(\frac{\pi}{5} + \frac{3\pi}{5} \right) = \frac{2\pi}{5}$ $ w = \sqrt{2} \cos \frac{\pi}{5}$	B1 M1 A1ft	Attempting to find length F.t. (positive) r from (i)
3		
(iv) $w = \sqrt{2} \cos \frac{\pi}{5} e^{\frac{2}{5}\pi j} \Rightarrow w^n = \left(\sqrt{2} \cos \frac{\pi}{5} \right)^n e^{\frac{2}{5}\pi nj}$ which is real if $\sin \frac{2\pi n}{5} = 0 \Rightarrow n = 5$ $ w^5 = \left(\sqrt{2} \cos \frac{\pi}{5} \right)^5$ $\Rightarrow a = 2^{\frac{5}{2}} \cos^5 \frac{\pi}{5}$	B1 M1 A1	Attempting the n th power of his modulus in (iii), or attempting the modulus of the n th power here Accept 1.96 or better
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3 (i)	$\det(\mathbf{M}) = 1(16 - 12) + 1(20 - 18) + k(10 - 12) \\ = 6 - 2k$ $\Rightarrow \text{no inverse if } k = 3$ $\mathbf{M}^{-1} = \frac{1}{6-2k} \begin{pmatrix} 4 & 4+2k & -6-4k \\ -2 & 4-3k & 5k-6 \\ -2 & -5 & 9 \end{pmatrix}$	M1 A1 A1 M1 A1 M1 A1	Obtaining $\det(\mathbf{M})$ in terms of k Accept $k \neq 3$ after correct determinant Evaluating at least four cofactors (including one involving k) Six signed cofactors correct (including one involving k) Transposing and dividing by $\det(\mathbf{M})$. Dependent on previous M1M1
(ii)	$\begin{pmatrix} 1 & -1 & 3 \\ 5 & 4 & 6 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$	M1 A1	Setting $k = 3$ and multiplying
(iii)	$\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$ is an eigenvector corresponding to an eigenvalue of 1	B1 B1	For credit here, 2/2 scored in (ii) Accept "invariant point"
(iv)	$3x + 6y = 1 - 2t, x + 2y = 2, 5x + 10y = -4t$ $(\text{or } 9x + 18z = 4t + 1, 5x + 10z = 2t, x + 2z = -1)$ $(\text{or } 9y - 9z = 1 - 5t, 5y - 5z = -3t, 2y - 2z = 3)$ For solutions, $1 - 2t = 3 \times 2$ $\Rightarrow t = -\frac{5}{2}$ $x = \lambda, y = 1 - \frac{1}{2}\lambda, z = -\frac{1}{2} - \frac{1}{2}\lambda$ Straight line	M1 A1 M1 A1 M1 A1 B1	Eliminating one variable in two different ways Two correct equations Validly obtaining a value of t Obtaining general solution by setting one unknown = λ and finding other two in terms of λ (accept unknown instead of λ) Accept "sheaf". Independent of all previous marks

