

Friday 1 June 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4764 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4764
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (24 marks)

- 1 A rocket in deep space has initial mass m_0 and is moving in a straight line at speed v_0 . It fires its engine in the direction opposite to the motion in order to increase its speed. The propulsion system ejects matter at a constant mass rate k with constant speed u relative to the rocket. At time t after the engines are fired, the speed of the rocket is v .

(i) Show that while mass is being ejected from the rocket, $(m_0 - kt) \frac{dv}{dt} = uk$. [6]

(ii) Hence find an expression for v at time t . [5]

- 2 A light elastic string AB has stiffness k . The end A is attached to a fixed point and a particle of mass m is attached at the end B. With the string vertical, the particle is released from rest from a point at a distance a below its equilibrium position. At time t , the displacement of the particle below the equilibrium position is x and the velocity of the particle is v .

(i) Show that

$$mv \frac{dv}{dx} = -kx. \quad [4]$$

(ii) Show that, while the particle is moving upwards and the string is taut,

$$v = -\sqrt{\frac{k}{m}(a^2 - x^2)}. \quad [5]$$

(iii) Hence use integration to find an expression for x at time t while the particle is moving upwards and the string is taut. [4]

Section B (48 marks)

- 3 A uniform rigid rod AB of length $2a$ and mass m is smoothly hinged to a fixed point at A so that it can rotate freely in a vertical plane. A light elastic string of modulus λ and natural length a connects the midpoint of AB to a fixed point C which is vertically above A with $AC = a$. The rod makes an angle 2θ with the upward vertical, where $\frac{1}{3}\pi \leq 2\theta \leq \pi$. This is shown in Fig. 3.

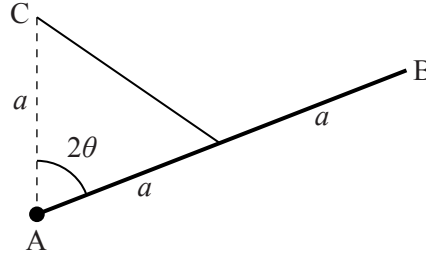


Fig. 3

- (i) Find the potential energy, V , of the system relative to A in terms of m , λ , a and θ . Show that

$$\frac{dV}{d\theta} = 2a \cos \theta (2\lambda \sin \theta - 2mg \sin \theta - \lambda). \quad (*) \quad [7]$$

Assume now that the system is set up so that the result (*) continues to hold when $\pi < 2\theta \leq \frac{5}{3}\pi$.

- (ii) In the case $\lambda < 2mg$, show that there is a stable position of equilibrium at $\theta = \frac{1}{2}\pi$. Show that there are no other positions of equilibrium in this case. [9]
- (iii) In the case $\lambda > 2mg$, find the positions of equilibrium for $\frac{1}{3}\pi \leq 2\theta \leq \frac{5}{3}\pi$ and determine for each whether the equilibrium is stable or unstable, justifying your conclusions. [7]

- 4 (i) Show by integration that the moment of inertia of a uniform circular lamina of radius a and mass m about an axis perpendicular to the plane of the lamina and through its centre is $\frac{1}{2}ma^2$. [6]

A closed hollow cylinder has its curved surface and both ends made from the same uniform material. It has mass M , radius a and height h .

- (ii) Show that the moment of inertia of the cylinder about its axis of symmetry is $\frac{1}{2}Ma^2\left(\frac{a+2h}{a+h}\right)$. [6]

For the rest of this question take the cylinder to have mass 8 kg, radius 0.5 m and height 0.3 m.

The cylinder is at rest and can rotate freely about its axis of symmetry. It is given a tangential impulse of magnitude 55 N s at a point on its curved surface. The impulse is perpendicular to the axis.

- (iii) Find the angular speed of the cylinder after the impulse. [3]

A resistive couple is now applied to the cylinder for 5 seconds. The magnitude of the couple is $2\dot{\theta}^2$ N m, where $\dot{\theta}$ is the angular speed of the cylinder in rad s^{-1} .

- (iv) Formulate a differential equation for $\dot{\theta}$ and hence find the angular speed of the cylinder at the end of the 5 seconds. [7]

The cylinder is now brought to rest by a constant couple of magnitude 0.03 N m.

- (v) Calculate the time it takes from when this couple is applied for the cylinder to come to rest. [3]

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