

**GCE**

# **Mathematics (MEI)**

Advanced GCE

Unit **4756**: Further Methods for Advanced Mathematics

## **Mark Scheme for June 2011**

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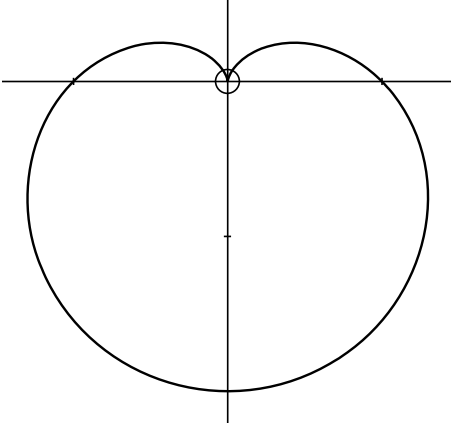
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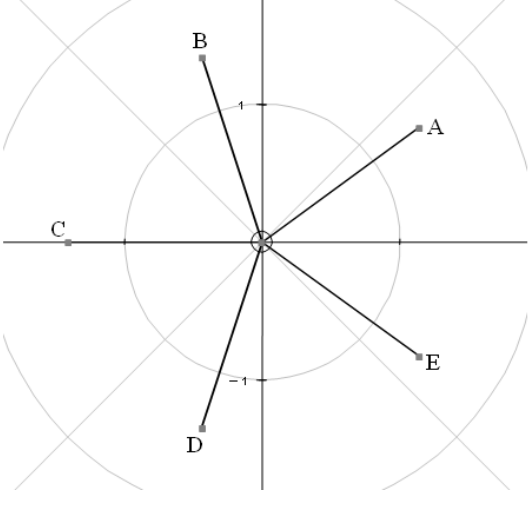
## 4756 (FP2) Further Methods for Advanced Mathematics

<b>1</b> <b>(a)(i)</b>		G1 G1	Correct general shape including symmetry in vertical axis Correct form at O and no extra sections. Dependent on first G1 For an otherwise correct curve with a sharp point at the bottom, award G1G0
<b>(ii)</b>	$\text{Area} = \frac{1}{2} a^2 \int_0^{2\pi} (1 - \sin \theta)^2 d\theta$ $= \frac{1}{2} a^2 \int_0^{2\pi} (1 - 2\sin \theta + \sin^2 \theta) d\theta$ $= \frac{1}{2} a^2 \int_0^{2\pi} \left( \frac{3}{2} - 2\sin \theta - \frac{1}{2} \cos 2\theta \right) d\theta$ $= \frac{1}{2} a^2 \left[ \frac{3}{2} \theta + 2 \cos \theta - \frac{1}{4} \sin 2\theta \right]_0^{2\pi}$ $= \frac{3}{2} \pi a^2$	M1 M1 A1 M1 A2 A1	Integral expression involving $(1 - \sin \theta)^2$ Expanding Correct integral expression, incl. limits (which may be implied by later work) Using $\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$ Correct result of integration. Give A1 for one error Dependent on previous A2
<b>(b)(i)</b>	$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1+4x^2} dx = \frac{1}{4} \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\frac{1}{4} + x^2} dx = \frac{1}{4} [2 \arctan 2x]_{-\frac{1}{2}}^{\frac{1}{2}}$ $= \frac{1}{2} \left( \frac{\pi}{4} - \left( -\frac{\pi}{4} \right) \right)$ $= \frac{\pi}{4}$	M1 A1 A1	arctan alone, or any tan substitution $\frac{1}{4} \times 2$ and $2x$ Evaluated in terms of $\pi$
<b>(ii)</b>	$x = \frac{1}{2} \tan \theta$ $\Rightarrow dx = \frac{1}{2} \sec^2 \theta d\theta$ $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{(\sec^2 \theta)^{\frac{3}{2}}} \times \frac{\sec^2 \theta}{2} d\theta$ $= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2} \cos \theta d\theta$ $= \left[ \frac{1}{2} \sin \theta \right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$ $= \frac{1}{2} \left( \frac{1}{\sqrt{2}} - \left( -\frac{1}{\sqrt{2}} \right) \right)$ $= \frac{1}{\sqrt{2}}$	M1 A1A1 M1 A1ft A1	Any tan substitution $\frac{1}{(\sec^2 \theta)^{\frac{3}{2}}}, \frac{\sec^2 \theta}{2}$ Integrating $a \cos b\theta$ and using consistent limits. Dependent on M1 above $\frac{a}{b} \sin b\theta$

