OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Examiners' Reports

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EXAMINERS' REPORTS

4751: Introduction to Advanced Mathematics (C1)

General Comments

This paper differentiated well across all abilities, with some questions accessible to most candidates, but some to challenge the best candidates. Few candidates scored very low marks. As expected, the questions found most difficult were 10, 12(iii) and 13(iii). Very few gained full marks; where just one or two marks were lost this was most often on question 10.

It is still the case that many candidates do not use brackets when they should. This was seen most clearly in question 5 with $2x^4$ instead of $(2x)^4$ and in question 13(iii) with $y-2+\sqrt{2}=-(x-4+\sqrt{2})$ for the tangent at A instead of $y-(2+\sqrt{2})=-(x-(4+\sqrt{2}))$, with consequent sign errors when simplifying.

Poor arithmetic was also evident at times from some candidates, particularly in question 5, where many did not manage to obtain the correct answer of 6000 after reaching 15 × 25 × 16, and question 11(i), where one root involved fractions.

There were three quadratic equations to solve, in questions 11(i), 12(ii) and 13(ii). The first was not difficult to factorise, but the formula involved large numbers and knowing the square root of 289. The second came out easily using the formula. In the third, the square was already completed, which made that method very simple. Candidates should be proficient in all three methods and thus able to choose whichever method is easiest in a particular question.

Some candidates possibly ran out of time, having lost time in question 5 (struggling to multiply), in question 10 (futile efforts) and in question 11(i) (having failed to factorise). However, the lack of an attempt at question 13(iii) may simply indicate that they did not know how to tackle this question, which was targeted at the better candidates.

Comments on Individual Questions

Section A

1 Most gained full marks for this question. Those who reached $13 > -4x$ rather than $4x > -13$ often ended up with the wrong inequality.

2 This was generally completed well with most candidates choosing to find the equation of the straight line ($y = 3x - 8$) before substituting to find k, instead of directly using the gradient of a line between two points. A small minority struggled with the negative number arithmetic involved, finding an incorrect y-intercept and hence k value.

3 Many candidates gained full marks on both parts of this question. Some candidates found part (i) of question challenging and did not have a clear idea of the meaning of fractional and negative indices. Candidates appeared to find part (ii) of the question easier than the first part, with a significant majority being able to find at least two of the three terms in the product correctly. Errors tended to be introduced by expanding $(2ac)^3$ as $2ac^6$ or $2a^3c^6$.

4 Many candidates gained all 4 marks, but there were some who translated in the wrong direction. Some candidates failed to read the question correctly and gave a description of the transformation without the coordinates, gaining no marks.

5 The stronger candidates produced a well-organised solution to finding the binomial coefficient, focusing directly on the term asked for. A few candidates calculated the fourth term in the binomial expansion instead of the coefficient of x^4. The expected error of using $2x^4$ instead of $(2x)^4$ was common. In spite of the 15 × 25 × 16 often being seen, many candidates involved themselves in long multiplication of 15 × 25 to work out the answer of 6000 instead of shortcuts of starting with 25 × 4 etc, with arithmetic errors common. Attempts to find the term by multiplying $(2x+5)$ by itself six times were nearly always unsuccessful.

6 This question was done well by most of those candidates who as a first step multiplied out two brackets. Those who attempted to multiply all three brackets at once were often unsuccessful. A few treated the expression as an equation = 0 and divided by 2 at the end, losing a mark.

7 Many candidates did this question well, knowing what a discriminant is, finding it, and knowing that a positive discriminant meant there were two real roots. Some used $\sqrt{b^2 - 4ac}$ as the discriminant and were able to gain two of the three marks. The majority of candidates obtained the mark for the number of real roots, but some thought that a discriminant of 1 meant there was one real root. A minority either did not read the question properly or did not know what a discriminant is, solved the equation by factorising and found the roots; some of these stated the number of roots and obtained 1 mark.

8 Many candidates gave the impression of being well practised in the steps required to change the subject of a formula and the correct answer was seen encouragingly often. Almost all candidates multiplied by $x + 3$ as their first step, but some later attempted to divide through by y and then did not divide every term by y; very few candidates completed successfully after dividing by y.

9 The less familiar form for the equation of the line caused some candidates problems as they were unable to correctly identify the gradient of the original line. Most realised that the area could be found easily by first finding the intercepts of L with the coordinate axes. A few attempted integration, often successfully.

10 The majority of the candidates factorised the given expression completely and correctly and obtained the first mark. However, few obtained more than 1 mark. Many simply showed that numbers divisible by 6 were obtained when several values of n were substituted, with some claiming proof by exhaustion! Some knew that it was not sufficient to try a few values of n and made unsuccessful attempts to use algebra. Some were able to obtain a second mark by a correct argument based on odd and even numbers. Only a small minority argued correctly that with three consecutive numbers, at least one must be a multiple of 2 and one a multiple of 3, making the expression divisible by 2 and by 3 and hence by 6. A few candidates who knew about proof by induction from FP1 attempted to use it but very few did so successfully.

Section B

11 (i) Most candidates knew how to tackle this question and made a good attempt. Many who reached the correct quadratic were unable to factorise it correctly or avoided factorising and resorted to using the formula, which was applied correctly by most. However few recognised that $\sqrt{289} = 17$ and so were unable to gain the correct simplified answers. Some had difficulty finding y from the fractional value of x. Some, presumably thinking that there must be an easier way of doing this, decided to try to eliminate x. However, they soon realised that this was less easy than before. Having correctly substituted for x into the

right hand side of the equation and simplified that quadratic expression, most ignored the *y* term on the left hand side and treated it as thought the left hand side was zero. As a consequence they were then left with the wrong quadratic equation which was even more difficult to solve.

(ii) The candidates' ability to deal with the method of completing the square seems to be improving and nearly half of the candidates gave fully correct answers. As would be expected, the main reason for some candidates failing to do this correctly was the fact that the x^2 term was a multiple of 4. Many of them successfully started off by taking out the factor of 4 from the $4x^2$, but they were then unable to determine how this affected the coefficients of the other terms, so it was quite common to see $4(x + 6)^2$ .... , or $4(x + 12)^2$ .... . When it came to determining the constant term it was quite common to determine the value of *c* as 31 – the value of their b^2 rather than 31 – 4 × their b^2 .So common incorrect final answers were $4(x + 3)^2 + 22$ or $4(x + 3)^2 - 113$.

(iii) Some candidates knew how to extract the required information from their completed square form, others clearly had no idea. Some started from scratch, using calculus. In part (*B*), some gave the coordinates of the minimum point rather than the minimum *y*-value that was requested. There were quite a few "No Responses" in part (iii).

12 (i) This was attempted well. Nearly all candidates were able to draw the line accurately on the diagram and most realised that they were required to read off the *x*-coordinates of the intersections, although some did not attempt to give roots. A few lost marks due to a loss of accuracy in one or more of the values – some did not realise that –2 was an exact answer, and others misread the *x*-scale. A very small minority of candidates appear to have attended the examination without a ruler or a sharp pencil and this affected their performance here.

(ii) The majority of candidates were able to derive the cubic and verify that –2 is a root. A range of methods was employed for the factorisation of the cubic and the majority of candidates did this successfully, although many then thought that the quadratic factor could be factorised. The phrasing 'exact form' and their answers to part (i) should have warned them not to expect this – and many of the better candidates did successfully find the other roots.

(iii) This was one of the least well done parts of any question on the paper. Most students did not know what to do, despite the similarity to part (i). Some drew curves, others did long calculations, attempting to find roots or to calculate a discriminant from the cubic equation.

13 (i) Nearly all candidates stated the centre and radius correctly.

(ii) There were a good number of fully correct solutions. A reasonable number realised that the equation could easily be rewritten in the completed square form and produced an efficient solution using this method. Those who multiplied out and used the quadratic formula were more prone to errors, but many did reach the correct answer. A few used $x = 0$ instead of $y = 0$; this gave the simple solution of $y = 2$ and so did not earn credit. Solutions using a geometric method were rare.

(iii) This was a challenging question, but there were many good attempts, although some struggled with manipulating the surds. A good number gained the mark for showing that A was on the circle, although some, having substituted the coordinates, did not simplify and attempted to square the unsimplified expression. Finding the gradient of the tangent caused some problems – most realised that they needed to start by finding the gradient of the radius at A, but were often unable to do so correctly. Some arrived straight at the correct tangent gradient by reasoning geometrically. Very many candidates failed to use brackets in their attempt at the equation of the tangent, resulting in sign errors.

4752: Concepts for Advanced Mathematics (C2)

General Comments

Most candidates presented their work well and made efficient use of the answer booklet. The question paper was accessible to the majority of candidates, and there was enough challenging material to stretch even the best candidates. However, some lost easy marks through using prematurely rounded values in convoluted methods, and a surprisingly high proportion of candidates evidently failed to read the question carefully and ignored specific demands – especially finding the volume in Q11 (iii). Most found section B more straightforward than section A.

Comments on Individual Questions

1 This was done very well by most candidates. However, a few either differentiated or simply calculated f(5) – f(2), thus scoring 0, and a few made arithmetic slips, often after a bracket error. The most common integration error was $\frac{3^2}{2}$ instead of 3*x*.

2 A small minority of candidates misunderstood what to do here, using a.p. or g.p. formulae, or attempting an algebraic definition. Most were able to correctly obtain the three terms demanded. However, the descriptions of the sequence were usually too vague to score, and even those who made use of the term divergent sometimes spoiled their answers with comments like "divergent to 0".

3 (i) Generally this was well done, but a surprising number of candidates ignored the instruction to present the answer to 4 s.f. and thus lost an easy mark. $\sqrt{1+2x} = \sqrt{1} + \sqrt{2x}$ was sometimes seen, and a few candidates evaluated $\frac{x_2 - x_1}{y_2 - y_1}$, which didn't score. A small minority of candidates attempted differentiation.

3 (ii) There were many good answers to this part: most realised that a shorter *x*-step was required, and only a few of these candidates spoiled their answer by only quoting 1 or 2 s.f. in their answer.

4 Many candidates were able to write down a correct pair of equations, but then automatically used logarithms and often went wrong. Gradient = - 2.4 was often seen. A significant minority only wrote down one correct equation and produced a large amount of futile work. Many of the minority who worked with the original equations were successful, however.

5 This was generally well answered, with a high proportion gaining full marks. Nearly all candidates found the correct *y*-value, but a few wrote $\frac{dy}{dx}$ = $32x^3$ + 4 or $24x^3$ and lost an easy mark. The gradient of the tangent was often found correctly, but some candidates then worked with this value, or with $\frac{1}{4}$ or in a few cases with - $\frac{1}{8}$.

6 A few candidates went straight to *y* = m*x* + c and didn't score, and some lost marks in integrating $6\sqrt{x}$. However, most knew what to do and obtained the first three marks. A few candidates substituted (4, 9) instead of (9, 4) and made no further progress, but a pleasing proportion of the cohort obtained full marks.

7 Many candidates were unable to make the correct initial step; a significant minority began with $\tan\theta = \frac{\cos\theta}{\sin\theta}$. Even those who did start correctly often made no further progress, and very few candidates found $\sin\theta = 0$ *and* $2\cos\theta - 1 = 0$. Generally it was the former which was lost in transit. A small minority of candidates managed to obtain all five roots by adopting a graphical approach.

8 There were many fully correct answers to this question. A few candidates spoiled their answer with an incorrect further simplification – cancelling out "log" was the usual one. The most common incorrect initial step was $n\log st$, but $\log s \times \log t^n$ was also seen often.

9 The assumption that $a = 10$ followed by efficient use of calculator was sometimes seen. Those who gave the answer as 12 scored full marks – anything else scored 0. Many candidates earned a method mark for $\log 16^{½}$ or $\log 5^2$, but a significant proportion were then unable to make a second correct move. Of those who did, the work was often then spoiled by subsequent incorrect working such as $\frac{\log 300}{\log 25} = \log 12$

Of those who failed to score, $16^{½} + 75 - 25 = 54$ was perhaps the most common answer.

10 Very few candidates understood this question, and convincing fully correct answers were seldom seen. The most common approach was $\sin(\theta + 180) = \sin\theta + \sin 180$ etc., which did not score.

11 (i) Most candidates appreciated the need to work from the area formula to produce the required result, and a considerable number of legitimate approaches were seen. The algebra was too much for many weaker candidates, and the award of just the first method mark was not uncommon. A small number of candidates tried to work backwards, and they were very rarely successful.

11 (ii) This part was done very well indeed. Some candidates lost marks by omitting π or occasionally 100, and surprisingly many had the wrong sign for the second derivative. A few candidates tried to differentiate $\pi r^2 h$ and made no progress.

11 (iii) Most managed at least M1, but a few candidates set $V = 0$ or $\frac{d^2V}{dr^2} = 0$, and a few worked with > instead of "=". A number of them were unable to evaluate $\sqrt{\frac{100}{3\pi}}$ correctly, r = 10.23 was often seen. A good number of candidates ignored the demand for an answer correct to 3 s.f., or neglected to go on and find the volume, this losing an easy mark. Many candidates wasted time testing the sign of the second derivative, and a small number then went on to give negative answers for the radius and volume if their sign for the second derivative was wrong.

12 (i)(A) This was very well done, although a few candidates gave the answer as £380.

12 (i)(B) This was generally done well. Many candidates made minor bracket errors but recovered with later work. A few laboured by writing out all the terms and some failed to score because they failed to show enough detail for a "show that" demand.

12 (ii)(A) This was done very well. A few candidates worked out all the terms one by one and failed to score due to a loss of accuracy during the process.

12 (ii)(B) This was very well done, although 313.38 was a common error for Mary's 20th payment.

12 (ii)(C) This was done very well by most candidates. A few used $r = 0.02$ or 1.02 and failed to score, a small minority used $n = 23$ and a few used the corresponding a.p. formula. Those who used a trial and error approach almost never scored.

