

Mathematics (MEI)

Advanced GCE

Unit **4757**: Further Applications of Advanced Mathematics

Mark Scheme for June 2011

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

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GENERAL MARKING INSTRUCTIONS

- 1 Please mark in RED.
- 2 Errors in the scripts should be clearly indicated (e.g. by ringing them). Please tick correct work. Some indication must be given on every page that the work has been assessed.
- 3 Marks for sections of questions should be written in the right-hand margin; detailed marks (e.g. M1A0M1A1) need only be shown when the breakdown of marks is not obvious.
- 4 The total mark for each question should be written in the right-hand margin at the end of the answer, and ringed.
- 5 Work crossed out by the candidate, and not replaced, should be marked wherever possible.
- 6 M: method marks
 A: accuracy marks which are dependent on the relevant previous method mark(s) having been awarded (e.g. marks M0A1 cannot be awarded; where appropriate, the correct answer obtained with no method shown is taken to imply the method marks).
 B: accuracy marks which are not dependent on method marks.
- 7 Marks are indivisible; if the scheme says M2, give either 2 or 0, except where indicated in the scheme.
- 8 Where there are two or more method marks for a section of work, the method marks can be awarded independently, except where indicated in the scheme.
- 9 cao: correct answer only (i.e. no follow through).
 All A and B marks are cao unless stated otherwise.
 ft: award mark for correct working following through from a previous error; use \checkmark to indicate correct ft work, and \surd^{\wedge} to indicate that a further error has been made. Ft marks are intended to ensure that a small arithmetic error is not unduly penalised; when the previous error is one of principle, particularly if the nature of the work is changed, or made considerably easier, the ft might not be given.
- 10 'Correctly obtained' means that all the working leading to that result must be correct.
- 11 Where the candidate is asked to show a given result, we expect the explanation to be reasonably clear.
- 12 If the candidate misreads a question, but in such a way that the nature and difficulty of the work is not changed, transfer all marks (including cao) to the new equivalent figures. Deduct the first A or B mark so earned (M marks are not lost for misread).

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1 (i)	Distance is $\frac{2(-2) - (-7) + 2(7) - 11}{\sqrt{2^2 + 1^2 + 2^2}}$ $= 2$	M1 A1 A1 3	Formula, or other complete method Numerical expression for distance
(ii)	$\begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 15 \\ 12 \\ -9 \end{pmatrix}$ Equation of AD is $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}$	M1 A1 A1 ft 3	Vector product of normals, or finding a point on AD, e.g. $(0, -2.6, 4.2)$, $(3.25, 0, 2.25)$, $(7, 3, 0)$ Correct direction Accept any form
(iii)	$\overline{\mathbf{CA}} \times \mathbf{d} = \begin{pmatrix} -5 \\ -6 \\ 2 \end{pmatrix} \times \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 10 \\ -5 \\ 10 \end{pmatrix}$ Distance is $\frac{ \overline{\mathbf{CA}} \times \mathbf{d} }{ \mathbf{d} } = \frac{\sqrt{10^2 + 5^2 + 10^2}}{\sqrt{5^2 + 4^2 + 3^2}} = \frac{\sqrt{225}}{\sqrt{50}}$ $= \sqrt{4.5} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2} \approx 2.12$	M1 A2 ft M1 M1 A1 6	Appropriate vector product Give A1 if just one error Formula for distance Finding magnitude <i>Both dependent on first M1</i>
(iv)	$\mathbf{d} \times \overline{\mathbf{BC}} = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix} \times \begin{pmatrix} 9 \\ 12 \\ -6 \end{pmatrix} = \begin{pmatrix} 12 \\ 3 \\ 24 \end{pmatrix} = 3 \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix}$ Distance is $\frac{\begin{pmatrix} -4 \\ -6 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix}}{\sqrt{4^2 + 1^2 + 8^2}} = \frac{10}{9}$	M1 A1 ft M1 A1 ft M1 A1 6	Vector product of directions Appropriate scalar product Evaluation of scalar product For denominator
(v)	$V = \frac{1}{6}(\overline{\mathbf{AB}} \times \overline{\mathbf{AC}}) \cdot \overline{\mathbf{AD}} = \frac{1}{6} \left[\begin{pmatrix} -4 \\ -6 \\ 4 \end{pmatrix} \times \begin{pmatrix} 5 \\ 6 \\ -2 \end{pmatrix} \right] \cdot \lambda \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}$ $= \frac{1}{6} \lambda \begin{pmatrix} -12 \\ 12 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix} = -5\lambda$ $V = \pm 20 \Rightarrow \lambda = \pm 4$ D is $(22, 15, -9)$ or $(-18, -17, 15)$	M1 M1 A1 ft M1 A1A1 6	Appropriate scalar triple product Evaluation of scalar triple product or $-2a + 2b + c + 3$ (simplified) for $D(a, b, c)$ Obtain a value of λ , or one of a, b, c

