RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4764:** Mechanics 4

Mark Scheme for June 2011

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1(i)	$\frac{dm}{dt} = -\lambda m \Rightarrow m = m_0 e^{-\lambda t}$	M1 A1		
				2
(ii)	$\frac{d}{dt}(mv) = mg - kmv$	B1	N2L	
	$\frac{dm}{dt}v + m\frac{dv}{dt} = mg - kmv$	M1	Expand derivative	
	$-\lambda mv + m\frac{dv}{dt} = mg - kmv$	M1	Substitute	
	$\frac{dv}{dt} = g + (\lambda - k)v$	A1		
	$\int \frac{dv}{g + (\lambda - k)v} = \int dt$	M1	Separate and integrate	
	$\frac{1}{\lambda - k}\ln(g + (\lambda - k)v) = t + c$	A1√		
	$g + (\lambda - k)v = Ae^{(\lambda - k)t}$			
	$v = 0, t = 0 \Rightarrow A = g$	M1	Use condition	
	$v = \frac{g}{\lambda - k}\left(e^{(\lambda - k)t} - 1\right)$ AG	E1	Convincingly shown	
				8
(iii)	$m = \frac{1}{2}m_0 \Rightarrow e^{-\lambda t} = \frac{1}{2}$	M1	Accept substituted into their expression in part (i)	
	$\Rightarrow t = \frac{1}{\lambda}\ln 2$			
	$v = \frac{g}{\lambda - k}\left(2^{\frac{\lambda - k}{\lambda}} - 1\right)$	A1	Any correct form	
				2

2(i)	$V = \frac{1}{2}k(2a - x - a)^2 + \frac{1}{2}k\left(\sqrt{a^2 + x^2} - a\right)^2$	M1 A1 A1	for $E = \frac{1}{2}kx^2$	
	$\frac{dV}{dx} = -k(a - x)$ $+k\left(\sqrt{a^2 + x^2} - a\right) \cdot 2x \cdot \frac{1}{2}(a^2 + x^2)^{-1/2}$ $= -k(a - x) + kx\left(1 - \frac{a}{\sqrt{a^2 + x^2}}\right)$	M1		
	$= 2kx - ka - \frac{kax}{\sqrt{a^2 + x^2}}$ AG	E1	Convincingly shown	
				5
(ii)	$\frac{d^2V}{dx^2} = 2k - \frac{ka\sqrt{a^2 + x^2} - kax \cdot x(a^2 + x^2)^{-1/2}}{a^2 + x^2}$	M1		
	$= 2k - \frac{ka^3}{(a^2 + x^2)^{3/2}}$	A1		
	$(a^2 + x^2)^{3/2} > (a^2)^{3/2} = a^3$	M1		
	$\Rightarrow \frac{ka^3}{(a^2 + x^2)^{3/2}} < k \Rightarrow V''(x) > 2k - k > 0$	E1	Convincingly shown	
				4
(iii)	$x = \frac{1}{2}a \Rightarrow V' = ka - ka - \frac{ka \cdot \frac{1}{2}a}{\sqrt{a^2 + \left(\frac{1}{2}a\right)^2}} < 0$	M1		
	$x = a \Rightarrow V' = 2ka - ka - \frac{ka^2}{\sqrt{a^2 + a^2}} = ka - \frac{ka}{\sqrt{2}} > 0$			
	Hence (as V' continuous) $V' = 0$ between $\frac{1}{2}a$ and a.	E1	Convincingly shown	
	So equilibrium. Stable as $V'' > 0$.	B1		
				3