12 (ii)(D) There were many good answers to this question from candidates who had done well in the previous part, but some candidates lost a mark through premature rounding.

13 (i) Those who worked in radians often scored full marks, although some candidates just found the area of the sector as if on autopilot and ignored the length of the greenhouse. A good number of candidates preferred to work in degrees, and a proportion of these inevitably made errors in the conversion process and lost marks. A few candidates converted θ to degrees and then multiplied it by 2.1.

13 (ii) This should have been a straightforward exercise in finding a length in a right angled triangle, but many went astray because they were unable to find angle *CBD* correctly. Others chose a convoluted method such as the Sine Rule or finding CD first and using Pythagoras, and then made algebraic or arithmetic slips. It was the exception, rather than the norm, to see the expected approach of $2.1\times\cos(\pi - 1.8)$ leading to the correct answer. Some candidates lost the accuracy mark because their calculator was in the wrong mode.

13 (iii) Most candidates were able to find the area of the sector correctly, but too many candidates made finding the area of the triangle far more complicated than necessary. The usual approach was to find *CD* and then use ½base × height, and marks were often lost due to mistakes in applying the Pythagoras formula. As with part (ii), the expected approach ($\frac{1}{2} \times 2.1 \times BD \times \sin(\pi - 1.8)$) was the exception rather than the norm. Most realised the need to sum the two areas and then multiply by 5.5.

4753/01: Methods for Advanced Mathematics (C3) (Written Examination)

General Comments

There was the usual wide range of responses to this paper, with some excellent scripts in the high 60s with very well-presented solutions, to scripts which failed to achieve 20 marks. In general, there were lots of opportunities for all suitably prepared candidates to show what they could do, and only one part question – the proof in 7(ii) – which failed to attract plenty of fully correct answers.

In general, questions on calculus are well answered, showing sound knowledge of product, quotient and chain rules and integration by parts and substitution. Questions involving algebra with e^x and ln x, the modulus function, and the language of functions are perhaps less well done.

Candidates seemed to have sufficient time to answer all the questions. It is perhaps worth pointing out that when they offer a number of different solutions, it is the last attempt which is marked, not the best! So, if offering more than one solution, they should decide which is their best attempt, and cross out others. It is also worth emphasizing that when they are required to 'show' a given result on the paper, they should make sure that all the relevant steps in the argument are included.

Scripts are now scanned in for marking. It aids the overall legibility of scanned-in work if candidates do not write over partially erased pencil, or over-write existing work, and to be especially careful when writing negative signs.

Comments on Individual Questions

1 Most attempted this by considering $\pm(2x - 1) = \pm x$, some thinking that this led to four different possibilities, and indeed finding more than two solutions by faulty algebra. A few squared both sides, found the correct quadratic, and solved this by factorising or formula. Some had no idea how to start and tried to manipulate the equations with modulus signs, often ending with the answers like $|x| = 1$ or $x = \pm 1$. Others thought the modulus signs implied inequalities and either replaced them with these or introduced them into their answers.

2 Most candidates managed to get 1 mark out of 3 by writing the composite function correctly as $e^{2\ln x}$. This was often simplified by cancelling the e with the ln to get $2x$. Occasionally, they formed the composite function in the wrong order, viz 2 ln e^x, simplifying this to $2x$. Another error was to write $gf(x) = g(x)\,f(x) = e^x.2\ln x$, though this was relatively rare.

3 (i) This was well answered, with the majority using the quotient rule correctly, though a significant number failed to cancel the common factor of x. The product rule works equally well for this question, and was seen occasionally.

(ii) The key to this integration by parts was selecting $u = \ln x$ and $dv/dx = 1/x^2$. Other choices gained no marks. There was evidence of 'stockpiling' negative signs in the subsequent parts formula – students could be encouraged to resolve these as they progress through the working. As the answer was given, we needed to see $\int v\,u'dx$ as $\int 1/x^2 dx$ before integrating this to get $-1/x$.

4 (i) Most students got either full marks, or gained one mark out of three for $a - b = 0.5$, thereafter making no further progress, as they failed to recognise that 'long-term' meant $e^{-kt} \to 0$, to get $a = 10.5$.

(ii) Generous follow-through was allowed on incorrect (even manufactured) values of *a* and *b*, and candidates usually showed good understanding of re-arranging the equation and taking lns of both sides to get $k = -\ln[(a - 6)/b] / 8$. Occasionally, though, their values led to the ln of a negative number, and they lost the second mark by fiddling this. Not all rounded to 2 decimal places correctly, and 0.01 after correct working was not uncommon.

5 Most candidates handled the product and chain rules well and gained three marks for a correct expression. Occasionally the multiplication by 4 was missing in the derivative of $(1 + 4x)^{1/2}$. In general, more aimed to find a common denominator than to factorise when simplifying: plenty of good work was seen. The most common reasons for loss of marks were:

- letting the 2 slide into the denominator: $x^2.2(1 + 4x)^{-1/2}$ becoming $x^2/ [2 (1+4x)^{1/2}]$;
- insufficient detail/evidence in the final stages.

6 (i) Failure to express sin(π/3) and cos (π/6) in exact form led to this mark being lost frequently: simply writing sin(π/3) + cos (π/6) = √3 was not sufficient.

(ii) Generally, the implicit differentiation of sin 2*x* + cos *y* was handled well. Errors included:

- omitting the '2' in 2cos 2*x* term,
- sign errors in the derivatives of sine and cosine,
- starting d*y*d*x* = 2 cos 2*x* – sin *y*d*y*/d*x* (= 0), and incorporating the superfluous d*y*/d*x* into the subsequent equation,
- faulty differentiation of the constant √3.

The calculation of the gradient at (π/6, π/6) by substituting for *x* and *y* in their derivative was well done, but awarded only if they made a reasonable attempt at the implicit differentiation.

7 (i) Most handled this expansion correctly, though $3^n \times 3^n = 9^{2n}$ was not uncommon.

(ii) Correct solutions to this question were the preserve of A* candidates. Most attempted it by verifying the result for various values of *n* (some quoting 'proof by exhaustion' after checking *n* = 1 to 9). Some indeed used negative numbers hoping to prove it *only* worked for positive. A popular argument was that as 3^{2n} is a multiple of 9, $3^{2n} - 1$ must therefore be a multiple of 8. Those considering the two factors made better progress, some getting as far as even x even, but failing to spot that one of two consecutive even numbers must be a multiple of 4. [The neatest solutions are using the binomial expansion of $(8 + 1)^{2n}$ and induction. Both of these were seen, and were highly commendable.]

8 (i) This was well done, with virtually all candidates being familiar with how to show that f(-*x*) = f(*x*), and the symmetry of the even function in the *y*-axis, though this property might on occasions have been expressed more accurately ('It reflects in the y-axis' being a typical example).

(ii) Of those candidates who did not start by saying f(*x*) = 1/e^x + 1/e^{-x} + ½, the rest divided equally between using a chain or quotient rule. A rather surprising minority multiplied e^x by $-e^{-x}$ to obtain the derivative of the bracket. A lack of brackets and/or careless positioning of minus signs led to some marks being lost, and some forgot the –2 power in their answer when using the chain rule. A few thought that f'(*x*) was the inverse rather than the derivative.

(iii) This little piece of algebra was often well done, provided that candidates did not start with f(x) =1/e^x + 1/e^{-x} + ½. They should be discouraged from starting with the given result, cross-multiplying it and then working towards 1 = 1. There were some errors in expanding the brackets, or multiplying out powers of e, e.g. e^x x $e^{-x} = e^{-2x}$.

(iv) This question was not quite so well answered. It is helpful if candidates start by writing down, in terms of *x*, the integral they are trying to work out. The integration by substitution to get $\int 1/u^2\, du$ suffered from a number of errors: some just replaced d*x* by d*u* (or, omitted both d*x* and d*u* entirely, with the same result). Others moved non-constant terms to the front of the integral, or left a mixture of *u*'s and *x*'s. The integration of $1/u^2$ then caught out others, with answers such as $\ln(u^2)$ or $u^{-3}/(-3)$. It was not uncommon to see the lower limit for *u* as 1, and a few candidates changed back to *x* but substituted limits for *u*. Finally, there were numerical answers, and exact answers but with e^1 not evaluated as e.

(v) Virtually all candidates scored the first mark for equating the two functions. Thereafter, sorting out the equation to get a quadratic in e^x was done with varying degrees of success. Some took lns incorrectly; many failed to divide through by e^x, and ended up with a cubic in e^x. A number of those who got as far as $(e^x + 1)^2 = 4$ failed to consider $e^x + 1 = -2$. Some arrived at (0, ¼) fortuitously, notwithstanding algebraic errors. This failed to secure any marks.

9 (i) Candidates were split between algebraic approaches, using two of the three points to find *a* and *b*, using transformation of the *y* = sin *x* graph, and a mixture of the two. However, as the values of *a* and *b* are given, we were fairly strict in requiring evidence. Thus, simply pointing out that the *y*-intercept was 2, so *a* = 2, was not enough, especially as a number of candidates quoted '*y* = *mx* + *c*' as their justification for this. If using a stretch to show that *b* = ½ , a scale factor of 2, not ½ , as well as the direction of the stretch was required. Verifying that *a* = 2 and *b* = ½ at points A and B was not quite enough to secure all three marks.

(ii) The gradient at (0,2) was generally well done, with most candidates scoring the first 3 marks. However, good justifications of this being a maximum were less common. Many candidates simply looked at the gradient at specific values of x either side of 0 rather than using the fact that cos (*x*/2) has a greatest value of 1, or using the second derivative.

(iii) The majority had no difficulty finding $f^{-1}(x)$, though some mistakes occurred in re-arranging the equation, e.g. $x - 2 = \sin y/2 \Rightarrow y = \arcsin [2(x - 2)]$. Precision in specifying the domain and range, with $\le$ rather than $<$ signs, and correct use of x and y or $f^{-1}(x)$ was required. This cost some candidates marks. The gradient of the inverse function was done with varying success, with -2 or ½ as common errors.

(iv) The first three marks were easy for many, but there was plenty to be done for the last mark, which was frequently lost because of poor handling of the lower limit, zero. Where other marks were lost it was usually for poor integration of $\sin (x/2)$ or, surprisingly frequently, forgetting the integral of the constant 2.

4754: Applications of Advanced Mathematics (C4)

General Comments

There were questions in this summer's paper that were accessible to all candidates and few very low scores were obtained. There were also sufficient questions to challenge the more able candidates and few fully correct scores were seen. Questions 5, 7 and 8 provided the greatest challenge.

The comprehension was well understood and good scores were obtained here by most candidates.

As usual, loss of marks often followed poor algebra including errors when using negative signs and the absence of appropriate brackets. When integrating, it is still disappointing to see so few candidates adding the constant and then showing its evaluation. In this case, in 8(v), candidates, on seeing the given answer, too often either ignored '+c' completely or just wrote c=0 without justification.

The standard of work was, in general, good and the presentation was of a satisfactory standard.

Questions that were based on familiar methods were answered better than those where candidates had to think more for themselves.

Comments on Individual Questions

Paper A

1 The partial fractions method was well known and many candidates scored the full five marks. Candidates seemed well prepared for this question. There were some arithmetic errors and a few only put a constant on the numerator of the quadratic factor. Although it was not penalised on this occasion, it was disappointing to see so many candidates who found A, B and C correctly could not then accurately assemble the final result.

2 This binomial expansion question proved to be highest scoring question in Paper A. Few used an index other than 1/3. It was pleasing to note that unlike on some previous papers, most candidates did attempt the validity. On this occasion the expansion was valid at the endpoints but we accepted both inequality and 'less than or equal to' for both marks.

3 The first four marks were usually obtained. Candidates seemed to be well prepared for the 'R-method' and the number achieving full marks has improved. The most common error was the use of degrees instead of radians.

The final part, when finding the greatest and least possible values, caused some confusion. Whilst there were many completely correct solutions some omitted the 1 and just gave $\pm\sqrt{13}$ and many others tried to find angles from say solving $\sqrt{13}\sin(\theta - 0.983) = -1$ for θ.

4 (i) Quite a number of candidates overlooked the requirement to find the coordinates of *x* and *y* and so unnecessarily lost two marks.

Most candidates understood the method for finding the gradient of the curve but there were errors in the differentiation. Good candidates scored well here. Common errors included, $y=\cos 2\theta$, $dy/d\theta=2\sin\theta$, or $-2\sin\theta$, or $-\sin 2\theta$, or $-1/2 \sin 2\theta$. Some of the incorrect differentiation lead to apparently the correct answer but obtained from wrong working.

(ii) Those that started with the correct double angle formula were usually successful here although some candidates left both *x* and θ in the answer.

5 This was a harder trigonometry question than on recent papers and few candidates scored the full six marks.

When using the first method, many failed to use the correct trigonometrical identity. Then, a common error was to 'lose' one of the factors eg $\cot^2\theta-2\cot\theta=0$, $\cot^2\theta=2\cot\theta$, $\cot\theta=2$. Others obtained $\cot\theta=0$ but did not find the solutions 90° and -90° (considering the graph of $\cot\theta$ or using $\cot\theta=\cos\theta/\sin\theta$ may have helped), often 180°, 0°, and -180° being given.

Those that chose to express their equation in terms of $\sin\theta$ and $\cos\theta$ from the outset usually obtained $\sin^2\theta+2\sin\theta\cos\theta-1=0$. Much then depended upon them using Pythagoras to obtain $2\sin\theta\cos\theta-\cos^2\theta=0$ and factorising. Once again, $2\sin\theta=\cos\theta$ was often seen and the $\cos\theta=0$ was forgotten.

Candidates would be advised to factorise instead of cancelling.