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Alternative methods for Question 1

1 (ii)	<p>Eliminating x</p> $3y + 4z = 9$ $x = 7 - \frac{5}{3}t, y = 3 - \frac{4}{3}t, z = t$	<p>M1</p> <p>A1A1</p> <p>3</p>	<p>Eliminating one of x, y, z</p> <p>or $3x + 5z = 21$ or $4x - 5y = 13$</p>
1 (iii)	$\left[\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \\ 1 \end{pmatrix} \right] \cdot \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix} = 0$ $25\lambda - 25 + 16\lambda - 24 + 9\lambda - 6 = 0$ $\lambda = 1.1, \text{ F is } (7.5, 3.4, -0.3)$ $CF = \sqrt{(0.5)^2 + (-1.6)^2 + (-1.3)^2}$ $= \sqrt{4.5}$	<p>M1</p> <p>A1 ft</p> <p>A1 ft</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>6</p>	<p>Appropriate scalar product</p> <p>Obtaining a value for λ</p> <p>Finding magnitude</p>
1 (iv)	$\left[\begin{pmatrix} -2 \\ -7 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 9 \\ 12 \\ -6 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \lambda \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix} \right] \cdot \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix} = 0$ <p>and</p> $\begin{pmatrix} -5\lambda + 9\mu - 4 \\ -4\lambda + 12\mu - 6 \\ 3\lambda - 6\mu + 4 \end{pmatrix} \cdot \begin{pmatrix} 9 \\ 12 \\ -6 \end{pmatrix} = 0$ $\lambda = \frac{4}{81}, \mu = \frac{128}{243}$ <p>Distance is $\sqrt{\left(\frac{40}{81}\right)^2 + \left(\frac{10}{81}\right)^2 + \left(\frac{80}{81}\right)^2}$</p> $= \frac{10}{9}$	<p>M1</p> <p>A1 ft</p> <p>A1 ft</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>6</p>	<p>Two appropriate scalar products</p> <p>Obtaining values for λ and μ</p> <p>Obtaining distance</p>