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4 (i)	$\cosh y = x \Rightarrow x = \frac{1}{2}(e^y + e^{-y})$	B1	Using correct exponential definition
	$\Rightarrow 2x = e^y + e^{-y}$	M1	Obtaining quadratic in e^y
	$\Rightarrow (e^y)^2 - 2xe^y + 1 = 0$	M1	Solving quadratic
	$\Rightarrow e^y = \frac{2x \pm \sqrt{4x^2 - 4}}{2} = x \pm \sqrt{x^2 - 1}$	A1	$x \pm \sqrt{x^2 - 1}$
	$\Rightarrow y = \ln(x \pm \sqrt{x^2 - 1})$	M1	Validly attempting to justify \pm in printed answer
	$(x + \sqrt{x^2 - 1})(x - \sqrt{x^2 - 1}) = 1$	A1 (ag)	
	$\Rightarrow y = \pm \ln(x + \sqrt{x^2 - 1})$	B1	Reference to arcosh as a function, or correctly to domains/ranges
	$\text{arcosh}(x) = \ln(x + \sqrt{x^2 - 1})$ because this is the principal value		
			7
(ii)	$\int_{\frac{4}{5}}^1 \frac{1}{\sqrt{25x^2 - 16}} dx = \frac{1}{5} \int_{\frac{4}{5}}^1 \frac{1}{\sqrt{x^2 - \frac{16}{25}}} dx$	M1	arcosh alone, or any cosh substitution
	$= \frac{1}{5} \left[\text{arcosh}\left(\frac{5x}{4}\right) \right]_{\frac{4}{5}}^1$	A1A1	$\frac{1}{5}, \frac{5x}{4}$
	$= \frac{1}{5} \left(\text{arcosh}\left(\frac{5}{4}\right) - \text{arcosh}(1) \right)$	M1	Substituting limits and using (i) correctly at any stage (or using limits of u in logarithmic form). Dep. on first M1
	$= \frac{1}{5} \ln\left(\frac{5}{4} + \sqrt{\left(\frac{5}{4}\right)^2 - 1}\right) - 0$	A1	
	$= \frac{1}{5} \ln 2$		
	OR $= \frac{1}{5} \left[\ln\left(x + \sqrt{x^2 - \frac{16}{25}}\right) \right]_{\frac{4}{5}}^1$	M	$\ln(kx + \sqrt{k^2x^2 + \dots})$
		A1A	Give M1 for $\ln(k_1x + \sqrt{k_2^2x^2 + \dots})$ $\frac{1}{5}, \ln\left(x + \sqrt{x^2 - \frac{16}{25}}\right)$ o.e.
	$= \frac{1}{5} \ln \frac{8}{5} - \frac{1}{5} \ln \frac{4}{5}$	A	
	$= \frac{1}{5} \ln 2$		
			5
(iii)	$5 \cosh x - \cosh 2x = 3$	M1	Attempting to express $\cosh 2x$ in terms of $\cosh x$
	$\Rightarrow 5 \cosh x - (2 \cosh^2 x - 1) = 3$	M1	Solving quadratic to obtain at least one real value of $\cosh x$
	$\Rightarrow 2 \cosh^2 x - 5 \cosh x + 2 = 0$	A1	Or factor $2 \cosh x - 1$
	$\Rightarrow (2 \cosh x - 1)(\cosh x - 2) = 0$	A1	
	$\Rightarrow \cosh x = \frac{1}{2}$ (rejected)	A1	
	or $\cosh x = 2$	A1	
	$\Rightarrow x = \ln(2 + \sqrt{3})$	A1ft	F.t. $\cosh x = k, k > 1$
	$x = -\ln(2 + \sqrt{3})$ or $\ln(2 - \sqrt{3})$	A1ft	F.t. other value. Max. A1A0 if additional real values quoted
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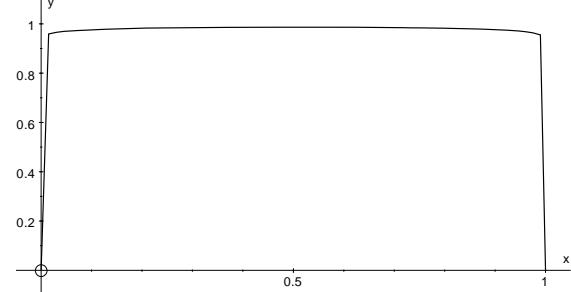
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5 (i)	(A) $m = 1, n = 1$	G1	Negative parabola from (0,0) to (1,0), symmetrical about $x = 0.5$
	(B) $m = 2, n = 2$		
	(C) $m = 2, n = 4$		
	(D) $m = 4, n = 2$		
(ii)	When $m = n$, the curve is symmetrical Exchanging m and n reflects the curve	B1 B1 2	
(iii)	If $m > n$, the maximum is to the right of $x = 0.5$ As m increases relative to n , the maximum point moves further to the right $y = x^m (1-x)^n \Rightarrow \frac{dy}{dx} = mx^{m-1} (1-x)^n - nx^m (1-x)^{n-1}$ $= x^{m-1} (1-x)^{n-1} [m(1-x) - nx]$ $\frac{dy}{dx} = 0 \Rightarrow \text{maximum at } x = \frac{m}{m+n}$	B1 B1 M1 A1 M1 A1 6	<p>o.e. Give B1B0 if the idea is correct but vaguely expressed Using product rule Any correct form</p> <p>Setting derivative = 0 and solving to find a value of x other than 0 or 1</p>

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(iv)	$y''(0) = 0$ provided $m > 1$ $y'(1) = 0$ provided $n > 1$	B1 B1 2	
(v)	For large m and n , the curve approaches the x -axis $\Rightarrow \int_0^1 x^m (1-x)^n dx \rightarrow 0$ as $m, n \rightarrow \infty$	B1 B1 2	Comment on shape Independent
(vi)	e.g. $m = 0.01, n = 0.01$  The curve tends to $y = 1$	M1 A1 2	Evidence of investigation s.o.i. Accept "three sides of (unit) square" 18

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