6 Many candidates obtained full marks in this volume of revolution question. Common mistakes included;

for the cone

- using the incorrect value of *h* (often *h*=3), or the incorrect value of *r*
- incorrectly expanding $\int_2^3 (3-y)^2 dy = \int_2^3 9+y^2 dy$
- omitting a negative sign $\int (3-y)^2 dy = (3-y)^3/3$
- using the wrong limits
- finding the area of a triangle

for the curve

- using the wrong limits (often 0-1 or 0-3)
- using the wrong substitution for x^2 (often $x^2=y^2/4$)

Most candidates did add their two parts together. The presence of the cone formula proved not to be a sufficient hint to deter many candidates from integrating.

7 (i) Many candidates scored full marks when finding the angle between the vectors. The vectors were almost always correct. The most common error was, after having obtained the vector AB correctly at the start then using it as $\begin{pmatrix} -4 \\ 0 \\ 2 \end{pmatrix}$ when calculating the angle. This was a surprisingly common error.

(ii) A variety of methods were used when verifying the equation of the plane. These were often successful. For those who substituted points (which was quite common) the most frequent error was to substitute fewer than three points. For those who tried to establish the result by finding the scalar product with two vectors in the plane, the most common errors were that either they only used one vector or that they failed to substitute a point in order to find d.

There were some confused candidates who tried to substitute the vectors in the equation for the plane or to use position vectors of the points when finding the scalar products. Some confused d with –d.

A few used the vector equation of the plane, often with success, especially when substituting the general point into the plane equation.

The normal vector was usually found and used but candidates did not often find its scalar product with $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ having misunderstood how to find the angle the given plane

(iii) makes with the horizontal plane.

In the first part there were some circular arguments where CD was assumed to find k and then k was used to find CD. Those who realised they were required to substitute the coordinates of D in the equation of the plane were usually successful.

A large number failed to understand how to show that ABDC was a trapezium. The majority attempted to find vector CD (or DC), but did not always state it explicitly, and then correctly found the ratio of CD to AB-often by finding the lengths and stating $\sqrt{5}$:$\sqrt{20}$. Unfortunately this was often cancelled to 1:4.

8 Some candidates found this question difficult even though they were lead through it in stages and some straightforward marks were available.

(i) If they differentiated V=1/3 x^3 they were usually successful here.

(ii) There were many correct solutions in this part but those who failed to include +c or failed to substitute x=10 at t=0 lost marks.

(iii) This was usually successful. The most common error was √ (100-100k) =√100 -√100k.

(iv) Few candidates realised that they needed to start with dV/dt =1-kx. Those that did were usually successful.

(v) The algebra at the start of this part was poor. Negative signs missing or fiddled were often seen. A common error was $\frac{1}{1-x} - x - 1 = \frac{1-(1-x)(x-1)}{1-x} = \frac{x^2}{1-x}$. For the integration, separation of the variables and $\int dt = \int \frac{1}{1-x} - x - 1dx$ was needed. Many failed to work with both sides of the equation. This then needed to be integrated and the constant of integration included, together with its clear evaluation. Most omitted the constant merely giving the given answer. Unnecessary marks were lost.

(vi) An appreciation that 1/0 or ln(1/0) is undefined was needed. There were many correct solutions. Some incorrectly gave ln (1/0) as ln 0 or 0.

Paper B

The Comprehension

1 This was very successful except when the evaluation was not shown.

2 (i) The most common error was not starting with 10,000. 10P4 was one of the commonest alternative starting points. Others included 10!,10!x4! and 10C4. Another common error was to assume that there was the same number of consecutive digits in both ascending and descending orders when these were not the same. The most common mark was M0 M1A0 followed by M1M1A0. There were few completely correct answers.

(ii) Candidates were allowed to follow through their result from (i) and still obtain full marks. Although the method was usually correct, answers were not given to an appropriate degree of accuracy as required in the question.

3 This was almost always correct.

4 Most candidates showed the correct method, with the failure to divide by 80 the most common mistake. However, there were a large number that did not correct this to the desired integer answer. There were also some interesting numbers of days in a year including 265.

5 The majority of candidates were not successful here and many omitted this part. The most common correct answer was that 'An attack can happen without a breach of the card's security'.

6 Candidates scored well in this question.

The most common error in the first part was putting 80 rather than 60 in the middle of the bottom row (and 60 not 40 above it).

In the second part answers were allowed as follow through marks from part(i) as were the method marks in the third part. The most common incorrect answer in the third part being 2425:5 with ratio 485:1. There were some unusual values in the tables following unclear methods. These occasionally included decimals, fractions and negative numbers.

4755: Further Concepts for Advanced Mathematics

General Comments

There were many scripts showing good understanding and ability. The majority of candidates were well prepared, and most had sufficient time to cover all the questions. There were some surprising lapses in elementary algebraic work, not confined to the lower-scoring candidates. On the whole solutions were clearly written, if not always fully logical. Diagrams were sometimes difficult to decipher. After scanning, alterations can make the intention obscure. Pencil for diagrams is really essential, together with a quality eraser, in case of mistakes.

Comments on Individual Questions

1 Generally this question was well done.

(i) A surprising number of candidates did not obtain the correct matrix for this rotation. Several candidates had learned the general form for a rotation, and wrote it in terms of sin 90 and cos 90, unsimplified, but usually correct. Some errors in signs occurred.

(ii) This was usually correctly answered, but there were instances when the identity matrix was given, which should have rung alarm bells in part (iii).

(iii) Follow through marking here allowed some recovery from earlier errors, but there were many instances of matrices written in the wrong order.

(iv) It was not essential to answer this from the result of (iii), and possible to check the previous result by thinking through the transformations. Some candidates thought that the description was achieved by writing down the matrix again.

2 This question was also well answered by many.

(i) A good response. Nearly all candidates knew how to use the complex conjugate, but some obtained 15 instead of 17 in the denominator.

(ii) The modulus of w was usually correct, but the argument was often found from arctan 0.25 or arctan(-0.25) without adjustment. Several candidates gave their argument in degrees, and some rounded it inappropriately. Not all candidates wrote down the required form for w after finding these components.

(iii) The mark for placing w at (-4, 1) was often the only one scored here, and sometimes this was lost through the lack of adequate annotation. It was not uncommon for the argument to be wrongly shown, and sometimes two angles were indicated, with the choice left for the examiner. A few diagrams showed the locus $\arg(z - w) = \arg w$, and had nothing available with which to indicate the modulus.

3 Almost all scored two marks for p, although some gave the wrong sign. Most candidates attempted $(\alpha+\beta+\gamma)^2$ in order to find q, but often with errors. A few candidates assumed that the roots were 1, 1 and 2, which satisfied the two equations given but did not fit with the constant term in the cubic.

4 This question was not well answered, except by a very few. Most candidates chose to try to solve the equation $\frac{5x}{(x^2+4)} = x$, but errors were frequent, for example $x^3 + 4x + 5x$ or $x^3 + 4x - 5$ instead of $x^3 + 4x - 5x$. The critical value of $x = 0$ was often lost because of division by x. It was also common to see $x^2 + 4$ factorised to $(x - 2)(x + 2)$ and asymptotes drawn when a sketch was attempted. Even when the correct critical values were found, many candidates could not follow a coherent method to obtain the intervals required.

5 This question was well answered. The main mistake was to forget to multiply by 1/3. Some multiplied by 3. Some candidates gave the sum of n terms without substituting $n = 20$. A small minority summed twenty fractions with their calculators which did not demonstrate familiarity with the specification. A smaller minority thought that the standard results for $\sum r$ and $\sum r^2$ could be used in the denominator.

6 Many good, carefully worded answers were seen. It is a shame when otherwise good work is spoiled by lack of attention to details, for example $\sum_{r=1}^{k} n^3$ or $\sum_{r=1}^{k} k^3$, or even the statement that $k^3 = \frac{1}{4}k^2(k+1)^2$. There are still candidates who do not use the formal "if…then…" statement in the concluding stages of the argument, and also those who do not appreciate that answering the question requires rather more than a sketchy indication of working.

Candidates who could not factorise the expression $\frac{1}{4}k^2(k+1)^2 + (k+1)^3$ and multiplied out gained credit if they also showed that this matched the expansion of the target expression, but some candidates lost marks by failing to show how the quartic they obtained then factorised to the target, quoting the result without working.

7 Overall this question was well done.

(i) When asked for co-ordinates, some candidates cannot bring themselves to write down pairs of numbers in brackets.

(ii) Some candidates do not like to write down clearly three separate equations.

(iii) Clear methods were usually shown, where working or a result of a calculation was needed beyond a statement of extreme values for x. Some candidates thought that "positive" or "negative" was enough to show how the curve approached y=3.

(iv) Many good carefully drawn sketches showed the salient features with full annotation. However, some candidates believed that "correct" scales were needed, which is not necessary. Through lack of space, in many cases this led to the curves not showing clearly the approaches to the asymptotes nor the precise values of x and y where the axes were crossed and the asymptotes positioned. Some diagrams suffered from alterations which were difficult to interpret.

8 (i) Most candidates had the right idea but many could not express it clearly. A full explanation needed to refer to complex conjugates, more than just "pairs". It was also important to make some mention of the number of roots. Although it may not have forfeited the mark, quite a lot of candidates believed that P(z), as yet unspecified, would have only four roots, rather than a minimum of four roots.

(ii) Candidates who used the factors and expanded were usually more successful than those who used the root relationships. Factors with incorrect signs were seen in some scripts. In both methods, there were commonly mistakes in multiplications, for example writing $(2j)^2$ as $2j^2$ and then obtaining -2. When the root relationships were used some candidates failed to use all six terms in $\sum\alpha\beta$.

(iii) Most candidates were able to show the positions of the four roots but a few gave no indication of scale. Again, alterations were sometimes confusing. Not many candidates were able to give the correct circle equation.

9 This question was well done by many but some candidates were probably running out of time.

(i) Generally well answered.

(ii) Most candidates gave the correct value of *k*, a few failed to get the right sign.

Some candidates gave $\mathbf{M}^{-1}$ using $k = 5$ instead of "in terms of *k*". Most candidates obtained the values of *x* and *y* through the correct method, with $\mathbf{M}^{-1}$ in the right place. A few lost marks through using another method.

(iii) A number of candidates wrote "no solution" or "can't be solved", without the other possibility.

(iv) The first situation (A) was usually correctly described, but (B) and (C) were often confused. A number of answers referred to equations instead of describing lines. There were several instances of "no response".

4756: Further Methods for Advanced Mathematics

General Comments

The total number of candidates was about the same as in Summer 2010, and the overall standard of work was also about the same. Some candidates were very well prepared for the paper and scored highly throughout; about 20% scored 60 marks or more. Others appeared well prepared for some of the paper, scoring highly on one or two questions but achieving much lower marks on the other questions. These questions varied, so there was little difference between the means: Questions 3 and 4 were the best done by a small margin, with Question 2 scoring the lowest. Only a very small minority of candidates seemed to be unprepared, with about 5% scoring fewer than 20 marks. Few candidates attempted Question 5.

Very few candidates appeared to run out of time, although inefficient methods often caused much waste of time and paper, especially in Question 3 (iv). Most candidates' work was presented coherently and legibly. From January 2012, this paper will have a printed answer book.

Comments on Individual Questions

1 (Polar curves, calculus with trigonometric functions)

About a fifth of candidates found this a most agreeable start and achieved full marks. Throughout the question, a very small number of candidates appeared unaware of the need to use radians when integrating with trigonometric functions.

(a) Most candidates were able to score both marks for the sketch of the cardioid in part (i), although some curves could have been more obviously symmetric, and others made the shape pointed at the bottom. A sketch which itself earns marks should be completed more carefully than a sketch which, for example, just aids the solution of an inequality.

For the area in part (ii), the vast majority of candidates knew what to do, but often failed to score all seven marks through inaccuracies such as losing the ½ or a^2 at some point, using an incorrect double angle formula to substitute for $\sin^2\theta$, using incorrect limits, or even failing to expand $(1 - \sin\theta)^2$ correctly. A small minority of candidates seemed unaware of how to deal with the integration of $\sin^2\theta$, giving as their result $\frac{1}{3}\sin^3\theta$ or similar.

(b) Part (i) was done well, with many candidates able to write down the result of the integration immediately, although a number omitted the factor of ½.

Part (ii) discriminated well. Most of those who began with the correct substitution were able to score most of the marks. It was interesting to see a few candidates successfully using the substitution $x = \frac{1}{2}\sinh u$. Far more common were substitutions which led nowhere, e.g. $u = 1 + 4x^2$. A small number of candidates gave the correct exact answer with no working: these had obviously used their calculators to perform the integration. No credit was given for this approach.

2 (Complex numbers)

A surprisingly large number of candidates attempted the question but scored no marks at all.

(a) The majority of candidates were able to score the first four marks in this part. They were able to expand $(\cos\theta + j\sin\theta)^5$ correctly and separate the real and imaginary parts, although a few were determined to use the $z \pm z^{-1}$ method. A significant proportion then went on to do substantial work to express $\cos 5\theta$ in terms of $\cos\theta$ and $\sin 5\theta$ in terms of $\sin\theta$. This attracted no further credit and had the unfortunate effect of making the demonstration of the given expression for $\tan 5\theta$ much more difficult, with those who took this approach having to use the identity $1 + \tan^2\theta = \sec^2\theta$.

(b) Candidates who recognised that the modulus and argument of $-4\sqrt{2}$ were $4\sqrt{2}$ and π respectively had little trouble with parts (i) and (ii). However, a substantial number gave the modulus as $-4\sqrt{2}$, and the argument as 0, which lost them a significant number of marks. The question did ask for the fifth roots in the form $re^{j\theta}$ with $r > 0$ and $0 \le \theta < 2\pi$. Arguments in the range $-\pi$ to π were also accepted.