4

3(a)(i)	$\frac{dy}{dx} = \frac{1}{2}e^{\frac{1}{2}x} - \frac{1}{2}e^{-\frac{1}{2}x}$ $\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}e^x - \frac{1}{2} + \frac{1}{4}e^{-x}$ $1 + \left(\frac{dy}{dx}\right)^2 = \frac{1}{4}e^x + \frac{1}{2} + \frac{1}{4}e^{-x} = \left(\frac{1}{2}e^{\frac{1}{2}x} + \frac{1}{2}e^{-\frac{1}{2}x}\right)^2$	B1 M1 A1 (ag)	3 Correct completion
(ii)	Length is $\int_0^{\ln a} \left(\frac{1}{2}e^{\frac{1}{2}x} + \frac{1}{2}e^{-\frac{1}{2}x}\right) dx$ $= \left[e^{\frac{1}{2}x} - e^{-\frac{1}{2}x} \right]_0^{\ln a}$ $= \left(\sqrt{a} - \frac{1}{\sqrt{a}} \right) - (1 - 1) = \frac{a-1}{\sqrt{a}}$	M1 A1 A1 (ag)	For $\int \left(\frac{1}{2}e^{\frac{1}{2}x} + \frac{1}{2}e^{-\frac{1}{2}x}\right) dx$ For $e^{\frac{1}{2}x} - e^{-\frac{1}{2}x}$ Correctly shown
(iii)	Curved surface area is $\int 2\pi y ds$ $= \int_0^{\ln a} 2\pi \left(e^{\frac{1}{2}x} + e^{-\frac{1}{2}x}\right) \left(\frac{1}{2}e^{\frac{1}{2}x} + \frac{1}{2}e^{-\frac{1}{2}x}\right) dx$ $= \pi \int_0^{\ln a} (e^x + 2 + e^{-x}) dx$ $= \pi \left[e^x + 2x - e^{-x} \right]_0^{\ln a}$ $= \pi \left(a + 2\ln a - \frac{1}{a} \right)$	M1 A1 M1 A1 A1	For $\int y ds$ Correct integral form <i>including limits</i> Obtaining integrable expression For $e^x + 2x - e^{-x}$
(b)(i)	$\frac{dy}{dx} = \frac{\cos \theta}{-2 \sin \theta}$ Gradient of normal is $\frac{2 \sin \theta}{\cos \theta} \quad (= 2 \tan \theta)$ Normal is $y - \sin \theta = 2 \tan \theta (x - 2 \cos \theta)$ $y = 2x \tan \theta - 3 \sin \theta$	B1 M1 A1 (ag)	3 Correctly shown
(ii)	Differentiating partially w.r.t. θ $0 = 2x \sec^2 \theta - 3 \cos \theta$ $x = \frac{3}{2} \cos^3 \theta$ $y = 3 \cos^3 \theta \tan \theta - 3 \sin \theta$ $= 3 \sin \theta (\cos^2 \theta - 1) = -3 \sin^3 \theta$ $(2x)^{\frac{2}{3}} + y^{\frac{2}{3}} = (3 \cos^3 \theta)^{\frac{2}{3}} + (-3 \sin^3 \theta)^{\frac{2}{3}}$ $= 3^{\frac{2}{3}} (\cos^2 \theta + \sin^2 \theta) = 3^{\frac{2}{3}}$	M1 A1 M1 A1 M1 A1 (ag)	Obtaining an expression for y Any correct form Using $1 - \cos^2 \theta = \sin^2 \theta$ Correctly shown
(iii) (A)	$(2, 0)$ has $\theta = 0$ Centre of curvature is $\left(\frac{3}{2}, 0\right)$ $\rho = \frac{1}{2}$	M1 A1	Using param eqn with $\theta = 0$ (or other method for ρ or cc)
(B)	$(0, 1)$ has $\theta = \frac{1}{2}\pi$ Centre of curvature is $(0, -3)$ $\rho = 4$	M1 A1	Using param eqn with $\theta = \frac{1}{2}\pi$ (or other method for ρ or cc)