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2 (a)	$\cos 5\theta + j \sin 5\theta = (\cos \theta + j \sin \theta)^5$ $= c^5 + 5c^4js - 10c^3s^2 - 10c^2js^3 + 5cs^4 + js^5$ $\Rightarrow \cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$ $\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$ $\Rightarrow \tan 5\theta = \frac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4}$ $= \frac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4}$ $= \frac{t(t^4 - 10t^2 + 5)}{5t^4 - 10t^2 + 1}$	M1 M1 A1 A1  M1 A1 (ag)	Expanding Separating real and imaginary parts. Dependent on first M1 Alternative: $16c^5 - 20c^3 + 5c$ Alternative: $16s^5 - 20s^3 + 5s$  Using $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and simplifying
(b)(i)	$\arg(-4\sqrt{2}) = \pi$ $\Rightarrow$ fifth roots have $r = \sqrt{2}$ and $\theta = \frac{\pi}{5}$ $\Rightarrow z = \sqrt{2}e^{\frac{1}{5}j\pi}, \sqrt{2}e^{\frac{3}{5}j\pi}, \sqrt{2}e^{j\pi}, \sqrt{2}e^{\frac{7}{5}j\pi}, \sqrt{2}e^{\frac{9}{5}j\pi}$	B1 B1 M1 A1	No credit for arguments in degrees Adding (or subtracting) $\frac{2\pi}{5}$ All correct. Allow $-\pi \leq \theta < \pi$
(ii)		G1 G1	Points at vertices of “regular” pentagon, with one on negative real axis Points correctly labelled
(iii)	$\arg(w) = \frac{1}{2} \left( \frac{\pi}{5} + \frac{3\pi}{5} \right) = \frac{2\pi}{5}$ $ w  = \sqrt{2} \cos \frac{\pi}{5}$	B1 M1 A1ft	Attempting to find length F.t. (positive) $r$ from (i)
(iv)	$w = \sqrt{2} \cos \frac{\pi}{5} e^{\frac{2}{5}j\pi} \Rightarrow w^n = \left( \sqrt{2} \cos \frac{\pi}{5} \right)^n e^{\frac{2}{5}jn\pi}$ which is real if $\sin \frac{2\pi n}{5} = 0 \Rightarrow n = 5$ $ w^5  = \left( \sqrt{2} \cos \frac{\pi}{5} \right)^5$ $\Rightarrow a = 2^{\frac{5}{2}} \cos^5 \frac{\pi}{5}$	B1 M1 A1	Attempting the $n$ th power of his modulus in (iii), or attempting the modulus of the $n$ th power here Accept 1.96 or better

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3 (i)	$\det(\mathbf{M}) = 1(16 - 12) + 1(20 - 18) + k(10 - 12)$ $= 6 - 2k$ $\Rightarrow$ no inverse if $k = 3$  $\mathbf{M}^{-1} = \frac{1}{6 - 2k} \begin{pmatrix} 4 & 4 + 2k & -6 - 4k \\ -2 & 4 - 3k & 5k - 6 \\ -2 & -5 & 9 \end{pmatrix}$	M1 A1 A1 M1 A1 M1 A1 <b>7</b>	Obtaining $\det(\mathbf{M})$ in terms of $k$  Accept $k \neq 3$ after correct determinant Evaluating at least four cofactors (including one involving $k$ ) Six signed cofactors correct (including one involving $k$ ) Transposing and dividing by $\det(\mathbf{M})$ . Dependent on previous M1M1
(ii)	$\begin{pmatrix} 1 & -1 & 3 \\ 5 & 4 & 6 \\ 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$	M1 A1 <b>2</b>	Setting $k = 3$ and multiplying
(iii)	$\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$ is an eigenvector  corresponding to an eigenvalue of 1	B1  B1 <b>2</b>	For credit here, 2/2 scored in (ii) Accept “invariant point”
(iv)	$3x + 6y = 1 - 2t, x + 2y = 2, 5x + 10y = -4t$ (or $9x + 18z = 4t + 1, 5x + 10z = 2t, x + 2z = -1$ ) (or $9y - 9z = 1 - 5t, 5y - 5z = -3t, 2y - 2z = 3$ ) For solutions, $1 - 2t = 3 \times 2$ $\Rightarrow t = -\frac{5}{2}$  $x = \lambda, y = 1 - \frac{1}{2}\lambda, z = -\frac{1}{2} - \frac{1}{2}\lambda$  Straight line	M1  A1 M1  A1  M1  A1 B1 <b>7</b>	Eliminating one variable in two different ways  Two correct equations Validly obtaining a value of $t$  Obtaining general solution by setting one unknown = $\lambda$ and finding other two in terms of $\lambda$ (accept unknown instead of $\lambda$ ) Accept “sheaf”. Independent of all previous marks  <b>18</b>