In part (iii), most candidates were able to give the argument of *w* correctly, but relatively few managed to connect the modulus with a length which could be found by elementary trigonometry. This length was sometimes negative. The value of *n* was often correct in part (iv), but a correct value of *a* was rare.

3 (Matrices and simultaneous equations)

Part (iv) was the discriminator: many candidates managed most of the first 11 marks without too much trouble.

(i) Candidates usually manage to find the determinant and inverse of a 3 × 3 matrix with little difficulty, and this series was no exception. Some interesting methods were seen, such as using a scalar triple product to find the determinant and the vector products of columns of **M**, but most candidates stuck to the more familiar method in which they appeared well versed. There were, as usual, slips in some of the cofactors. A few candidates, having obtained the determinant correctly as $6 - 2k$, "simplified" it to $3 - k$.

(ii) Multiplying **M** by a column vector presented very few problems.

(iii) Again, this part presented few problems, with the vast majority of candidates noticing that the column vector in part (ii) represented an invariant point (which was frequently expressed as e.g. "transforms to itself") or was an eigenvector with eigenvalue 1. The last of these observations was occasionally omitted.

(iv) A variety of approaches was possible here. The majority of candidates began by eliminating unknowns, and a high proportion made elementary errors trying to do this: they would have done better to write out all their working, rather than trying to do it mentally. Errors here often led to candidates substituting incorrect expressions into other expressions, which in turn led to a downward spiral which wasted a great deal of time. An alternative approach which was sometimes taken was to find a point on planes (2) and (3) and substitute it into plane (1): this found the value of t efficiently, but candidates could not always develop this method to find the general solution, which most did recognise as a line, or "sheaf" which was also accepted. Some candidates were determined to use the inverse matrix, even though they also correctly stated that the solution was not unique. Having identified one eigenvector and eigenvalue earlier in the question, a few candidates were determined to use this part to find the other eigenvalues (and, sometimes, eigenvectors): this was not required and gained no credit.

4 (Hyperbolic functions)

(i) Most candidates were able to score the first four marks here by deriving a quadratic equation in e^y and giving both roots, although one or two candidates just stated that 'the result appears in *Examination Formulae and Tables*, so it must be true'. The remaining three marks were gained much less frequently. The majority of candidates took the ± sign outside the log expression without an attempt to explain why, and the explanation of why we take arcosh x as the positive root was often spurious.

(ii) This part was done well. As in Question 1(b), many candidates were able to write down the result of the integration immediately, using arcosh or the logarithmic form given in *Examination Formulae and Tables*. A number even went straight to an expression involving $\ln(5x + \sqrt{25x^2 - 16})$. The most common error was in dealing with the factor of $\frac{1}{5}$; many missed it out entirely, or gave $\frac{1}{4}$ or $\frac{4}{5}$ or $\frac{5}{4}$.

(iii) This part was also done well. Most candidates substituted for cosh $2x$, solved the resulting quadratic and used part (i) or an equivalent method to obtain the root $\ln(2 + \sqrt{3})$. This scored five marks out of six. The other root, $-\ln(2 + \sqrt{3})$, was seen much less frequently. A number of candidates used an incorrect substitution for cosh $2x$, while others changed everything to exponentials: this resulted in a quartic equation in e^x, which candidates had to factorise into two quadratic factors to gain any credit. A few managed this, but correct answers by this route were very rare.

5 (Investigations of Curves)

Fewer than 2% of candidates answered this question, and their attempts were often fragmentary: only a handful scored 10 marks or more.

(i) Candidates often ignored the instruction to consider values of x between 0 and 1 and used the default setting for their x values on their graphical calculators. The result was that it was very difficult to see differences between their four curves. We expect sketches to be clear enough to show important features.

(ii) Most were able to say that the graph was symmetric when $m = n$ and could describe the effect of interchanging m and n.

(iii) The description of how the maximum varied was rarely sufficiently precise. Most could use the product rule to differentiate, but only a few were able to obtain the value $\frac{m}{m+n}$.

(iv) Very few candidates attracted credit here. Most asserted that *m* and/or *n* should be zero, ignoring the fact that the question states that they are positive.

(v) Most were able to see that the curve approached the *x*-axis and (especially) that the integral tends to zero.

(vi) There were some appropriate sketches and descriptions.

4757: Further Applications of Advanced Mathematics (FP3)

General Comments

The work on this paper was generally of a high standard, and there were many excellent scripts, with about a third of the candidates scoring 60 marks or more (out of 72). Almost all candidates made substantial attempts at three questions, with just a very few answering an additional fourth question. The most popular question was question 1, followed by question 2 then question 5, question 4 and question 3. The average marks achieved on the five questions were similar, ranging from about 16 (out of 24) on questions 1, 3 and 4 to about 18 on question 5. A fair number of candidates were clearly rushed at the end and were unable to complete their final question; this was almost always caused by spending too long using over-complicated methods in question 1.

Comments on Individual Questions

1 *(Vectors)*
In part (i) the perpendicular distance from a point to a plane was usually found correctly. Part (ii) was very often not immediately recognised as the intersection of two planes, with lengthy, and often incorrect, methods being attempted instead. This work was frequently crossed out and a more appropriate method, such as finding the vector product of the two normals, was then adopted. However, several candidates were unable to obtain any answer for this part, and many gave a wrong equation; then using their incorrect answer they could obtain most of the marks in the remainder of the question. In parts (iii) and (iv) most candidates used efficient methods to find the shortest distances from a point to a line and between two skew lines, although in part (iii) a fairly common error was to use a scalar product instead of the vector product in the formula $\left|\overrightarrow{\mathbf{AC}}\times\overrightarrow{\mathbf{AD}}\right| / \left|\overrightarrow{\mathbf{AD}}\right|$. In part (v) almost all candidates knew that the volume of the tetrahedron was given by the scalar triple product $\frac{1}{6}(\overrightarrow{\mathbf{AB}}\times\overrightarrow{\mathbf{AC}}).\overrightarrow{\mathbf{AD}}$ (although the $\frac{1}{6}$ was sometimes omitted). However, $\overrightarrow{\mathbf{AD}}$ was often put equal to the position vector of D. Very many candidates used the coordinates of D$(x,\ y,\ z)$ to obtain the correct equation $-2x+2y+z+3=\pm 20$ but were unable to proceed beyond this.

2 *(Multi-variable calculus)*
The section sketches and stationary points in parts (i) and (ii) were generally well understood, although careless slips spoilt many answers. In part (iii) the partial derivatives were almost always found correctly and equated to zero. The equation $24y^2-6x^2=0$ was quite often simplified to $x=2y$ only (omitting the case $x=-2y$), and arithmetic and sign errors were fairly common. Several candidates used the quadratic substitution and obtained a quartic equation, which was sometimes solved successfully. In part (iv) most candidates used the partial derivatives to obtain two correct equations, but only a few managed to score full marks in this part. Some did not state that the quadratic equation obtained from $x=2y$ had no real roots, and those who omitted the case $x=-2y$ were unable to find any values. Some obtained values for x or y, but did not go on to find the values of k.

3 *(Differential geometry)*
Part (a) was answered well, with the arc length and surface area usually being found correctly, although there were many algebraic errors, for example when multiplying out. In part (b)(i) the equation of the normal was usually obtained correctly. Part (b)(ii) was quite often omitted, but when attempted the parametric equations for the evolute were generally found correctly and frequently used to obtain the cartesian equation. The final part (b)(iii) was very often not attempted at all. It was intended that candidates would use the parametric equations of the evolute to find the centres of curvature, but most preferred to use the standard formulae and many were successful.

4 *(Groups)*
Most candidates showed very good understanding in parts (i), (iv) and (vii). Only about half gave a correct isomorphism in part (iii) and the final part (viii) defeated most candidates. The explanations in parts (ii), (v), and especially (vi), were very often inadequate, and only a handful of candidates scored full marks on this question. In part (i) explicit reference to the group properties of closure, identity and inverses was expected, and most candidates did this nicely by exhibiting the complete composition table and listing the inverses. Those for example who showed that all the elements could be written as powers of 3 were expected to explain why this implies closure and the existence of inverses. In part (v) the step $(x, y)^5 = (x^5, y^5)$ was quite often omitted, and in part (vi) the essential point that $(x, y)^5 = (1, 1)$ implies the order of (x, y) is a factor of 5 was rarely stated clearly.

5 *(Markov chains)*
The techniques were generally well understood and calculators were used competently throughout this question; parts (i), (ii), (iii), (v), (vi) and (vii) were all answered very well. In part (iv) many candidates used the probabilities for days 10 and 14 instead of the probabilities for day 10 and the diagonal elements of $\mathbf{P}^4$. In part (viii) many candidates used the original transition matrix $\mathbf{P}$ instead of the limiting matrix found in part (vi); and a surprising number obtained correct equations for the new probabilities but could not solve them accurately.

4758/01: Differential Equations (Written Examination)

General Comments

The overall performance on this paper was very good. Many candidates scored high marks and very few scored less than half of the available marks. As usual, the familiar topics tested in Questions 1 and 4 were attempted by almost all of the candidates, with Question 3 the least popular choice. Most candidates have a very good working knowledge of the topics on this syllabus, the exception being, on this occasion, an understanding of the terminology of isoclines and tangent fields.

A high standard of algebraic and arithmetical accuracy of solutions is expected on this paper, and it is pleasing to note an improvement in this aspect.

Comments on Individual Questions

1 This question was attempted by all candidates and many earned the majority of the available marks.

(i) The method was well-understood by all, but a minority of candidates made arithmetical or algebraic errors in solving the linear simultaneous equations in finding the particular integral.

(ii) As in part (i), there were some algebraic errors.

(iii) Most candidates scored one out of the two marks available here, by recognising that one differential equation was the integral/differential of the other. Few candidates were able to go on to give a convincing argument to show that z was equal to $y + c$.

(iv) Apart from arithmetical and algebraic errors, a minority of candidates worked with the particular solution to the original differential equation, rather than the general solution.

2 Candidates usually answered part (a) well, but many seemed unclear of the terminology and/or methods involved in part (b).

(a)(i) Candidates showed a good understanding of the integrating factor method of solving this first order differential equation, and they applied it with accuracy.

(ii) Again, this use of an initial condition was well-executed.

(iii) Most candidates were able to find the stationary point of the curve and the values of y and its derivative as x approaches zero, but they did not always go on to use this information to help them sketch the curve.

(b) There were a few excellent solutions to this part of the question, but the work of many candidates suggested that they were not familiar with the words "isocline" and "tangent field."

(i) A statement that the isocline is a circle with centre at the origin and with unit radius was required here.

(ii) Many candidates showed confusion in their attempted solutions here, indicating that they were unsure of what was required in the requests for three isoclines and corresponding tangent fields. The isoclines, which were concentric circles in this case, were often not drawn.

(iii) The majority of candidates were able to recover here and sketch the solution curve through (0,1).

(iv) Again, candidates recognised the general shape of the solution curve through the origin, but relatively few indicated its gradient at the origin to be zero.

3 This was the least popular choice of question, but those who selected it were usually successful in scoring the majority of the marks.

(a)
(i) This was invariably answered well.

(ii) The separation of variables and integration was done well, but the justification for taking the negative sign in the final expression was not always present.

(iii) Again, the separation of variables and integration was handled correctly by the candidates.

(b)
(i) There were many fully correct solutions here, though a minority of candidates made a sign error in the trigonometric integration. Again, the justification for the negative sign in the given expression was omitted by some candidates.

(ii) Almost all candidates applied the Euler method accurately to obtain the requested estimation. Very few candidates were able to explain that it was not helpful to continue using the given algorithm, because of the non-constancy of θ.

4 The vast majority of candidates attempted this question and many scored full marks.

(i) Solutions were almost always convincing and correct.

(ii) Candidates were clearly very familiar with the method and worked through it accurately.

(iii) Occasionally there were arithmetical errors in finding the general solution for y. It was particularly pleasing that only a handful of candidates failed to use their general solution for x as the starting-point.

(iv) Again, there were a few arithmetical slips, but the method was well-known by all.

(v) This request presented no problems to candidates and with follow through from previous incorrect answers, almost all candidates scored the three marks available.

4761: Mechanics 1

General Comments

This paper was well answered and there were few very low scores. Most candidates were clearly well prepared for it. Many of them used the conventions for writing mathematics well, and so were able to communicate their intentions effectively. There were, however, some cases of poor arithmetical and algebraic manipulation.

While many candidates predictably lost marks on the more mathematically challenging parts of the paper, it was also the case that many, including some who were clearly very strong, lost marks through not reading the paper carefully; this was particularly evident in questions 2, 3, 4 and 5.

There was no evidence of candidates being under time pressure.

Comments on Individual Questions

1 **Motion in a vertical straight line**

The majority of candidates got this question right but a few made errors such as giving the wrong sign to *g*, attempting to apply an incorrect formula or not distinguishing between the speed of the pellet on the way up and that which it would have had on the way down if it had not hit the ceiling.

While the simplest way to answer this question involved using the formula $v^2 = u^2 + 2as$, it was also possible to use a sequence of *suvat* equations, for example $s = ut + \frac{1}{2}at^2$ followed by $v = u + at$. It was noticeable that many candidates who adopted such less efficient strategies also made arithmetical and algebraic errors.

2 **Motion along a straight line with constant acceleration**

Many candidates scored 4 out of the 5 marks for this question by finding the correct values of the initial velocity and the acceleration of the particle. However, it was only a minority who obtained the final mark for making the directions clear; many candidates made no attempt to answer this part of the question. Algebraic and arithmetical errors on this question were not uncommon among generally weaker candidates.

3 **Forces given as 3-dimensional column vectors**

In part (i) of this question candidates were asked to find the missing components of a force parallel to a given force, when they were given one component. Many candidates did this successfully but there were also many who did not realise that a scalar multiple was needed to answer such a question and tried to use addition instead.

