OCR
RECOGNISING ACHIEVEMENT

Thursday 21 June 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4756 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

* 4 7 1 5 7 4 0 6 1 2 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

Answer all the questions

1 **(a)** **(i)** Differentiate the equation $\sin y = x$ with respect to x, and hence show that the derivative of $\arcsin x$ is $\dfrac{1}{\sqrt{1-x^2}}$. **[4]**

(ii) Evaluate the following integrals, giving your answers in exact form.

(A) $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt{2-x^2}}\,dx$ **[3]**

(B) $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1-2x^2}}\,dx$ **[4]**

(b) A curve has polar equation $r = \tan\theta$, $0 \leqslant \theta < \frac{1}{2}\pi$. The points on the curve have cartesian coordinates (x, y). A sketch of the curve is given in Fig. 1.

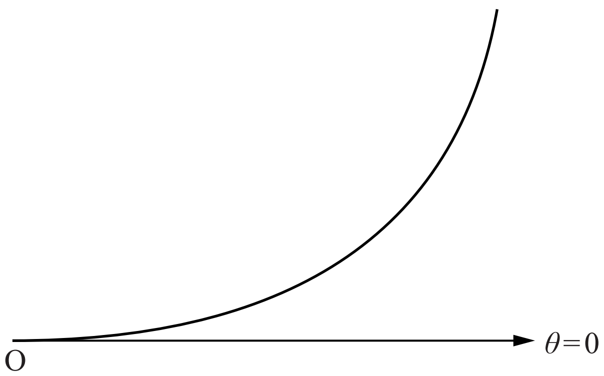

Fig. 1

Show that $x = \sin\theta$ and that $r^2 = \dfrac{x^2}{1-x^2}$.

Hence show that the cartesian equation of the curve is

$$y = \frac{x^2}{\sqrt{1-x^2}}.$$

Give the cartesian equation of the asymptote of the curve. **[7]**

2 **(a)** **(i)** Given that $z = \cos\theta + \mathrm{j}\sin\theta$, express $z^n + \frac{1}{z^n}$ and $z^n - \frac{1}{z^n}$ in simplified trigonometric form. **[2]**

(ii) Beginning with an expression for $\left(z + \frac{1}{z}\right)^4$, find the constants A, B, C in the identity

$$\cos^4\theta \equiv A + B\cos 2\theta + C\cos 4\theta.$$ **[4]**

(iii) Use the identity in part **(ii)** to obtain an expression for $\cos 4\theta$ as a polynomial in $\cos\theta$. **[2]**

(b) **(i)** Given that $z = 4\mathrm{e}^{\mathrm{j}\pi/3}$ and that $w^2 = z$, write down the possible values of w in the form $r\mathrm{e}^{\mathrm{j}\theta}$, where $r > 0$. Show z and the possible values of w in an Argand diagram. **[5]**

(ii) Find the least positive integer n for which z^n is real.

Show that there is no positive integer n for which z^n is imaginary.

For each possible value of w, find the value of w^3 in the form $a + \mathrm{j}b$ where a and b are real. **[5]**

3 **(i)** Find the value of a for which the matrix

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & 3 \\ -1 & a & 4 \\ 3 & -2 & 2 \end{pmatrix}$$

does not have an inverse.

Assuming that a does not have this value, find the inverse of $\mathbf{M}$ in terms of a. **[7]**

(ii) Hence solve the following system of equations.

$$\begin{aligned} x + 2y + 3z &= 1 \\ -x \qquad + 4z &= -2 \\ 3x - 2y + 2z &= 1 \end{aligned}$$ **[4]**

(iii) Find the value of b for which the following system of equations has a solution.

$$\begin{aligned} x + 2y + 3z &= 1 \\ -x + 6y + 4z &= -2 \\ 3x - 2y + 2z &= b \end{aligned}$$

Find the general solution in this case and describe the solution geometrically. **[7]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Prove, from definitions involving exponential functions, that

$$\cosh 2u = 2\sinh^2 u + 1.$$ **[3]**

(ii) Prove that, if $y \geqslant 0$ and $\cosh y = u$, then $y = \ln\left(u + \sqrt{(u^2 - 1)}\right)$. **[4]**

(iii) Using the substitution $2x = \cosh u$, show that

$$\int \sqrt{4x^2 - 1}\,\mathrm{d}x = ax\sqrt{4x^2 - 1} - b\,\mathrm{arcosh}\,2x + c,$$

where a and b are constants to be determined and c is an arbitrary constant. **[7]**

(iv) Find $\displaystyle\int_{\frac{1}{2}}^{1} \sqrt{4x^2 - 1}\,\mathrm{d}x$, expressing your answer in an exact form involving logarithms. **[4]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 This question concerns curves with polar equation $r = \sec\theta + a$, where a is a constant.

(i) State the set of values of θ between 0 and 2π for which r is undefined. **[2]**

For the rest of the question you should assume that θ takes all values between 0 and 2π for which r is defined.

(ii) Use your graphical calculator to obtain a sketch of the curve in the case $a = 0$. Confirm the shape of the curve by writing the equation in cartesian form. **[3]**

(iii) Sketch the curve in the case $a = 1$.

Now consider the curve in the case $a = -1$. What do you notice?

By considering both curves for $0 < \theta < \pi$ and $\pi < \theta < 2\pi$ separately, describe the relationship between the cases $a = 1$ and $a = -1$. **[6]**

(iv) What feature does the curve exhibit for values of a greater than 1?

Sketch a typical case. **[3]**

(v) Show that a cartesian equation of the curve $r = \sec\theta + a$ is $(x^2 + y^2)(x - 1)^2 = a^2x^2$. **[4]**