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4 (i)		1	3	4	5	9	B2	Give B1 if not more than 4 errors
	1	1	3	4	5	9		
	3	3	9	1	4	5		
	4	4	1	5	9	3		
	5	5	4	9	3	1		
	9	9	5	3	1	4		
	Composition table shows closure Identity is 1						B1 B1	Dependent on B2 for table
	Element	1	3	4	5	9	B2	Give B1 for 3 correct
	Inverse	1	4	3	9	5	6	
	So every element has an inverse							
(ii)	Since 5 is prime, a group of order 5 must be cyclic Two cyclic groups of the same order must be isomorphic						B1 B1 B1	3
(iii)	H	1	$e^{\frac{2}{5}\pi j}$	$e^{\frac{4}{5}\pi j}$	$e^{\frac{6}{5}\pi j}$	$e^{\frac{8}{5}\pi j}$	B1 B2	For $1 \leftrightarrow 1$ For non-identity elements
	G	1	3	9	5	4		
	or	1	4	5	9	3		
	or	1	5	3	4	9		
	or	1	9	4	3	5		
(iv)	Identity is (1, 1) Inverse of (9, 3) is (5, 4)						B1 B1	2
(v)	$(x, y)^5 = (x^5, y^5)$ Since G has order 5, $x^5 = 1$ and $y^5 = 1$ Hence $(x, y)^5 = (1, 1)$						M1 M1 A1 (ag)	3
(vi)	Order of (x, y) is a factor of 5 (so must be 1 or 5) Only identity (1, 1) can have order 1 Hence all other elements have order 5						M1 B1 A1 (ag)	3
(vii)	{ (1, 1), (9, 3), (4, 9), (3, 5), (5, 4) }						B2	2 Give B1 ft for 5 elements including (1, 1), (9, 3), (5, 4)
(viii)	An element of order 5 generates a subgroup, and so can be in only one subgroup of order 5 Number is $24 \div 4 = 6$						M1 A1	2 Or for $24 \div 4$ Or listing at least 2 other subgroups Give B1 for unsupported answer 6

Pre-multiplication by transition matrix

5 (i)	$\mathbf{P} = \begin{pmatrix} 1 & 0.07 & 0 & 0 \\ 0 & 0.8 & 0.15 & 0 \\ 0 & 0.13 & 0.75 & 0 \\ 0 & 0 & 0.1 & 1 \end{pmatrix}$	B2 2	<p>Allow tolerance of ± 0.0001 in probabilities throughout this question</p> <p>Give B1 for two columns correct</p>
(ii)	<p>If system enters an absorbing state, it remains in that state</p> <p>A and D are absorbing states</p>	B1 B1 2	
(iii)	$\mathbf{P}^9 \begin{pmatrix} 0 \\ 0.4 \\ 0.6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.2236 \\ 0.2505 \\ 0.1998 \\ 0.3261 \end{pmatrix}$ <p>Prob(owned by B) = 0.2505</p>	M1M1 A1 3	<p>For \mathbf{P}^9 (or \mathbf{P}^{10}) and $\begin{pmatrix} 0 \\ 0.4 \\ 0.6 \\ 0 \end{pmatrix}$</p>
(iv)	$\mathbf{P}^4 = \begin{pmatrix} 1 & \dots & \dots & \dots \\ \dots & 0.4818 & \dots & \dots \\ \dots & \dots & 0.3856 & \dots \\ \dots & \dots & \dots & 1 \end{pmatrix}$ <p>$0.2236 + 0.2505 \times 0.4818 + 0.1998 \times 0.3856 + 0.3261 = 0.7474$</p>	M1 M1 A1 3	<p>Using diagonal elements from \mathbf{P}^4</p> <p>Using probabilities for 10th day</p>
(v)	$(1 \ 0 \ 0 \ 1) \mathbf{P}^n \begin{pmatrix} 0 \\ 0.4 \\ 0.6 \\ 0 \end{pmatrix}$ <p>$= (0.8971)$ when $n = 26$ $= (0.9057)$ when $n = 27$</p> <p>i.e. on the 28th day</p>	M1 M1 A1 ft A1 4	<p>Considering \mathbf{P}^n for some $n > 20$</p> <p>Evaluating Prob(A or D) for some values of n</p> <p>Identifying $n = 26$ or $n = 27$ <i>(Implies M1M1)</i></p>
(vi)	$\mathbf{P}^n \rightarrow \begin{pmatrix} 1 & \mathbf{0.5738} & \mathbf{0.3443} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \mathbf{0.4262} & \mathbf{0.6557} & 1 \end{pmatrix} = \mathbf{Q}$	B2 2	<p>Give B1 for two bold elements correct (to 3 dp)</p>
(vii)	$\mathbf{Q} \begin{pmatrix} 0 \\ 0.4 \\ 0.6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.4361 \\ 0 \\ 0 \\ 0.5639 \end{pmatrix}$ <p>Prob(eventually owned by A) = 0.4361</p>	M1M1 A1 3	<p>Using \mathbf{Q} and $\begin{pmatrix} 0 \\ 0.4 \\ 0.6 \\ 0 \end{pmatrix}$</p>
(viii)	$\mathbf{Q} \begin{pmatrix} 0 \\ p \\ q \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0 \\ 0 \\ 0.5 \end{pmatrix} \quad (\text{where } q = 1 - p)$ <p>$0.5738p + 0.3443q = 0.5$</p> <p>$p = 0.6786, \quad q = 0.3214$</p>	M1M1 A1 ft M1 A1 5	<p>For LHS and RHS</p> <p>Or $0.4262p + 0.6557q = 0.5$</p> <p>Solving to obtain a value of p</p> <p>Allow 0.678 - 0.679, 0.321 - 0.322</p>