In part (ii), candidates were asked to find the acceleration when two of the forces were applied to a particle. This was well answered, with most candidates knowing just what to do. Many candidates, however, lost marks by misreading either figures in the given vectors or sometimes the whole of this part-question.

4 **Forces in equilibrium**

Many candidates lost marks on this question.

In part (i), they were asked to draw a force diagram but only a minority were successful in doing this. The most common mistake was to draw the horizontal force parallel to the sloping plane. Another common mistake was to omit the normal reaction to the plane, or to draw it acting vertically upwards.

Most of those candidates who had drawn the horizontal force parallel to the plane went on to lose further marks in part (ii) where they were required to find the magnitude of that force. Continuing on from that earlier mistake simplified the problem and so some of the marks were not available to them; however, some credit was given for a sensible attempt at resolution in this situation: for example, considering the component of the weight parallel to the slope.

5 **Projectile**

This question attracted a full range of scores; many candidates got it fully right, including some who obtained few marks elsewhere on the paper.

A few candidates did not distinguish between horizontal and vertical motion and they lost most, and sometimes all, of the marks. A more common, but much less expensive, mistake was not to read the final request for the time after projection at which the impact was heard, and instead to give the time after the projectile hit the ground.

6 **Motion described in vectors**

In part (i) of this question, candidates were required to use given information to find expressions for the velocity and position vector of a skater at a general time. Most knew what was expected of them, but a minority did not realise that vectors were required or found it difficult to use vectors in this context. Others did not know how to use the information in the question about the initial position vector of the skater.

In part (ii) the direction of motion at a particular time was asked for as a compass bearing. Many candidates made the mistake of finding the bearing of the skater's position instead of his direction of motion.

7 **Models for motion involving constant and non-constant acceleration**

This question was well answered and it allowed many candidates who had, until then, obtained rather few marks, to show their understanding of this part of the syllabus.

Parts (i) and (ii) were based on a velocity-time graph and asked for the acceleration in part (i) and displacement in part (ii). Most candidates knew what they had to do, but there were some careless mistakes in carrying it out. Some candidates tried to do this using constant acceleration formulae, rather than the properties of the graph, and they were usually unsuccessful.

The question then switched to a different model using an expression for the velocity based on non-constant acceleration. Almost all candidates were able to use the requisite calculus and so there were many correct answers to parts (iii), (iv) and (v). However, in part (iv) some weaker candidates did not recognise that the least value of the velocity occurred when the acceleration was zero and spent time trying out various informal methods.

8 **Connected particles**

This question involved first one and then two (connected) trolleys being pulled by a string. The early parts, (i) to (iv), dealing with relatively straightforward situations, were well answered. In part (iii), the string was at an angle to the horizontal and candidates were asked to calculate the normal reaction of the floor on that trolley; a particularly common mistake was to forget about the vertical component of the tension in the string.

The final part, (v), carrying half of the marks for the question, allowed the stronger candidates to show their expertise. There was a wide spread of marks. It involved a more complicated situation in which the trucks were on a slope and this proved a challenge to most candidates. Many did not fully analyse the situation and this often led to inconsistent use of signs. Some candidates failed to realise that the two trolleys must have the same acceleration. It was common for those candidates who were able to get started at all to find the acceleration correctly and substitute into $v = u + at$ to find the velocity; and so obtain 5 of the 9 marks. In the last part, where candidates were asked to find the force in the coupling, sign errors were common as was the omission of relevant forces. However, many strong candidates obtained full marks for perfect answers.

4762: Mechanics 2

General Comments

This paper proved to be very accessible to candidates. It gave opportunity for them to demonstrate their knowledge and understanding of Mechanics and many did so, to very good effect. The presentation of answers was generally of a high standard. As usual, some marks were lost unnecessarily through arithmetical slips and rounding errors from calculator work. It is worth noting that candidates must remain diligent about giving sufficient evidence when they are working towards a given answer. The examiner needs to be convinced that the candidate is able to work through to the answer, without error.

Comments on Individual Questions

1

(a)(i) Most candidates scored full marks, the only loss of marks being due to either a sign error in calculating the change in momentum or an arithmetical slip.

(a)(ii) This was almost always correctly answered.

(a)(iii) The principle of conservation of linear momentum was clearly known and it was applied successfully by the vast majority of candidates. Again, there were some sign errors. Newton's experimental law was understood and there were pleasingly few errors in applying it.

(b) There were a pleasing number of clearly-presented and accurate answers to this question, displaying a sound understanding of the mechanics of the situation under consideration. A common error, however, was to assume that the ball travelled in a straight line and hit the ground at the given angle of projection, rather than considering first its motion under gravity. Most candidates knew that only the vertical component of the impact velocity of the ball is changed at the collision. It was a pity that a minority of candidates noted that the horizontal velocity was unchanged, but then proceeded to multiply it by the coefficient of restitution. It is worth pointing out that those candidates who drew clear diagrams, indicating velocity components before and after each bounce, were usually more successful than those who appeared to work in an unordered way. A significant minority of candidates either made little attempt beyond writing down the initial vertical component of the velocity of the ball, or seemed to write down every equation that they knew, involving time, distance, velocity in the hope that something would emerge.

2

(i) Candidates rarely had any problem in taking moments to find the correct magnitude of the force at B, but more than half did not indicate its direction, either in words or clearly on a diagram.

(ii) The vast majority of candidates showed again their good understanding of the principle of moments. A few took moments about B rather than A but did not take into account the forces at the hinge.

(iii) Again, there were many clear and concise answers, with candidates who chose to resolve horizontally and vertically almost always successful. The minority who opted for taking moments were more liable to make errors, usually by using an incorrect length or, occasionally, omitting a distance in one of the terms.

(iv) About 20% of the candidates were able to give the correct angle with either an appropriate explanation noting that the lines of actions of the three forces must meet at the point P, or by using the trigonometry of the situation. Many candidates falsely gave the answer as 40° supported by a variety of geometrical properties such as equality of alternate angles.

3

(i) The majority of candidates scored full marks here, with clear and sufficient working shown. As the answer was given in the question, a few lost one or two marks by not giving sufficient evidence of how they had achieved it.

(ii) Again, there were many well-presented solutions, showing a sound understanding of how to apply the principle of moments.

(iii) As in part (i), answers were usually clearly-presented and sound, with the bending of the shape causing no problem to most.

(iv) The majority of candidates were able to calculate the correct angle and earn full marks, though the diagrams drawn were not of the same good standard. A significant minority were not able to identify the required angle; others had incorrect lengths for their triangle.

4

(a) Almost all candidates realised that they needed to use the work energy equation and many did so very well. Some made sign errors, particularly by assigning the same sign to the Work Done and Gravitational Potential Energy terms. Others omitted either the initial kinetic energy term or the Gravitational Potential Energy term.

(b)(i) There were many solutions presented with the clarity required when an answer is given in the question. The minority who did not overtly indicate their use of Newton's second law often made sign errors along the way, though the correct result was still miraculously obtained.

(b)(ii) The majority of candidates did not appreciate the fact that the sledge had the same acceleration as the vehicle, detailed in the stem of the question. It was common to see an assumption of zero acceleration. The normal reaction was usually found successfully and credit was given for the use of $F = \mu R$ for most calculated values of a frictional force.

4763: Mechanics 3

General Comments

The work on this paper was generally of a high standard and presented clearly. The marks were somewhat lower than last year, but even so about 40% of the candidates scored 60 marks or more (out of 72). The topic which caused the most difficulty was circular motion.

Comments on Individual Questions

1 *(Simple harmonic motion)*

This question was well answered and about a quarter of the candidates scored full marks.

(i) Most candidates derived the two given results confidently. Some did not see how to use $\cos^2 \omega t = 1 - \sin^2 \omega t$ to obtain the second result, and there were a few sign errors in the differentiations.

(ii) Almost all candidates knew that they should substitute into the formula $v^2 = \omega^2 (A^2 - x^2)$, and this was often completed successfully. However, many took *x* to be the height above the sea-bed instead of the displacement from the centre of oscillation. Some omitted to find the period.

(iii) The maximum speed was usually found correctly.

(iv) The acceleration, and its direction, were usually given correctly. Here again the height 6.4 m was sometimes used instead of the displacement $(-)1.6\,\text{m}$.

(v) Most candidates had an appropriate displacement-time equation, but about half did not select an interval during which the ball is moving upwards. There were also a few working in degrees instead of radians.

2 *(Circular motion)*

This was the worst answered question and only about 10% of candidates scored full marks.

(a)(i) This was usually answered correctly. Sometimes there was a sign error in the equation of motion, and the mass was sometimes omitted.

(a)(ii) The tangential component of acceleration ($g \sin 60°$) was usually given correctly, although quite a number gave it as $mg \sin 60°$. Many candidates attempted to find the radial acceleration, and the tension in the string, without using the conservation of energy to find the speed first.

(b)(i) A surprising number of candidates omitted the normal reaction when resolving vertically, or tried to resolve in the direction of the string.

(b)(ii) In this part most candidates did consider the horizontal components of both the tension and the normal reaction, and used the given period to find the angular velocity. Only a few obtained the correct length for the string, as this required a correct value for the tension and accurate work in this part. Some found the radius of the circle but did not proceed to find the length of the string.

3 *(Elasticity and dimensional analysis)*

This question was quite well answered and about 20% of candidates scored full marks.

(i) Almost all candidates calculated the elastic energy correctly. Just a few took 573.3 to be the stiffness instead of the modulus of elasticity.

(ii) Although some treated the lowest point as a position of equilibrium rather than one of instantaneous rest, the majority considered elastic and gravitational potential energy and obtained the given mass convincingly.

(iii) The work in this part was often less convincing; the tension was not always clearly stated, a factor 2 was sometimes missing, and the resolution of the tension in the vertical direction was often unclear.

(iv) Almost every candidate could explain why the given expression is dimensionless.

(v) The method for finding the powers was very well understood, and most candidates obtained the correct values. Common errors arose from having the wrong dimensions for λ (usually MT^{-2} instead of MLT^{-2}) or the period (usually T^{-1} instead of T).

(vi) This part was also well understood, and a good number obtained the correct answer. Some candidates omitted the dimensionless part and just evaluated $m^{\alpha}a^{\beta}\lambda^{\gamma}$.

4 *(Centres of mass)*

About a quarter of the candidates scored full marks on this question.

(i) The methods for finding the centre of mass of a lamina were very well understood and usually carried out accurately. A factor ½ was sometimes missing from the *y*-coordinate, and there were a few careless algebraic slips.

(ii) Most candidates knew how to find the centre of mass of a solid of revolution. Not many errors were made in this part, but some used the wrong limits for the integration.

(iii) Many candidates did not make any progress in this part. Those who put the pieces together to form a cylinder very often obtained the correct answer, but those who tried to find the centre of mass of *S* directly by integration very rarely earned any credit.

4764: Mechanics 4

General Comments

In general the performance of the candidates was very good. As in recent sessions, the standards of presentation and communication were high, though some candidates failed to include necessary detail when establishing given answers.

Comments on Individual Questions

1 (*Variable mass – evaporating raindrop)*

(i) Many candidates treated $\frac{\mathrm{d}m}{\mathrm{d}t}$ as a function of *m* instead of *t*, leading them to *m* as a linear instead of exponential function of *t*. Candidates who took the time to integrate carefully almost always found the correct expression.

(ii) This was generally not well answered. Many candidates used *F* = *ma*, treating *m* as a constant, rather than $F = \frac{\mathrm{d}}{\mathrm{d}t}(mv)$. However, most candidates separated variables and integrated accurately from their differential equation.

(iii) Nearly all candidates stated that $m = \frac{1}{2}m_0$, though some were not able to use the relationship to eliminate *t*.

2 (*Equilibrium*)

(i) This was well answered by the majority of the candidates. Some did not give sufficient detail to earn the final mark.

(ii) Most candidates earned the first two marks of this part and many took the time to simplify their expression for $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$ before considering its sign.

Very few candidates gave good attempts at showing that $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$ is positive, with many manipulating inequalities incorrectly or failing to give sufficient detail.

(iii) While most candidates knew that a position of equilibrium required $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$, many assumed that the value of *x* had to be found and attempted to solve the resulting equation, not realising that it was not easily soluble and that a change of sign approach was all that was necessary.

Almost all the candidates correctly identified the equilibrium as stable.

3 (*Variable Force*)

This question was answered well by most candidates. The use of different forms of Newton's second law for different contexts was well understood. In parts (i) and (iii) the question specifically asks for expressions for *v* in terms of *x* and *t* respectively; many candidates failed to give the required form and simply showed the given numerical answer.

(i) The application of Newton's second law was well done; very few candidates used power instead of force.

Most then proceeded to separate the variables and integrate using partial fractions and were able to show that x = 100ln1.9 accurately.

(ii) Again, this was answered well. Full marks were awarded more often here than in (i) and (iii) because the requested relationship did not require rearrangement. However, a minority of candidates found *v* in terms of *t* and could only be awarded 4 out of 6.

(iii) This was generally well answered. The change in force was understood and the resulting integral found accurately. Some candidates changed *t* to be the time after the driving force was removed; one mark was withheld in this case.

4 (*Rotation – moments of inertia and dynamics*)

(i) Most candidates gave working that led to the given answer, but the detail of the perpendicular axes theorem was often lacking.

(ii) This was not well answered by many candidates. Many candidates found the integral of $I = \frac{17}{16}mx^2$ instead of carefully considering the sum of the moments of inertia of slices of the cone. Candidates that worked in terms of ρ and substituted at the end often failed to give enough detail for a given answer; those that worked in terms of *M* having already made the substitution were more successful.

(iii) This was well answered by those candidates that attempted it, with good derivation of the various terms. Many different approaches were seen involving energy considerations.

(iv) The idea of this seemed to be understood well by the stronger candidates and the techniques applied accurately. Some interesting approaches were seen using the small angle approximation for cosine before differentiating.

