RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

4776/01

Numerical Methods

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Wednesday 18 May 2011
Morning

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

1 The equation $f(x) = 0$, where $f(x)$ is a continuous function, is known to have a single root in the interval [0.4, 1.8].

(i) Suppose the root is to be found using the bisection method. State the best possible estimate of the root at the start of the process. State also the maximum possible error associated with that estimate.

Determine how many iterations of the bisection process would be required to reduce the maximum possible error to less than 0.05. **[4]**

(ii) Given now that $f(0.4) = -0.2$ and $f(1.8) = 0.5$, find an estimate of the root using the false position method. **[3]**

2 The function $g(x)$ has the values shown in the table.

x	1.80	2.00	2.20
$g(x)$	2.66	2.85	3.02

(i) Taking the data to be exact, use the central difference formula to estimate $g'(2)$. **[2]**

(ii) Suppose instead that the x values are exact but the values of $g(x)$ are rounded to 2 decimal places. Find an appropriate range of estimates of $g'(2)$. **[3]**

(iii) Now suppose that all the values in the table have been rounded to 2 decimal places. Find the appropriate range of estimates of $g'(2)$ in this case. **[3]**

3 The function $Q(x)$ is known to be quadratic and it has the values shown in the table.

x	-1	1	5
$Q(x)$	-4	-12	20

(i) Write down the estimate of $Q(0)$ obtained by linear interpolation. **[1]**

(ii) Use Lagrange's method to write down an expression for $Q(x)$. [You are not required to simplify this expression.] **[5]**

(iii) Find the exact value of $Q(0)$. **[2]**

4 **(i)** Show that the equation $x = 1 - x^4$ has a root in the interval [0.7, 0.8]. **[2]**

(ii) Show, by considering the derivative of $1 - x^4$, that the iteration $x_{r+1} = 1 - x_r^{\ 4}$, with a starting value in the interval [0.7, 0.8], will diverge. **[4]**

5 **(i)** Find the absolute error and the relative error when $X = 3.162$ is used as an approximation to $x = \sqrt{10}$. **[3]**

(ii) Find the relative error if X^4 is used as an approximation to x^4. **[3]**

(iii) State, in terms of k, the approximate relative error if X^k is used as an approximation to x^k. **[1]**

Section B (36 marks)

6 The integral $I = \int_2^{2.8} \sqrt{1 + x^3}\, dx$ is to be determined numerically. You should give all your answers to 7 decimal places unless instructed otherwise.

(i) Find mid-point rule and trapezium rule estimates of I, taking $h = 0.8$.

Use these two estimates to find a second trapezium rule estimate and a Simpson's rule estimate of I. **[8]**

(ii) Find the mid-point rule estimate with $h = 0.4$, and hence obtain a second Simpson's rule estimate of I. **[3]**

(iii) You are now given that the mid-point rule estimate of I with $h = 0.2$ is 3.091 429 8, correct to 7 decimal places.

Find a third Simpson's rule estimate. Show by considering ratios of differences that Simpson's rule is of order h^4.

Give the value of I to the accuracy that appears justified. **[7]**

[Question 7 is printed overleaf.]

7 The function f(x) has the exact values shown in the table.

x	1	3	5
f(x)	4	–2	10

(i) Use Newton's forward difference interpolation formula to find the quadratic function that fits the data. (There is no need to simplify your answer.) **[6]**

(ii) Hence estimate the values of f(2) and f(6). State, with a reason, which of these estimates is likely to be more accurate. **[3]**

(iii) Now suppose that f(7) = 11. Find the cubic function that fits all the data. Use this cubic to estimate f(2) and f(6). **[7]**

(iv) Comment on (A) the absolute changes and (B) the relative changes in the estimates of f(2) and f(6) from part **(ii)** to part **(iii)**. **[2]**