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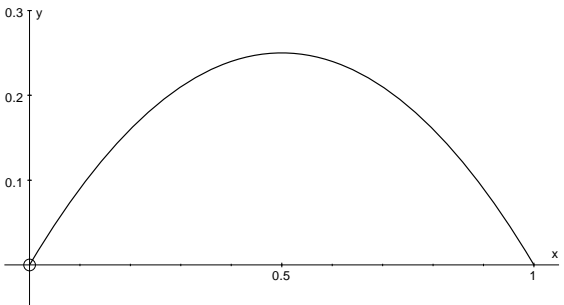
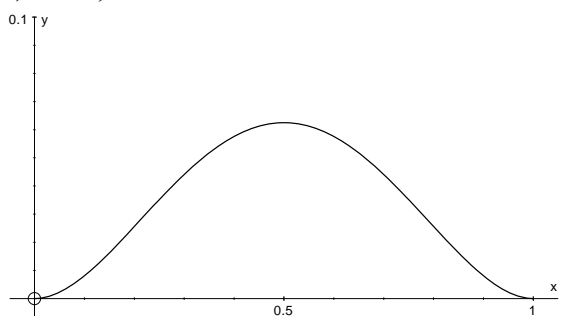
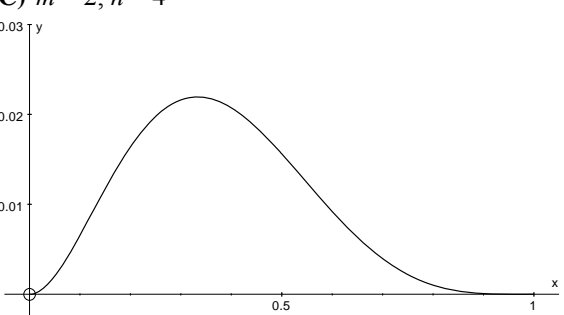
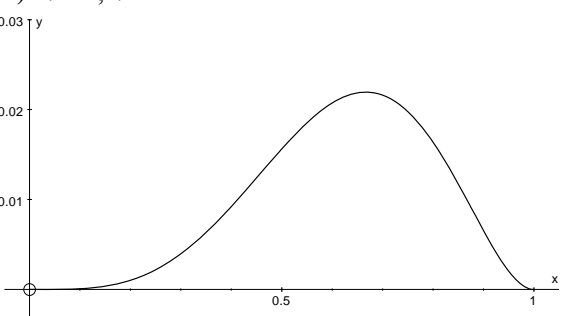
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4 (i)	$\cosh y = x \Rightarrow x = \frac{1}{2}(e^y + e^{-y})$ $\Rightarrow 2x = e^y + e^{-y}$ $\Rightarrow (e^y)^2 - 2xe^y + 1 = 0$ $\Rightarrow e^y = \frac{2x \pm \sqrt{4x^2 - 4}}{2} = x \pm \sqrt{x^2 - 1}$ $\Rightarrow y = \ln(x \pm \sqrt{x^2 - 1})$ $(x + \sqrt{x^2 - 1})(x - \sqrt{x^2 - 1}) = 1$ $\Rightarrow y = \pm \ln(x + \sqrt{x^2 - 1})$ <p>arcosh(x) = <math>\ln(x + \sqrt{x^2 - 1})</math> because this is the principal value</p>	B1  M1 M1 A1  M1 A1 (ag)  B1	Using correct exponential definition  Obtaining quadratic in $e^y$ Solving quadratic $x \pm \sqrt{x^2 - 1}$  Validly attempting to justify $\pm$ in printed answer  Reference to arcosh as a function, or correctly to domains/ranges
		7	
(ii)	$\int_{\frac{4}{5}}^1 \frac{1}{\sqrt{25x^2 - 16}} dx = \frac{1}{5} \int_{\frac{4}{5}}^1 \frac{1}{\sqrt{x^2 - \frac{16}{25}}} dx$ $= \frac{1}{5} \left[ \operatorname{arcosh} \left( \frac{5x}{4} \right) \right]_{\frac{4}{5}}^1$ $= \frac{1}{5} \left( \operatorname{arcosh} \left( \frac{5}{4} \right) - \operatorname{arcosh}(1) \right)$ $= \frac{1}{5} \ln \left( \frac{5}{4} + \sqrt{\left( \frac{5}{4} \right)^2 - 1} \right) - 0$ $= \frac{1}{5} \ln 2$ <hr/> <p>OR</p> $= \frac{1}{5} \left[ \ln \left( x + \sqrt{x^2 - \frac{16}{25}} \right) \right]_{\frac{4}{5}}^1$ $= \frac{1}{5} \ln \frac{8}{5} - \frac{1}{5} \ln \frac{4}{5}$ $= \frac{1}{5} \ln 2$	M1 A1A1  M1 A1  M A1A  A	arcosh alone, or any cosh substitution  $\frac{1}{5}, \frac{5x}{4}$  Substituting limits and using (i) correctly at any stage (or using limits of $u$ in logarithmic form). Dep. on first M1  $\ln(kx + \sqrt{k^2x^2 + \dots})$ Give M1 for $\ln(k_1x + \sqrt{k_2^2x^2 + \dots})$ $\frac{1}{5}, \ln \left( x + \sqrt{x^2 - \frac{16}{25}} \right)$ o.e.
		5	
(iii)	$5 \cosh x - \cosh 2x = 3$ $\Rightarrow 5 \cosh x - (2 \cosh^2 x - 1) = 3$ $\Rightarrow 2 \cosh^2 x - 5 \cosh x + 2 = 0$ $\Rightarrow (2 \cosh x - 1)(\cosh x - 2) = 0$ $\Rightarrow \cosh x = \frac{1}{2} \text{ (rejected)}$ <p>or <math>\cosh x = 2</math></p> $\Rightarrow x = \ln(2 + \sqrt{3})$ $x = -\ln(2 + \sqrt{3}) \text{ or } \ln(2 - \sqrt{3})$	M1  M1 A1 A1 A1ft A1ft	Attempting to express $\cosh 2x$ in terms of $\cosh x$  Solving quadratic to obtain at least one real value of $\cosh x$ Or factor $2 \cosh x - 1$  F.t. $\cosh x = k, k > 1$ F.t. other value. Max. A1A0 if additional real values quoted
		6	18