Chief Examiner's Report (Statistics)

Attention is again drawn to issues of accuracy of numerical answers. As has been explained previously, while sufficient accuracy is of course essential, gross over-specification betrays a fundamental lack of understanding of statistical processes. For example, it is not useful to quote the value of a test statistic as, say, 1.8413879 merely because that is the number that happens to fall off a candidate's calculator. Such over-specification is now normally penalised on each occurrence by withholding the final accuracy mark. This also applies to probability calculations.

Hard-and-fast rules for specification of accuracy cannot be laid down. Any attempt to do so would often be misleading in individual circumstances and would be liable to cause many "hard cases", which is certainly not the intention. Rather, candidates are expected to act sensibly and intelligently in the light of the problem in hand. Thus in most cases 2 decimal places are likely to be appropriate for calculated values of test statistics. Probabilities however may need to be given to up to 5 significant figures, depending on the problem in hand. Other final numerical answers will probably rarely need to be given to more than 4 significant figures, if that.

It must be emphasised that these guidelines do not apply to intermediate stages of working. Candidates should be alert to the dangers of premature approximation, and always be sure to carry sufficient accuracy in intermediate stages to be confident in the final answer at the end. For example, values of the sample mean and standard deviation are required in the calculation of many test statistics, and these should certainly not be calculated, or reported, to only 2 decimal places. Another example is found in the contributions from individual cells to the usual statistic for a chi-squared test; these contributions may well need to be calculated, and reported, to at least 4 decimal places, even though the final value is reported to only two.

It should also be emphasised that these remarks apply in detail only to units in the statistics strand.

4766: Statistics 1

General Comments

The level of difficulty of the paper appeared to be appropriate for the candidates and there was no evidence of candidates being unable to complete the paper in the allocated time. Most candidates appeared to be well prepared for the paper with relatively few unable to gain many marks. Most candidates supported their numerical answers with appropriate explanations and working. Presentation was generally good although, when explanations were required, some candidates made it difficult for us to apply the mark scheme because of poor handwriting and poor use of English. Fortunately only a small minority of candidates attempted parts of questions in answer sections intended for a different question/part and most candidates had adequate space in the answer booklet without having to use additional sheets. Once again many candidates over-specified some of their answers, despite recent Examiner's reports warning against this. Please see the comments about this in the Chief Examiner's report.

The hypothesis testing in question 7 caused problems for many candidates and there was still quite a lot of use of point probabilities in their arguments. Question 6 and the first three parts of question 8 provided valuable sources of marks for most candidates. However question 8(iv)B proved too difficult for all but the most able. There were few correct answers to question 1(iii), suggesting that 'midrange' is a measure of average which receives little attention. In question 4(ii) many candidates failed to understand what is required for a probability argument, despite this phrase having been used in past papers. Candidates should also be reminded to label any graphs that they draw and make sure their scales are uniform. Many lost vital marks by violating these basic requirements.

Comments on Individual Questions

1 In part (i) candidates were able to make a successful start to the paper by realising that the frequency was equal to the frequency density $\times$ class width. Most gained the expected answer of 13 but occasionally the examiners saw 14 (due to a misread of the vertical scale) or 130 (due to not being able to multiply by 1000 correctly).

In part (ii), the vast majority of candidates recognised that the distribution was positively skewed but some still insist on using the unacceptable terms of 'right skew' or 'symmetrical skew'.

Part (iii) defeated many candidates. Whilst many understood the idea of the mid-range, few were able to apply it in the context of the question. Very few appreciated that the maximum mid-range could only be found by averaging the highest value in the last class with highest value in the first class to give (4000 + 1000)/2 = 2500. Many wrote (4000 + 0)/2 = 2000 as their response here. Similarly, the minimum mid- range could only be found by averaging the lowest value in the last class with lowest value of the first class to yield (3000 + 0)/2 = 1500.

2 Part (i) was successfully answered by most candidates.

Many candidates gained only 1 mark out of 3 in part (ii), giving an answer of (1/5) x (1/4) = 1/20, failing to realise that Austen could be picked first followed by Brontë, or vice versa, hence requiring their answer to be multiplied by 2.

3 Part (i) was usually answered correctly, the standard correct response being $(0.75)^6$ = 0.178. However a surprising number of candidates thought that the probability was just 0.75.

In part (ii) the expectation was usually found correctly by multiplication of 0.178 by 50 but occasionally some used 300 instead. Candidates should be reminded not to round their final answer in an expectation calculation. There were too many cases of 8.9 being rounded to 9 which lost the final mark. Candidates who had got the wrong answer to part (i) were allowed a full follow through in part (ii).

4 Part (i) was generally well answered. Candidates who used fractions (in multiples of 1/18) on their probability scale usually scored full marks. Candidates who used decimals made the question more difficult, which often led to inaccurate heights and a loss of one mark. Some candidates lost the first mark due to failure to label both axes.

In part (ii) parts A and B, a significant number of candidates failed to understand the questions by thinking that they had to use the probability distribution given, subtracting the other probabilities from 1, but there was no actual probability argument evident. Those who did begin to identify combinations with a difference of one often did not recognize that the order mattered and then claimed that there were only 18 possible outcomes in order to make the numbers fit the given answer. Most candidates who were successful compiled a two way table of all of the possibilities. A correct numerical method which lacked the essential explanation of where it had come from was fairly commonly seen.

In part (iii), a large majority used a correct method, but a surprising number did not realise that expectation and mean are interchangeable in this context and consequently they divided by 6 or some other number.

5 Many candidates got full marks for their Venn diagram in part (i). A minority failed to subtract 0.11 from 0.41 and 0.14 but even these usually produced two intersecting circles labelled correctly to get the first mark. A few candidates did not work out the probability for the fourth region (0.56).

Part (ii) was answered fairly well and showed that many candidates know how to test for independence, although surprisingly candidates often used the probabilities from their Venn diagram rather than those from the question. Some candidates failed to evaluate 0.14 x 0.41 and consequently lost the accuracy mark. A minority of candidates, having correctly completed the working, then got the conclusion the wrong way round. A small number of candidates used a conditional probability method, not always correctly.

Part (iii) was also answered fairly well, but again a significant number of candidates used the wrong figures from their Venn diagram. An impressively large proportion of candidates did get the correct explanation of what this probability represented but several missed out this mark because they did not explain the conditional probability in the context of the question.

6 In part (i) most candidates found the mean correctly and whilst decimal answers were frequently over-specified candidates gained the mark for giving the answer in fractional form, even if subsequently over-specified. Most candidates made a good attempt at the standard deviation; the main errors were the usual ones - calculating the rmsd instead of the standard deviation, incorrect squaring when calculating $\sum fx^2$ or using n=4 instead of n=70. The number of candidates who simply used the statistical functions on their calculator was fairly small, despite this being the easiest way to do the question.

In part (ii) many candidates found the mean correctly but thought that the standard deviation remained unchanged. Those candidates who understood that standard deviation is a measure of deviation were usually able to see clearly that the deviations would (tend to) be increased. Those who tried to reason their way through the formula usually came to the wrong conclusion. A very few very able candidates correctly said that if the number of gulls laying no eggs was very large (over around 500) then the standard deviation would decrease.

7 Part (i)A was generally answered correctly, although when using the binomial formula, a few candidates forgot to round off sensibly.

Part (i)B was found to be slightly more difficult. Most candidates used tables but some went wrong by calculating 1 – P(X < 1) or 1 – P($X \leq$ 2). A reasonable number of candidates first found P(X = 0) and then usually went on to finish off the question correctly.

In part (ii) most candidates correctly stated their hypotheses in terms of *p,* but then often lost the available mark for defining *p*. Most were able to give an explanation of the reason for the nature of the alternative hypothesis.

In parts (iii), (iv) and (v), too many candidates forgot to state their conclusions in context. This is required in every exam and so teachers should be careful to instruct their students to do this.

Part (iii) was a relatively easy hypothesis test, since it was a lower tail test. However, many candidates (almost half of the candidature) used point probabilities and thus gained no marks. Of those who gained some credit, most either got full marks, or lost the final mark for conclusion in context.

In part (iv) some candidates wasted a lot of time for these 3 marks, testing out trial distributions for large n. Candidates should appreciate that, with only 3 marks at stake, there must be a more tractable solution. In fact all that was required was a comparison of the test statistic with the critical value, followed by a conclusion in context. In fact, only one third of the candidature gained any marks at all.

Part (v) was expecting candidates to give a valid reason for the critical region being empty. A number of fully correct solutions were seen, and the question was generously marked, so that candidates who got some way to an explanation gained one mark.

8 In part (i) most candidates scored at least three marks. Many did not explicitly write down their calculations for the cumulative frequency, preferring to plot their points directly on the graph, but still gained the mark. Scales were usually correct and sensible but some candidates used a linear horizontal scale starting from zero, which made a very cramped graph. Labels were often forgotten altogether and the vertical scale was often seen as just 'frequency', losing the mark. Points were usually plotted correctly at the right height but far too often were plotted at the mid-points i.e. 9.2, 9.4 etc. losing the final 2 marks. Only a few candidates used the lower class boundaries. Many candidates lost the final mark by not joining (9.3, 5) to (9.1, 0). Cumulative frequency bars were sometimes seen as were lines of best fit. Occasionally no attempt was made at a cumulative frequency graph at all, with some candidates just plotting frequency against midpoints or attempting to find frequency density.

Part (ii) was generally well answered often from a follow through from a 'sensible' graph. Some of the scales used in part (i) meant that it was very difficult to read the figures if they fell outside the allowed ranges. The 12th value was often used instead of the 12.5th value, perhaps because it was easier to read as there was a point plotted there. A few candidates failed to calculate the IQR even though both quartiles were found.

There were many correct answers to part (iii) (or correct ft answers) but many candidates tried to use the median or twice the IQR. A few candidates reverted to calculating and using the mean and standard deviation, gaining up to 2 marks out of 3 although they could not gain the last mark because, with this method, outliers could exist.

In part (iv)(A) only the better candidates obtained the correct answer. Many used $(38/50)^3$ scoring one mark only. Others candidates had more complicated incorrect versions of binomial probabilities. Occasionally the numerators decreased but the denominators did not. Some candidates did not find the correct value of 38 from the table.

Part (iv)(B) was found very challenging and only about ten percent of candidates gained full credit. Many candidates scored one mark for adding their answer from part (A), but otherwise a common incorrect answer of 0.8549 was often seen, which scored SC2. Some candidates thought that they had to multiply only two probabilities when finding the probability of two being more than 9.5. Many candidates did not realise that there were three different ways of getting two more than 9.5. Those candidates who drew tree diagrams fared better here, and in realising that the probabilities diminished. Those candidates attempting 1 - (P(0) + P(1)) were on the whole not as successful, sometimes not including both probabilities or failing to include the factor of 3.

4767: Statistics 2

General Comments

Once again, the overall performance of the majority of candidates was very good. There was no evidence that candidates found it difficult to complete the paper in the time available. With the majority of candidates able to handle the numerical side of statistical tests accurately it was the questions, or parts thereof, requiring comments and interpretation that discriminated between candidates.

Comments on Individual Questions

1 (i) This was well-answered as expected.

1 (ii) Well-answered, but mistakes such as using $\Sigma x \Sigma y$ for Σxy were fairly common. A few candidates slipped up when rearranging the equation into the form $y = mx + c$.

1 (iii) Well done, but there was little evidence of candidates with clearly inaccurate equations going back to correct their work in part (ii).

1 (iv) Most candidates managed to use their equation to predict a value for y but many slipped up with the sign of their residual.

1 (v) Well done, although many gave their answers to 5 or more significant figures. In this question, this was deemed to be 'over-specification'.

1 (vi) Most managed to make a sensible comment relating to extrapolation, but additional, acceptable comments were few in number.

2 (i) Candidates of all abilities found it difficult to provide clear explanations to differentiate between independence, randomness and uniform average rate.

2 (ii) Well answered, though a small proportion of candidates lost accuracy through premature rounding.

2 (iii) Well done on the whole. Some candidates found $P(X \geq 5)$ instead of $P(X > 5)$ either by misinterpreting the question or by misunderstanding the method.

2 (iv) Some candidates stated the parameter, $\lambda = 37.2$, rather than specifying the correct distribution, Poisson(37.2), as requested. Several candidates also stated what was probably intended as the approximating distribution required in part (v), thus making it unclear if they had actually understood the question posed in part (iv).

2 (v) Well answered. Some candidates seemed unsure how to apply a continuity correction but otherwise handled the Normal calculation appropriately. There was some confusion over variance and standard deviation shown by a minority of candidates.

3 (i) Well answered, but spurious continuity corrections were seen frequently.

3 (ii) Well answered.

3(iii) A The majority of candidates correctly identified the Binomial distribution with a correct follow through from their value in part (ii). However some candidates lost marks by identifying parameters only. Several candidates identified either the Normal or Poisson distributions in error.

3(iii) B Most candidates who had the correct answer in part (A) answered this part correctly using a suitable Poisson approximation, though some candidates lost marks by ignoring the need for a "suitable approximating" distribution and continued to use the Binomial from part (A). Other candidates either used the Normal distribution inappropriately or failed to use the Normal distribution when their value of np was greater than 10.

3(iii) C This was poorly answered revealing a poor understanding of the conditions required to apply a Poisson approximation. Many candidates appeared to think that 'n is large and p is small' was justification to use a Normal approximation to the Binomial distribution.

3(iv) A This was usually correct with the odd sign error leading to an incorrect equation, though most candidates managed to use the correct z value.

3(iv) B Marks were frequently lost by candidates failing to appreciate that normal curves should be asymptotic to the horizontal axis and symmetrical about the mean. Many candidates failed to show the 'Cox' curve with a higher vertex than the 'Braeburn' curve.

4 (a)(i) This was well answered with few candidates failing to provide context in their hypotheses and with few using the word 'correlation'.