Post-multiplication by transition matrix

5 (i)	$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.07 & 0.8 & 0.13 & 0 \\ 0 & 0.15 & 0.75 & 0.1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	B2 2	<p>Allow tolerance of ± 0.0001 in probabilities throughout this question</p> <p>Give B1 for two rows correct</p>
(ii)	<p>If system enters an absorbing state, it remains in that state</p> <p>A and D are absorbing states</p>	B1 B1 2	
(iii)	$(0 \ 0.4 \ 0.6 \ 0) \mathbf{P}^9$ $= (0.2236 \ 0.2505 \ 0.1998 \ 0.3261)$ <p>Prob(owned by B) = 0.2505</p>	M1M1 A1 3	$(0 \ 0.4 \ 0.6 \ 0)$ and \mathbf{P}^9 (or \mathbf{P}^{10})
(iv)	$\mathbf{P}^4 = \begin{pmatrix} 1 & \dots & \dots & \dots \\ \dots & 0.4818 & \dots & \dots \\ \dots & \dots & 0.3856 & \dots \\ \dots & \dots & \dots & 1 \end{pmatrix}$ $0.2236 + 0.2505 \times 0.4818 + 0.1998 \times 0.3856 + 0.3261 = 0.7474$	M1 M1 A1 3	<p>Using diagonal elements from \mathbf{P}^4</p> <p>Using probabilities for 10th day</p>
(v)	$(0 \ 0.4 \ 0.6 \ 0) \mathbf{P}^n \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ $= (0.8971) \text{ when } n = 26$ $= (0.9057) \text{ when } n = 27$ <p>i.e. on the 28th day</p>	M1 M1 A1 ft A1 4	<p>Considering \mathbf{P}^n for some $n > 20$</p> <p>Evaluating Prob(A or D) for some values of n</p> <p>Identifying $n = 26$ or $n = 27$ (Implies M1M1)</p>
(vi)	$\mathbf{P}^n \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ \mathbf{0.5738} & 0 & 0 & \mathbf{0.4262} \\ \mathbf{0.3443} & 0 & 0 & \mathbf{0.6557} \\ 0 & 0 & 0 & 1 \end{pmatrix} = \mathbf{Q}$	B2 2	<p>Give B1 for two bold elements correct (to 3 dp)</p>
(vii)	$(0 \ 0.4 \ 0.6 \ 0) \mathbf{Q} = (0.4361 \ 0 \ 0 \ 0.5639)$ <p>Prob(eventually owned by A) = 0.4361</p>	M1M1 A1 3	<p>Using $(0 \ 0.4 \ 0.6 \ 0)$ and \mathbf{Q}</p>
(viii)	$(0 \ p \ q \ 0) \mathbf{Q} = (0.5 \ 0 \ 0 \ 0.5)$ <p>(where $q = 1 - p$)</p> $0.5738p + 0.3443q = 0.5$ $p = 0.6786, \ q = 0.3214$	M1M1 A1 ft M1 A1 5	<p>For LHS and RHS</p> <p>Or $0.4262p + 0.6557q = 0.5$</p> <p>Solving to obtain a value of p</p> <p>Allow 0.678 - 0.679, 0.321 - 0.322</p>

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