OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4758:** Differential Equations

Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

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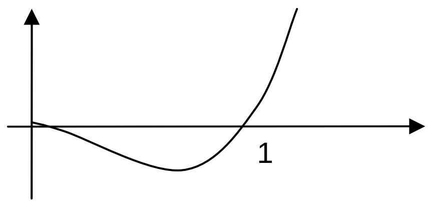
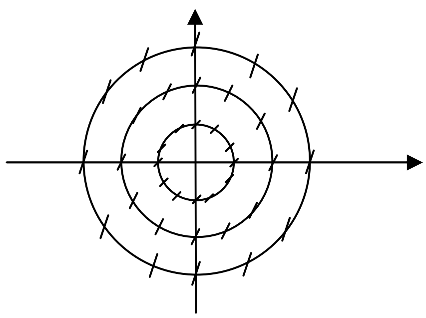
1(i)	$\lambda^2 + 4\lambda + 3 = 0$	M1	Auxiliary equation
	$\lambda = -1$ or -3	A1	
	CF $Ae^{-t} + Be^{-3t}$	F1	CF for their roots
	PI $y = a\cos 2t + b\sin 2t$	B1	
	$\dot{y} = -2a\sin 2t + 2b\cos 2t$	M1	Differentiate twice and substitute
	$\ddot{y} = -4a\cos 2t - 4b\sin 2t$		
	$-4a\cos 2t - 4b\sin 2t - 8a\sin 2t + 8b\cos 2t + 3a\cos 2t + 3b\sin 3t = 13\cos 2t$	M1	Compare coefficients
	$8b - a = 13$ $-b - 8a = 0$	A1	
	$a = -\frac{1}{5},\ b = \frac{8}{5}$	A1	
	GS $y = \frac{1}{5}(8\sin 2t - \cos 2t) + Ae^{-t} + Be^{-3t}$	F1	PI + CF with two arbitrary constants
			9
(ii)	$t = 0,\ y = 0 \Rightarrow 0 = -\frac{1}{5} + A + B$	M1	Use condition
	$\dot{y} = \frac{1}{5}(16\cos 2t + 2\sin 2t) - Ae^{-t} - 3Be^{-3t}$	M1	Differentiate
		F1	
	$t = 0,\ \dot{y} = 0 \Rightarrow 0 = \frac{16}{5} - A - 3B$	M1	Use condition
	$\Rightarrow A = -\frac{13}{10},\ B = \frac{3}{2}$	A1	
	$y = \frac{1}{5}(8\sin 2t - \cos 2t) - \frac{13}{10}e^{-t} + \frac{3}{2}e^{-3t}$	A1	Cao
			6
(iii)	If $z = y + c$, differentiating (*) gives new DE	M1	Recognise derivative
	and has 3 arbitrary constants so must be GS	A1	
	or		
	Integrating gives (*) with $+k$ on RHS	*M1*	
	PI will be previous PI $+\frac{1}{3}k$, CF as before, so GS $y + c$	*A1*	
	SC1 for showing that correct *y* from (i) + *c* satisfies new DE		2
(iv)	$z = \frac{1}{5}(8\sin 2t - \cos 2t) + De^{-t} + Ee^{-3t} + c$		
	$t = 0,\ z = 2 \Rightarrow 2 = -\frac{1}{5} + D + E + c$	M1	Use condition
	$\dot{z} = \frac{1}{5}(16\cos 2t + 2\sin 2t) - De^{-t} - 3Ee^{-3t}$	F1	Derivative
	$t = 0,\ \dot{z} = 0 \Rightarrow 0 = \frac{16}{5} - D - 3E$	M1	Use condition
	$\ddot{z} = \frac{1}{5}(-32\sin 2t + 4\cos 2t) + De^{-t} + 9Ee^{-3t}$	F1	Second derivative: condone, for this mark only, +*c* appearing
	$t = 0, \ddot{z} = 13 \Rightarrow 13 = \frac{4}{5} + D + 9E$	M1	Use condition
	$D = -\frac{13}{10},\ E = \frac{3}{2},\ c = 2$	B1	
	$z = \frac{1}{5}(8\sin 2t - \cos 2t) - \frac{13}{10}e^{-t} + \frac{3}{2}e^{-3t} + 2$	A1	Cao
			7

2(a)(i)	$I = \exp(\int -\frac{2}{x}\,dx)$	M1	Attempt integrating factor	
	$= \exp(-2\ln x)$	A1		
	$= x^{-2}$	A1		
	$x^{-2}\frac{dy}{dx} - 2x^{-3}y = x^{-3/2}$	M1	Multiply both sides by IF	
	$\frac{d}{dx}(x^{-2}y) = x^{-3/2}$	M1		
	$x^{-2}y = -2x^{-1/2} + A$	M1	Integrate both sides	
		A1		
	$y = -2x^{3/2} + Ax^2$	F1	Must divide constant	8
(ii)	$0 = -2 + A$	M1		
	$y = 2x^2 - 2x^{3/2}$	A1		2
(iii)	$x \to 0,\ y \to 0$	F1		
	$\frac{dy}{dx} = 4x - 3x^{1/2} = 0 \Leftrightarrow x = \frac{9}{16}$ (as $x > 0$)	M1		
	$x \to 0,\ \frac{dy}{dx} \to 0$	F1		
		B1	Behaviour at origin	
		B1	Through (1,0) and shape for $x > 1$	
		B1	Stationary point at $\left(\frac{9}{16}, -\frac{27}{128}\right)$	6
(b)(i)	Circle centre origin	B1		
	Radius 1	B1		2
(ii)		B1	One isocline correct	
		B1	All three isoclines correct	
		B1	Reasonably complete and accurate direction indicators	3
(iii)		B1	Solution curve	1
(iv)		B1	Solution curve	
		B1	Zero gradient at origin	2