4(a) (ii) Well answered.

4(a) (iii) Well answered. A minority of candidates failed to provide context in their conclusions and/or failed to provide acceptable conclusions.

4(b) Well answered. Notably, many candidates failed to define μ as the population mean. Most managed to carry out the test correctly, with the preferred method being the one outlined in the main body of the mark scheme.

4768: Statistics 3

General Comments

There were 462 candidates from 88 centres (June 2010: 428 from 83) for this sitting of the paper. The overall standard of many of the scripts seen remains high. Once again it was pleasing to note that most candidates remembered to state the hypotheses (Questions 1 and 2) despite not been instructed to do so.

There was a noticeable improvement in respect of the use of clear and accurate notation, concise and accurate computation and the overall presentation of the work. However, there were two parts in Question 3 that required the recall and application of topics from Statistics 1 that were rather badly answered.

Invariably all four questions were attempted. There was no evidence to suggest that candidates found themselves unable to complete the paper in the available time.

Comments on Individual Questions

1 (i) Most candidates knew exactly what was required here. It helps to clarify matters if they can be explicit about whether they are referring to population or sample properties.

(ii) There was much good work seen here. In particular, candidates remembered to state their hypotheses correctly and they appeared to be making good, efficient use of their calculators in obtaining the test statistic. There was just one widespread issue: the final conclusion, in context, is expected to include an explicit reference to "on average" or equivalent.

(iii) This part was not at all well answered. Few candidates appeared to have a clear and correct understanding of the meaning of a significance level.

(iv) On the whole this part was usually answered correctly, although there was the familiar problem of remembering to remain with the *t* distribution for the relevant percentage point in the construction of the confidence interval.

2 There were many good and fully or nearly fully correct solutions to this question.

(i) There was a noticeable improvement in the statement of hypotheses – there were far fewer instances of "the data fit the model" (or equivalent) than in the past. The expected frequencies were almost always found correctly, but then many candidates forgot to combine the first two classes and so ended up with the wrong test statistic. There was less success when it came to the critical value. Despite being told in the question that the mean for the model had been obtained from the data, some candidates forgot to make allowance for that in their number of degrees of freedom.

(ii) Here too there was a pleasing improvement in the way that the hypotheses were stated, though there still seems to be a reluctance by some to use the symbol "*m*" for the median and then define it as the <u>population</u> median. The subsequent test was carried out with little difficulty by most candidates.

3 This question seemed to cause the most problems for candidates, and yet the issues over which they struggled were largely to do with topics from Statistics 1 and with solving quadratic equations. Those candidates who could recall the relevant facts were able to obtain the required results quickly and easily and were well rewarded as a consequence.

(i) (A) Sketches were fairly poor. Many simply drew a straight line from the origin to some (unspecified) point on $y = 1$. There was often no attempt to show the asymptotic behaviour of G(x).

(B) Candidates made very heavy weather of this part and clearly spent a lot of time making little, if any, progress. Time was wasted in finding the median (unnecessarily) and in trying to integrate the CDF. It was surprising how many seemed to have forgotten about quartiles and interquartile range. To find each quartile it was necessary to solve a quadratic equation which, with a little thought, could be done quickly and easily. Many just pressed ahead regardless and often ended up making things more complicated for themselves. Candidates were then expected to apply the well-known criteria for outliers in sample data. A very large proportion did not appear to have remembered the rule of thumb that outliers are more than 1.5 × the interquartile range from the nearer quartile. "Median + 2 × the interquartile range" was a very common, but incorrect, alternative to this.

(ii) (A) Generally, this was answered satisfactorily by most candidates, though some may not have fared so well had they not known the result they were aiming for. Notation was often sloppy.

(B) As in the previous part, the answer, given in the question, may well have helped some candidates, but having said that the vast majority of responses were perfectly adequate.

(C) This part seemed to confound most candidates. Many had forgotten how to find a conditional probability. Furthermore, they overlooked the clues, given by the structure of the question, about how they might find $P(X > 400)$ and $P(X > 450)$.

4 (i) This part was almost invariably answered correctly.

(ii) Many candidates tackled this part well and successfully. However, it was somewhat surprising to see just how many candidates did not think to combine the distributions for supply and demand: they attempted to find the probability that the demand exceeded 40 kg. Thus for these candidates this part was trivialised with costly consequences.

(iii) The same was true in this part. There were many correct solutions but those whose attempts were flawed in part (ii) repeated the same mistake here. A further complication was that some candidates forgot to increase the number of cheeses supplied to 5.

(iv) There was much correct work in the construction of the confidence interval, although there were many candidates who appeared not to trust the information given in the question about the population standard deviation and so used the percentage point 2.201 from t_{11} instead of 1.96.

A large proportion of candidates neglected to make any comment about the interval. Explanations of the meaning of "a 95% confidence interval" were fairly mixed in quality.

4769: Statistics 4

General Comments

There were only 18 candidates for this module this year, thinly spread over 9 centres. This is a much smaller entry than last year.

There was much good work with many candidates scoring highly, but comparatively little that was really outstanding.

As usual, the paper consisted of four questions, each within a defined "option" area of the specification. The rubric requires that three be attempted. Two candidates in fact attempted all four. The best three attempts counted. In general, attempting all four questions is not a good strategy; it is better to try to complete three questions. All the questions attracted a reasonable number of attempts, with question 4, on design and analysis of experiments, the least popular though not by very much.

Comments on Individual Questions

1 This was on the "estimation" option. It was based on maximum likelihood estimation.

Part (i), on finding a maximum likelihood estimator, inevitably involved quite a lot of technical work. This was mostly well done, though a few candidates did not know how to form the likelihood to start with (a few follow-through marks were available for subsequent methods). Some candidates had difficulty showing that the obtained turning-point is indeed a maximum; in this case, the actual estimator has to be inserted in the second derivative.

Part (ii) required candidates to show that (in this case – it is *not* true in general) the maximum likelihood estimator is unbiased. Mostly this was done well, but a few candidates became badly lost in confusion between sample and population quantities.

Part (iii) required candidates to obtain an approximate 95% confidence interval for the parameter, using a given result for the variance of the estimator. Again this was mostly done well, but there were some very bad errors of introducing "$\sigma / \sqrt{n}$" in the denominator.

2 This was on the "generating functions" option and was concerned with moment generating functions of chi-squared distributions.

There was good technical work here. It was pleasing to see integrals carefully set out in correct and full notation and with proper attention to insertion of limits, and likewise pleasing to see careful differentiation in part (ii) (not too many cases of a disappearing minus sign). Part (iii) required candidates to invoke fairly explicitly the uniqueness of the relationship between a distribution and its moment generating function. Part (iv) was an application of the Central Limit Theorem; most candidates knew how to use the Normal distribution here, but there were some strange errors with the parameters.

3 This question was on the "inference" option.

It started by requiring definitions of Type I error, Type II error, operating characteristic and power. Sadly there were still candidates who had Type I and Type II errors the wrong way round, which is a bad mistake at this level. In the case of the power, a statement that "power = 1 – operating characteristic" was not accepted as a *definition* of power.

Part (ii) required a critical value and the minimum sample size to be found for a Normal test for the mean, given some criteria for the errors. This was commonly done well.

Part (iii) required a sketch of an ideal power function. Some quite extraordinary sketches came forward here, completely wrong and in some cases simply bizarre, even from candidates who had met with reasonable success in the earlier parts.

4 This was on the "design and analysis of experiments" option.

The first part required candidates to discuss and compare the randomised blocks design and the Latin square design, giving an example for each situation. Mostly this was done fairly well, but candidates were not always completely sound about how these designs can "allow for" one or two extraneous factors.

The second part required an analysis of variance to be carried out, which was usually done efficiently and correctly.

4771: Decision Mathematics 1

General Comments

A feature of this year's paper was the frequency of scripts on which good solutions to some questions were accompanied by poor answers to others.

Questions 1(v), 2, 3 and 6(ii) were found to be the most challenging.

Clarity of thought and good literacy skills are vital in producing well-argued answers for questions requiring an explanation.

Comments on Individual Questions

1 **Graph Theory**

Most candidates answered parts (i), (ii) and (iii) correctly. Both points were needed for the mark in (iv). Some did not understand what was meant by a "point".

Many answers of a vaguely statistical nature were seen in (v), with references to a "lack of correlation" et al.

2 **Algorithms**

Most candidates could compute *c/a* and *f/d* for the given values, but many then failed to identify the minimum of the two.

Very few candidates indeed scored the single mark in part (ii). For it to be awarded there had to be reference to the size of the feasible region when graphed.

3 **Simulation**

In each paper it is the turn of a part B topic to appear, in reduced form, in part A. This time it was the turn of simulation.

Whereas usually the majority of candidates can generally define appropriate rules for simulating a random variable, in this question the majority could not.

A common error in part (ii) was a rule which mapped die scores 1, 2 and 3 to the output 1, score 4 to output 2, score 5 to output 6, and rejected score 6. Many candidates gave a rule using 2-digit random numbers and a considerable number made no attempt at the question.

Common incorrect criticisms of John's methodology in (iii) were:

- because he subtracted one from the result on the die, there was no chance of his procedure producing a 6.
- a throw would be wasted if it produced a 1, since when John subtracted 1 from it there would be nothing left.

Very few marks indeed were awarded here.

Mention must be made of the very many candidates who also criticised John for not repeating his simulation often enough. They were determined to earn the "repetition" mark, even though John was not doing a simulation, so that there was no such mark to earn.

4 **LP**

The majority of candidates made a decent attempt at the modelling in part (i). Note that examiners were required to be very particular in requiring definitions to refer to "the number of", since one of the root causes of difficulties in algebraic understanding is a failure to realise that letters stand for numbers, and not objects.

In part (ii), and subsequently, the mark scheme was generous to those confusing 2x and x/2. Candidates can avoid such errors by checking specific values – do they satisfy the question/do they satisfy the inequality?

There were only 2 marks for part (iii), and about 50% of candidates were successful with it.

Part (iv) was more difficult with a little modelling, followed by the need for a comparison, and then a slightly tricky identification of the best point.

This was not an easy question, and many performances on it were good or acceptable.

5 **CPA**

As in January, most candidates were able to give immediate predecessors in part (i). Part (ii), the forward and backward passes, was also answered well. Part (iii) was a mixed bag, with critical activities, duration and float, and again answers were generally acceptable.

Part (iv) was reckoned to be a difficult explanation, and so it proved. Part (v) was found to be relatively easy. Few could marshal both critical activities and the revised duration for the mark in (vi).

6 **Networks**

Prim in tabular form caught out a good proportion of candidates. Not many candidates could produce the correct minimum connector for this small network. One would have expected it to be correct by inspection, and to be used to check the operation of the algorithm.

Some explanations in part (ii) revealed a confusion between direct connectivity and shortest routes. Typically candidates would refer to the minimum connector not using particular direct connections, when there were no such direct connections to use! Those candidates meant shortest routes. Of course, that was the point of the question, but few seemed to notice it.

As in part (i) one would have expected most candidates to have produced the required shortest route and distance in part (iii), independently of their work on Dijkstra. That was not the case – very few got it right.

In part (iv), and in other parts of this question, arithmetic errors abounded.

4772: Decision Mathematics 2

General Comments

Candidates found parts of this paper very much to their liking, for instance the decision analysis in question 3. They had been well prepared and had learned well, and all are to be congratulated for that.

Comments on Individual Questions

1 **Logic**

(a) Most candidates understood the issue, although the negativity of "ban the protest" created confusion for some. They analysed that a triple negative was involved.

(b) Prior knowledge of the cricket context was of no advantage.

The Boolean algebra was tackled well. Note that the first part of the published solution takes the longer route, opening brackets, to demonstrate how that works. Most candidates, very acceptably, took the much shorter route of factorisation. Marks were mostly lost in appending the correct name to the rule being used.

The last section of the question was more challenging. Candidates gave the batsman out in part (iii), spotted the issue in part (iv) and agreed that the logic was still valid, but did not really get to the nub of the issue in their analysis.

2 **Networks**

(i)(ii) It is not usual for a complete application of Floyd to be set – usually just one or two iterations are tested. Naturally, the network must be small for this.

The questions worked well, and most candidates scored well.

(iii) The unit is very careful to draw the distinction between the practical "TSP" problem and the classical "minimum Hamiltonian in a complete network" problem. The real life travelling salesman doesn't care too much about where his best route takes him, revisiting, retracing etc.

These two problems are reconciled by working in the complete network of shortest distances which is constructed from the original network. So candidates were expected, in part (iii), to be working in the network which they had just drawn in part (ii). However, it is impossible not to refer here to the "TSP problem defined in the original network", and understandable that some candidates were confused by this.

The mark scheme on this occasion allowed candidates to work in either network, but not both. Indeed several candidates seemed unperturbed when they produced a lower bound of 13 by working in the original network, and an upper bound of 12 by working in the complete network of shortest distances.

(iv) Interpretation is part of modelling, and modelling is difficult. Many candidates failed to realise that this question was related to modelling, and that their comments needed to be related to the physical situation and not too fanciful, i.e. not too far removed from the question.

(v) Easy to mark ... a digraph ... right or wrong. Some candidates scored the point.

3 **Decision Analysis**

There is not much to say about this question, other than "Well done!"
Candidates sorted out the complex situation, carefully attended to the distinction between decision nodes and chance nodes, and knew that utility is not linear, i.e. that $p \times \text{utility}(a) + q \times \text{utility}(b) \neq \text{utility}(p \times a + q \times b)$.
Part (iv) was also very well answered.

4 **Linear Programming (Simplex)**

(i) Most succeeded with the formulation. Note that examiners were required to be very particular in requiring definitions to refer to "the number of", since one of the root causes of difficulties in algebraic understanding is a failure to realise that letters stand for numbers, and not objects.