3(i)	$800v\frac{dv}{dx} = \frac{8v^4}{v} - 8v^2$	M1	N2L with P/v	
		A1		
	$\int \frac{100dv}{v^2 - v} = \int dx$	M1	Separate	
	$\int 100\left(\frac{1}{v-1} - \frac{1}{v}\right)dx = \int dx$	M1	Partial fractions	
	$100\left(\ln(v-1) - \ln v\right) = x + c$	A1		
	$x = 0, v = 2 \Rightarrow c = -100\ln 2$	M1	Use condition	
	$100\ln\left(\frac{2(v-1)}{v}\right) = x$	A1	AEF, condone m	
	$v = 20 \Rightarrow x = 100\ln\left(2 \times \frac{19}{20}\right) = 100\ln 1.9$	E1		
	$\frac{2(v-1)}{v} = e^{0.01x}$			
	$2v - 2 = ve^{0.01x}$	M1	Rearrange	
	$v = \frac{2}{2 - e^{0.01x}}$	A1	Cao without m	
				10
(ii)	$\frac{dx}{dt} = \frac{2}{2 - e^{0.01x}}$	M1		
	$\int (2 - e^{0.01x})dx = \int 2dt$	M1	Separate and integrate	
	$2x - 100e^{0.01x} = 2t + c_2$	A1		
	$x = 0, t = 0 \Rightarrow c_2 = -100$	M1	Use condition	
	$2x - 100e^{0.01x} = 2t - 100$	A1	Any correct form	
	$x = 100\ln 1.9 \Rightarrow t \approx 19.2$ AG	E1		
				6
(iii)	$800\frac{dv}{dt} = -8v^2$	M1	N2L	
		A1		
	$\int 100v^{-2}dv = \int -1dt$	M1	Separate and integrate	
	$-100v^{-1} = -t + c_3$	A1		
	$t = 19.2, v = 20 \Rightarrow -5 = -19.2 + c_3$	M1	Use condition	
	$c_3 = 14.2$			
	$v = \frac{100}{t - 14.2}$	M1	Rearrange	
		A1	CAO	
	$2 = \frac{100}{t - 14.2} \Rightarrow t = 64.2$	B1	Accept $t = 45$ (time for this part of motion)	
				8

4(i)	$I_x = \frac{1}{2}my^2$	B1	
	$2I_{\text{diameter}} = I_x$	M1	Use perpendicular axes theorem
	$I_{\text{diameter}} = \frac{1}{4}my^2$	B1	
	$I = \frac{1}{4}my^2 + mx^2$	M1	Use parallel axes theorem
	$= m\left(\frac{1}{4}\left(\frac{1}{2}x\right)^2 + x^2\right)$	M1	Use $y = \frac{1}{2}x$
	$= \frac{17}{16}mx^2$ AG	E1	Complete argument
			6
(ii)	Mass of slice $\approx M\left(\frac{\pi y^2 \delta x}{\frac{1}{3}\pi a^2 \cdot 2a}\right)$	M1	
	$= \frac{3M}{2a^3}y^2\delta x$	B1	Deal correctly with mass/density
	$I_{\text{slice}} \approx \frac{17}{16}\left(\frac{3M}{2a^3}y^2\delta x\right)x^2$	M1	
		A1	
	$= \frac{51M}{128a^3}x^4\delta x$	M1	Substitute for y
	$I = \int_0^{2a}\frac{51M}{128a^3}x^4\,\mathrm{d}x$	M1	
	$= \frac{51M}{128a^3}\left[\frac{1}{5}x^5\right]_0^{2a}$	A1	
	$= \frac{51}{20}Ma^2$ AG	E1	Complete argument
			8
(iii)	$\frac{1}{2}I\dot{\theta}^2 - Mg\cdot\frac{3}{2}a\cos\theta = -Mg\cdot\frac{3}{2}a\cos\alpha$	M1	Energy equation
		B1	Position of centre of mass
		A1	KE term
		F1	GPE terms ft their CoM only
	$\dot{\theta}^2 = \frac{3Mga}{I}(\cos\theta - \cos\alpha)$		
	$= \frac{20g}{17a}(\cos\theta - \cos\alpha)$	E1	Complete argument
			5
(iv)	$2\dot{\theta}\ddot{\theta} = -\frac{20g}{17a}\sin\theta\,\dot{\theta}$	M1	Differentiate or use $I\ddot{\theta}$ = torque
	$\ddot{\theta} = -\frac{10g}{17a}\sin\theta$	A1	
	$\approx -\frac{10g}{17a}\theta$ for small θ	M1	Use $\sin\theta \approx \theta$
	Hence SHM	E1	
	Period $2\pi\sqrt{\frac{17a}{10g}}$	B1	
			5

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