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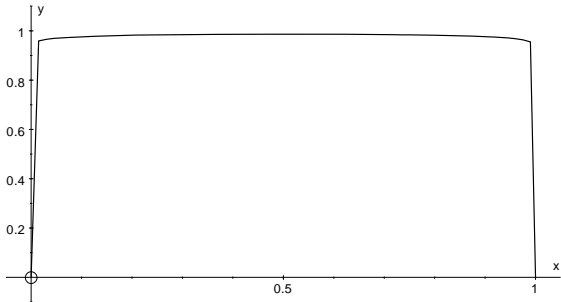
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<p><b>5 (i)</b></p>	<p><b>(A)</b> <math>m = 1, n = 1</math></p>  <p><b>(B)</b> <math>m = 2, n = 2</math></p>  <p><b>(C)</b> <math>m = 2, n = 4</math></p>  <p><b>(D)</b> <math>m = 4, n = 2</math></p> 	<p>G1</p> <p>G1</p> <p>G1</p> <p>G1</p> <p>4</p>	<p>Negative parabola from (0,0) to (1,0), symmetrical about <math>x = 0.5</math></p> <p>Bell-shape from (0,0) to (1,0), symmetrical about <math>x = 0.5</math>; flat ends, and obviously different to (A)</p> <p>Skewed curve from (0,0) to (1,0), maximum to left of <math>x = 0.5</math></p> <p>Skewed curve from (0,0) to (1,0), maximum to right of <math>x = 0.5</math></p>
<p><b>(ii)</b></p>	<p>When <math>m = n</math>, the curve is symmetrical Exchanging <math>m</math> and <math>n</math> reflects the curve</p>	<p>B1 B1</p> <p>2</p>	
<p><b>(iii)</b></p>	<p>If <math>m &gt; n</math>, the maximum is to the right of <math>x = 0.5</math> As <math>m</math> increases relative to <math>n</math>, the maximum point moves further to the right</p> $y = x^m (1-x)^n \Rightarrow \frac{dy}{dx} = mx^{m-1}(1-x)^n - nx^m(1-x)^{n-1}$ $= x^{m-1}(1-x)^{n-1} [m(1-x) - nx]$ $\frac{dy}{dx} = 0 \Rightarrow \text{maximum at } x = \frac{m}{m+n}$	<p>B1 B1 M1 A1</p> <p>M1 A1</p> <p>6</p>	<p>o.e. Give B1B0 if the idea is correct but vaguely expressed Using product rule Any correct form</p> <p>Setting derivative = 0 and solving to find a value of <math>x</math> other than 0 or 1</p>

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(iv)	$y'(0) = 0$ provided $m > 1$ $y'(1) = 0$ provided $n > 1$	B1 B1 <b>2</b>	
(v)	For large $m$ and $n$ , the curve approaches the $x$ -axis $\Rightarrow \int_0^1 x^m (1-x)^n dx \rightarrow 0$ as $m, n \rightarrow \infty$	B1 B1 <b>2</b>	Comment on shape Independent
(vi)	e.g. $m = 0.01, n = 0.01$  The curve tends to $y = 1$	M1 A1 <b>2</b>	Evidence of investigation s.o.i. Accept "three sides of (unit) square"

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