(ii) The basic simplex algorithm was well understood and executed by most candidates. Candidates who made slips often ended up with only one of x, y and z being non-zero. Whilst this is possible in exceptional cases (it would correspond to the best vertex of the feasible polyhedron lying on an axis), it should have been a warning that something might well have gone wrong.

(iii) This was challenging, and was quite often very well done. Some candidates successfully used the solution to (ii) in their formulation, and had an easier job in their manipulation, others formulated more easily ab initio, but had more manipulation.

(iv) Candidates were expected to produce the integer solution (5 15 6) and to compute the number of rooms there. Some missed the point.

(v) Candidates were expected to note that this integer solution is better, but is not revealed by Simplex. Not all noted this.

4773: Decision Mathematics Computation

General Comments

Performances were varied. The paper requires good modelling skills above all else. It also helps to have good communication skills. Candidates who produce a ream of output for the examiner are missing this point a little.

Comments on Individual Questions

1 **Networks**

This question was answered well by most candidates. They were able to draw the graph correctly in part (i). Few were able to find the correct number of maximal matchings, but that was difficult and it had no consequences.

In part (iii) not all candidates understood what a set of alternating paths should look like, and that the same vertex could appear in more than one path. Few drew all three paths.

The LP in (iv) was generally well done, although the full interpretation was often either weak or missing.

Very few candidates attempted the network flow modelling in part (v).

2 **LP Modelling**

Part (i) was generally done well. Some candidates followed the look rather than the meaning of the constraints for earlier years, and thus had x7+x8 on the left hand side of a new constraint.

Most were able to re-arrange in part (ii) and knew that they were maximising, but a significant proportion tried to maximise the sum of all of the variables. This led, in part (iv), to attempts to interpret a solution which did not make sense. In part (v) not all candidates were able to find all 5 strategies and evaluate them correctly in order to choose the best.

Very few candidates scored the final mark in part (vi).

3 **Recurrence Relations**

This was a poorly done question by most candidates. There were problems with producing the recurrence relation in part (i), and only a few candidates were able correctly to produce the solution in part (ii).

Many candidates only printed the numbers from their spreadsheet in part (iv). The rubric states that candidates should show their working, and that requires a sample of the formulae which are used.

The final advice on not taking a double dose was often given without clear analysis or justification.

4 **Simulation**

Many candidates were able to set up appropriate lookup tables and the correct simulation of customers was usually seen.

Only some followed the advice to simulate across rather than down the spreadsheet.

The choosing of dishes was sometimes not modelled by using a random variable, and the total meals prepared did not always correspond to the number of customers.

All of the above led to difficulty in creating the 100 rows needed for part (ii). Part (iii) was poorly answered and advice in (iv) rarely came with clear reasoning.

4776/01: Numerical Methods (Written Examination)

General Comments

Most candidates found this a straightforward paper, and there were many high marks. Candidates showed a good grasp of the basic ideas of numerical mathematics, but as in previous years the standard of presentation of work was frequently not good. The point has to be made yet again that numerical mathematics is systematic and algorithmic. Setting down work in a logical order, frequently in tabular form, makes it easier to see whether a solution is correct – easier, that is, for both the candidate and the marker. Jumbles of unidentified numbers scattered across a page are unlikely to receive credit.

Comments on Individual Questions

1 **Solution of an equation**
In part (i), the idea of halving the maximum possible error at each iteration was well understood, but many candidates counted the starting position of the bisection method as an iteration. In part (ii), false position was usually done correctly.

2 **Numerical differentiation**
Parts (i) and (ii) were generally done well, but part (iii) defeated most candidates. The idea in part (iii) is to combine the largest possible numerator with the smallest possible denominator, and vice versa.

3 **Lagrange's interpolation formula**
This was a straightforward question, but quite a number of candidates were not able to apply linear interpolation in part (i). In part (ii) there were the usual confusions between x values and function values. The question said, quite clearly, that no algebraic simplification was required; some candidates chose to simplify anyway.

4 **Fixed point iteration**
Locating the root in part (i) was very easy, but the majority of candidates were unable to use the magnitude of the derivative in a region surrounding the root to show that the iteration will diverge.

5 **Absolute and relative errors**
The numerical work in parts (i) and (ii) was done well by almost all. In part (iii), candidates were expected to know that when a number with a small relative error is raised to a power k the relative error will be increased (approximately) by a factor of k. This part was not done well.

6 **Numerical integration**
The numerical work in first two parts was done well by the vast majority of candidates. Part (iii) was more challenging. Not all candidates appeared to know how to recognise fourth order convergence. Some knew what to look for but arithmetical errors prevented them finding it. The value of I was often given to fewer decimal places than the work warranted. It is *not* correct to look for the number of figures of agreement in the two best Simpson's rule values. Thinking about differences and ratios of differences will show that the last Simpson's rule value will be very much more accurate than the one before.

7 **Newton's forward difference interpolation formula**

Almost all candidates found the quadratic in part (i) successfully, though some insisted on doing algebraic simplification which was not required. Part (i) was found easy too. In part (iii), the best approach is to add on the cubic term to the quadratic already found: this is one of the virtues of Newton's method. Some candidates worked from scratch here. The answers to part (iv) were frequently poor. Either the wrong numerical values found earlier prevented sensible comments, or what candidates said was muddled and unclear. The intended point was that the absolute change is greater in f(6) but the relative change is greater in f(2).

4777: Numerical Computation

General Comments

As usual, there was only a small entry for this paper. About half the candidates produced three good or very good solutions. Of the remainder almost all were able to demonstrate a decent ability to do numerical work on a spreadsheet.

Candidates' organisation of their work remains something of a problem. Solutions should not consist just of print-outs of numbers: the formulae used should be printed too. And it is only common sense that solutions should be labelled and properly ordered. If candidates do not indicate what a sheet of numbers represents or even which question they are from, there is every chance that correct work will go unrewarded.

Comments on Individual Questions

1 **Solution of an equation by relaxation**
This was a popular question, attracting some excellent solutions. Those candidates who did not score high marks generally fell down on the algebra rather than the numerical work.

2 **Gaussian integration**
This was the least popular question, but again there was very good work seen. A mistake, perhaps arising from unfamiliarity, was failing to realise that when a Gaussian rule is applied to an integral over the interval [0, 1] the formula needs to be adjusted so that it is centred on 0.5 rather than 0.

3 **Runge-Kutta methods**
This question was generally very well done, with candidates showing a good grasp of technique (implementing the given algorithms on a spreadsheet) and an understanding of principle (comparing the accuracy of the two methods).

4 **Least squares approximation**
Again, this question attracted some very good solutions – though there were a couple of candidates who made very limited progress. Obtaining the given normal equation was a challenge to one or two, but writing down the normal equations proved easy enough. The numerical work was often successful, though a couple of candidates wanted the sum of the residuals to be zero. (In this case there is no constant term in the fitted equation so the sum of the residuals will *not* be zero.)

Coursework

Administration

Most centres adhered to the deadline set by OCR very well and if the first despatch was only the MS1 then they responded rapidly to the sample request. Centres are asked to ensure that OCR has the current email address for the Examinations Officer; some coursework samples were sent late because this was not the case.

Acknowledgement is made of the amount of work that is involved to mark and internally moderate the work of candidates. This results, in the vast majority of cases, in marks being awarded by the centre that are appropriate. The unit specific comments are offered for the sake of centres that have had their marks adjusted for some reason.

Internal Moderators are asked to ensure that they adjust the criteria marks in such a way that the final mark on the cover sheet agrees with the submitted mark on the MS1 and is the sum of criteria marks.

Additionally, some assessors only give domain marks. This might be fine if the candidate deserves full marks (or zero) for a domain, but it makes it very difficult for External Moderators to understand the marking if a mark has been withheld – in this case we do not know which of the criteria have, in the opinion of the assessor, not been met adequately.

Assessors are reminded that they should not award marks in the domains as a result of the oral communication as this is work that is not presented and therefore not available to the Moderator. Any missing work also produces a mismatch of the mark recorded and the value of the work. This includes and spreadsheet work.

Teachers should note that all the comments offered have been made before. These reports should provide a valuable aid to the marking process and we would urge all Heads of Departments to ensure that these reports are read by all those involved in the assessment of coursework.

Core 3 – 4753/02

There continue to be a significant number of centres where so many of the points outlined below are not being penalised appropriately that the mark submitted is too generous.

The following points should typically be penalised by half a mark – failure to penalise four or more results in a mark outside tolerance. Centres are reminded that this list has been offered before so we wonder whether these reports are being disseminated to the assessors.

Change of Sign

- Graphs of the function being used do not constitute an illustration of the method.
- Equations which can be solved analytically should not be used for demonstration of success. This includes cubic equations with one integer root.
- The answer should be given as a value rather than given as an interval.
- Trivial equations to demonstrate failure should not be used.
- Tables of values which actually find the root merely demonstrate that the method has succeeded, not failed.
- Graphs which the candidate claims crosses the axis or just touch but don't should be checked.

Newton Raphson

- Equations with only one root fail to address the second criterion and so the whole mark should be lost.
- The root stated with no iterates given should not be accepted.
- Inadequate illustrations (for example, an "Autograph" generated tangent with no annotation or just a single tangent) should be penalised.
- Graphs should match the iterates.
- Error bounds should be established by a change of sign.
- Starting values too far away from the root or too artificial should be penalised.
- Failure examples should include iterates.

Rearrangement

- We often see incorrect rearrangements that have not been spotted and sometimes ticked as correct.
- As in previous domains, illustrations should match iterates.
- Weak discussions of $g'(x)$. Candidates should not just quote the criterion without linking it to their function and its graph.

Comparison

- Different starting values for the iterative methods mean that any comparison of speed of convergence is invalid.
- Sometimes different roots are found, contrary to the criteria
- Thin discussions.

Notation

- Equations, functions and expressions need to be distinguished. Candidates who assert that they are going solve $y = x^3 + 3x - 4$ or that they are going to solve $x^3 + 3x - 4$ should be penalised. Likewise, for candidates who assert that they are going to find roots of a graph.

Oral

The specification asks for a written report.

We would emphasise that this is not a comment on the work of candidates, who are entitled to write what they like. It is to point out that we find these errors in the reports that are being credited with marks for which there is no justification.

Differential Equations – 4758/02

The moderation process for this series appears to have resulted in more large changes than is generally the case.

Most centres that were moderated downwards tended to be penalised in the first domain. When simplifying and setting out the model a simple list of assumptions is not sufficient. The relative importance and relevance of the assumptions needs to be discussed. Although this is an essential and fundamental part of the modelling process, it is often given superficial treatment. When setting up the mathematical model, any variables or parameters should be clearly defined. This is particularly relevant when setting up the differential equations for 'Interacting Species'.

In the second domain, apart from the usual problem of not modelling the 'Aeroplane Landing' for the whole of the motion (ie $0 \le t \le 26$), the weakest area tended to be a consideration of the variation in parameters. This is particularly true for 'Aeroplane Landing'. A consideration of the accuracy of the provided data could help in this respect.

Where possible, a comparison of the predicted and collected data should be in table and graphical form. Graphs should be titled and axes labelled clearly, otherwise the narrative becomes difficult to follow, especially if there are several graphs.

Note, also, that the marking in domains 1 to 4 are allocated for the initial model. If the revised model meets some of these criteria, eg by considering a variation in parameters, marks cannot be allocated in these earlier domains.

When revising the model there can be a tendency to curve fit at this stage rather than model. For example, assuming that say air resistance is proportional to v^n then finding the value of n which produced the best fit, is not modelling. On the other hand if air resistance is assumed proportional to v^2 then some justification for assuming this particular power, is expected.

Numerical Methods – 4776/02

The most popular task is to find the value of an integral numerically. The following comments are offered – it is to be hoped that those teaching and assessing use these comments to inform their teaching and their assessment of the work.

There were several cases where incorrect work had been ticked. Assessors are requested not to tick work unless it has been checked thoroughly.
A significant error is a failure to enter into the spreadsheet the function that has been identified.

A typical example might be to state that the task is to find $\int_0^{\pi/2} (\sin x)^2 \, dx$ but then to enter into the spreadsheet the function sinx^2 meaning that it is the integral $\int_0^1 \sin(x^2) \, dx$ that is being found.

Where this error occurs it is not found and penalised very often.

There were also many cases where there was no annotation at all; leaving the Moderators unable to discern what work had been checked.

In domain 1, a formal statement of the problem in correct mathematical notation is expected.

In domain 2, there is no need to repeat book work. The second mark is earned by explaining why the algorithm selected is appropriate for the specified problem. It should be made clear by reference to the graph of the function whether the Trapezium Rule under- or over- estimates the integral, rather than simply stating the general case.

In domain 3, the usual minimum is subdivision of the interval into 1, 2, 4, 8, 16, 32 , 64 strips for a "substantial application". In some instances incorrect work still attracted full credit, or a different integral to the one specified was found.

In domain 4, the second mark is awarded for an explanation of how the algorithm has been implemented. An annotated print out of spreadsheet cell formulae works well, but please note that a printout of cell formulae with no annotations does not fulfil this criterion.

In domain 5, the convergence of r to its theoretical value – or another value – should be demonstrated before extrapolation takes place. It is not valid to use these extrapolated values of Trapezium Rule and Midpoint Rule estimates as bounds. Extrapolation should lead to a clear improvement in accuracy for the award of the third mark. Once Simpson's Rule estimates have been found, there is little point in extrapolating Trapezium Rule and Midpoint Rule estimates.

In domain 6, "6 significant figures" is a guideline (and as minimum), not the aim, as such. Candidates are expected to state the most accurate value that they can justify from their error analysis for the second mark, and a correct justification of the precision quoted earns the third mark. There is no credit for commenting on limitations relating to the precision the spreadsheet package works to. Rather, the commentary should focus on difficulties such as the failure of *r* to converge to its theoretical value, and how this was addressed.

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