3(a)(i)	N2L: $ma = -2k^2 x$	M1		
	$2v\frac{dv}{dx} = -2k^2 x$	M1	Acceleration $= v\frac{dv}{dx}$	
	$v\frac{dv}{dx} = -k^2 x$	E1		
				3
(ii)	$\int v\,dv = \int -k^2 x\,dx$	M1	Separate and integrate	
	$\frac{1}{2}v^2 = -\frac{1}{2}k^2x^2 + A$	A1	LHS	
		A1	RHS	
	$x = a,\ v = 0 \Rightarrow A = \frac{1}{2}k^2a^2$	M1	Use condition	
	$v^2 = k^2(a^2 - x^2)$	A1		
	So for $v < 0$, $\frac{dx}{dt} = -k\sqrt{a^2 - x^2}$	E1		
				6
(iii)	$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \int -k\,dt$	M1	Separate and integrate	
	$\arcsin\frac{x}{a} + B = -kt$	A1	LHS	
		A1	RHS	
	$x = a,\ t = 0 \Rightarrow B = -\frac{1}{2}\pi$	M1	Use condition	
	$x = a\sin(\frac{1}{2}\pi - kt) = a\cos kt$	A1	Either form	
				5
(b)(i)	$\int \omega\,d\omega = \int -9\sin\theta\,d\theta$	M1	Separate and integrate	
	$\frac{1}{2}\omega^2 = 9\cos\theta + C$	A1	LHS	
		A1	RHS	
	$\theta = \frac{1}{3}\pi,\ \omega = 0 \Rightarrow C = -\frac{9}{2}$	M1	Use condition	
	So $\omega^2 = 9(2\cos\theta - 1)$	A1		
	$\frac{d\theta}{dt} = -3\sqrt{2\cos\theta - 1}$ (decreasing)	E1		
				6
(ii)	$\theta = \frac{1}{3}\pi \Rightarrow \dot{\theta} = 0$	M1		
	So estimate $= \frac{1}{3}\pi + 0 = \frac{1}{3}\pi$	A1		
	The algorithm will keep giving $\theta = \frac{1}{3}\pi$	B1		
	but θ is not constant so not useful	B1		
				4

4(i)	$y = -\frac{1}{2}\dot{x} - \frac{3}{2}x + \frac{3}{2}t$	M1		
	$\dot{y} = -\frac{1}{2}\ddot{x} - \frac{3}{2}\dot{x} + \frac{3}{2}$	M1		
	$-\frac{1}{2}\ddot{x} - \frac{3}{2}\dot{x} + \frac{3}{2} = 2x + (-\frac{1}{2}\dot{x} - \frac{3}{2}x + \frac{3}{2}t) + t + 2$	M1	Eliminate y	
		M1	Eliminate $\dot{y}$	
	$\ddot{x} + 2\dot{x} + x = -5t - 1$	E1		
				5
(ii)	$\lambda^2 + 2\lambda + 1 = 0$	M1	Auxiliary equation	
	$\lambda = -1$ (repeated)	A1	Root	
	CF: $(A + Bt)e^{-t}$	F1	CF for their root(s) (with two constants)	
	PI: $x = at + b$	B1		
	$\dot{x} = a, \ddot{x} = 0$			
	In DE: $0 + 2a + at + b = -5t - 1$	M1	Differentiate and substitute	
	$a = -5$			
	$2a + b = -1$	M1	Compare and solve	
	$a = -5, b = 9$	A1		
	GS: $x = 9 - 5t + (A + Bt)e^{-t}$	F1	GS = PI + CF with two arbitrary constants	
				8
(iii)	$y = -\frac{1}{2}\dot{x} - \frac{3}{2}x + \frac{3}{2}t$	M1		
	$= -\frac{1}{2}[-5 + Be^{-t} - (A + Bt)e^{-t}]$	M1	Differentiate (product rule)	
	$-\frac{3}{2}[9 - 5t + (A + Bt)e^{-t}] + \frac{3}{2}t$	M1	Substitute	
	$= 9t - 11 - (A + \frac{1}{2}B + Bt)e^{-t}$	A1		
				4
(iv)	$t = 0, x = 9 \Rightarrow A = 0$	M1	Use condition	
	$t = 0, y = 0 \Rightarrow 0 = -11 - \frac{1}{2}B \Rightarrow B = -22$	M1	Use condition	
	$x = 9 - 5t - 22te^{-t}$	A1		
	$y = 9t - 11 + (11 + 22t)e^{-t}$	A1		
				4
(v)	$e^{-t} \to 0$	M1		
	$x \approx 9 - 5t$	F1		
	$y \approx 9t - 11$	F1		
				3

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