OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4769

Statistics 4

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Thursday 26 May 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Option 1: Estimation

1 The random variable X has the Normal distribution with mean 0 and variance θ, so that its probability density function is

$$f(x) = \frac{1}{\sqrt{2\pi\theta}} e^{-x^2/2\theta}, \quad -\infty < x < \infty,$$

where θ ($\theta > 0$) is unknown. A random sample of n observations from X is denoted by $X_1, X_2, \ldots, X_n$.

(i) Find $\hat{\theta}$, the maximum likelihood estimator of θ. **[14]**

(ii) Show that $\hat{\theta}$ is an unbiased estimator of θ. **[4]**

(iii) In large samples, the variance of $\hat{\theta}$ may be estimated by $\frac{2\hat{\theta}^2}{n}$. Use this and the results of parts **(i)** and **(ii)** to find an approximate 95% confidence interval for θ in the case when $n = 100$ and $\Sigma X_i^2 = 1000$. **[6]**

Option 2: Generating Functions

2 The random variable X has the χ_n^2 distribution. This distribution has moment generating function $M(\theta) = (1 - 2\theta)^{-\frac{1}{2}n}$, where $\theta < \frac{1}{2}$.

(i) Verify the expression for $M(\theta)$ quoted above for the cases $n = 2$ and $n = 4$, given that the probability density functions of X in these cases are as follows. **[10]**

$n = 2$: $\quad f(x) = \frac{1}{2}e^{-\frac{1}{2}x} \quad (x > 0)$

$n = 4$: $\quad f(x) = \frac{1}{4}xe^{-\frac{1}{2}x} \quad (x > 0)$

(ii) For the general case, use $M(\theta)$ to find the mean and variance of X in terms of n. **[7]**

(iii) $Y_1, Y_2, \ldots, Y_k$ are independent random variables, each with the χ_1^2 distribution. Show that $W = \sum_{i=1}^{k} Y_i$ has the χ_k^2 distribution. **[4]**

(iv) Use the Central Limit Theorem to find an approximation for $P(W < 118.5)$ for the case $k = 100$. **[3]**

Option 3: Inference

3 **(i)** Explain the meaning of the following terms in the context of hypothesis testing: Type I error, Type II error, operating characteristic, power. **[8]**

(ii) A market research organisation is designing a sample survey to investigate whether expenditure on everyday food items has increased in 2011 compared with 2010. For one of the populations being studied, the random variable X is used to model weekly expenditure, in £, on these items in 2011, where X is Normally distributed with mean μ and variance σ^2. As the corresponding mean value in 2010 was 94, the hypotheses to be examined are

H_0: $\mu = 94$,

H_1: $\mu > 94$.

By comparison with the corresponding 2010 value, σ^2 is assumed to be 25.

The following criteria for the survey are laid down.

- If in fact $\mu = 94$, the probability of concluding that $\mu > 94$ must be only 2%
- If in fact $\mu = 97$, the probability of concluding that $\mu > 94$ must be 95%

A random sample of size n is to be taken and the usual Normal test based on $\overline{X}$ is to be used, with a critical value of c such that H_0 is rejected if the value of $\overline{X}$ exceeds c. Find c and the smallest value of n that is required. **[13]**

(iii) Sketch the power function of an ideal test for examining the hypotheses in part **(ii)**. **[3]**

Option 4: Design and Analysis of Experiments

4 **(a)** Provide an example of an experimental situation where there is one factor of primary interest and where a suitable experimental design would be

(i) randomised blocks,

(ii) a Latin square.

In each case, explain carefully why the design is suitable and why the other design would not be appropriate. **[12]**

(b) An industrial experiment to compare four treatments for increasing the tensile strength of steel is carried out according to a completely randomised design. For various reasons, it is not possible to use the same number of replicates for each treatment. The increases, in a suitable unit of tensile strength, are as follows.

Treatment A	Treatment B	Treatment C	Treatment D
10.1	21.1	9.2	22.6
21.2	20.3	8.8	17.4
11.6	16.0	15.2	23.1
13.6		15.0	19.2
		12.4	

[The sum of these data items is 256.8 and the sum of their squares is 4471.92.]

Construct the usual one-way analysis of variance table. Carry out the appropriate test, using a 5% significance level. **[12]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE