OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4751:** Introduction to Advanced Mathematics

Mark Scheme for June 2012

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions: GCE Mathematics (MEI) Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$y = -2x + 7$ isw (0, 7) and (3.5, 0) oe or ft their $y = -2x + c$	2 1 **[3]**	M1 for $y - 1 = -2(x - 3)$ or $1 = -2 \times 3 + c$ oe	condone lack of brackets and eg $y = 7$, $x = 3.5$ or ft isw but 0 for poor notation such as (3.5, 7) and no better answers seen
2			$[b =] \pm \sqrt{\frac{3a}{2c}}$ oe www	3 **[3]**	M2 for $[b^2 =] \frac{3a}{2c}$ soi or M1 for other $[b^2 =] \frac{ka}{c}$ or $[b^2 =] \frac{a}{kc}$ oe and M1 for correctly taking the square root of their b^2, including the ± sign;	eg M2 for $[b =] \sqrt{\frac{3a}{2c}}$ allow M1 for a triple-decker or quadruple-decker fraction or decimals eg $\frac{1.5a}{c}$, if no recovery later square root must extend below the fraction line
3	(i)		25	2 **[2]**	M1 for $\frac{1}{\frac{1}{25}}$ or $\left(\frac{1}{25}\right)^{-1}$ or 5^2 or $\frac{25}{1}$	
3	(ii)		$\frac{4}{9}$	2 **[2]**	M1 for 4 or 9 or $\frac{1}{9}$ or $\frac{2}{3}$ or $\left(\frac{2}{3}\right)^2$ or $\sqrt[3]{\frac{64}{729}}$ seen	0 for just $\left(\frac{64}{729}\right)^{\frac{1}{3}}$
4			$\frac{x-3}{x+2}$ or $1 - \frac{5}{x+2}$ as final answer www	3 **[3]**	B2 for correct answer seen and then spoilt M1 for $(x + 3)(x - 3)$ and M1 for $(x + 2)(x + 3)$	

Question			Answer	Marks	Guidance	
5	(i)		30	3 [3]	M1 for $\left(\sqrt{6}\right)^3 = 6\sqrt{6}$ soi and M1 for $\sqrt{24} = 2\sqrt{6}$ soi or allow SC2 for final answer of $5\left(\sqrt{6}\right)^2$ or $5\sqrt{36}$ or $10\sqrt{9}$ etc	M0 for $6000\sqrt{6}$ ie cubing 10 as well for those using indices: M1 for both $10 \times 6^{3/2}$ and $2 \times 6^{1/2}$ oe then M1 for 5×6 oe award SC2 for similar correct answer with no denominator
5	(ii)		$\frac{8}{11}$	2 [2]	M1 for common denominator $\left(4+\sqrt{5}\right)\left(4-\sqrt{5}\right)$ soi - may be in separate fractions or for a final answer with denominator 11, even if worked with only one fraction	condone lack of brackets
6	(i)		10 cao	1 [1]		
6	(ii)		-720 [x^3]	4 [4]	B3 for 720 [x^3] or for $10 \times 9 \times -8$ [x^3] or M2 for $10 \times 3^2 \times (-2)^3$ oe or ft from (i) or M1 for two of these three elements correct or ft; condone x still included	condone $-720x$ etc allow equivalent marks for the x^3 term as part of a longer expansion eg M2 for $3^5\left(\ldots 10 \times \left(\frac{-2}{3}\right)^3 \ldots\right)$ or M1 for $10 \times \left(\frac{-2}{3}\right)^3$ etc

Question			Answer	Marks	Guidance	
7			$4k^2 - 4 \times 1 \times 5$ or $k^2 - 5$ [< 0] oe or $[(x + k)^2 +] 5 - k^2$ [> 0] oe	M2	allow =, > , ≤ etc instead of < or M1 for $b^2 - 4ac$ soi (may be in formula) or for attempt at completing square	allow M2 for $2k^2 < 20$, $2k^2 - 20 = 0$ etc but M1 only for just $2k^2 - 20$ ignore rest of quadratic formula ignore $\sqrt{b^2 - 4ac} < 0$ seen if $b^2 - 4ac < 0$ then used, otherwise just M1 for $\sqrt{b^2 - 4ac} < 0$
			$-\sqrt{5} < k < \sqrt{5}$	A2 **[4]**	may be two separate inequalities or A1 for one 'end' correct or B1 for 'endpoint' = $\sqrt{5}$	allow SC1 for $-\sqrt{10} < k < \sqrt{10}$ following at least M1 for $2k^2 - 20$ oe
8			$16 + 2b + c = 0$ oe	M1	need not be simplified; condone 8 or 32 as first term if 2^4 not seen	in this question use annotation to indicate where part marks are earned
			$81 - 3b + c = 85$ oe	B2	M1 for f(–3) seen or used, condoning one error except $+3b$ – need not be simplified or for long division as far as obtaining $x^3 - 3x^2$ in quotient	eg M1 for $81 - 3b + c = 0$ 'long division' may be seen in grid or a mixture of methods may be used eg B2 for $c - 3(b - 27) = 85$
			$20 + 5b = 0$ oe	M1	for elimination of one variable, ft their equations in b and c, condoning one error in rearrangement of their original equations or in one term in the elimination	correct operation must be used in elimination
			$b = -4$ and $c = -8$	A1 **[5]**	allow correct answers to imply last M1 after correct earlier equations	for misread of x^4 as x^3 or x^2 or higher powers, allow all 3 Ms equivalently

Question			Answer	Marks	Guidance	
9			$6n + 9$ isw or $3(2n + 3)$	B1		
			$6n$ is even [but 9 is odd], even + odd = odd or $2n + 3$ is odd since even + odd = odd and odd × odd = odd	B1 dep	this mark is dependent on the previous B1 accept equiv. general statements using either $6n + 9$ or $3(2n + 3)$	
			'n is a multiple of 3' or 'n is divisible by 3' without additional incorrect statement(s)	B2	B2 for 'it is divisible by 9, so n is divisible by 3' M1 for '$6n$ is divisible by 9' or '$2n + 3$ is divisible by 3' or for 'n is a multiple of 3' oe with additional incorrect statement(s)	B2 for just 'it is divisible by 3' but M1 for 'it is divisible by 9, so it is divisible by 3' eg M1 for 'n is divisible by 9, so n is divisible by 3' N.B. 0 for 'n is a factor of 3' (but M1 may be earned earlier)
				[4]		

Question			Answer	Marks	Guidance	
10	(i)		$AB^2 = (1-(-1))^2 + (5-1)^2$	M1	oe, or square root of this; condone poor notation re roots; condone $(1+1)^2$ instead of $(1-(-1))^2$ allow M1 for vector AB = $\begin{pmatrix}-2\\-4\end{pmatrix}$, condoning poor notation, or triangle with hyp AB and lengths 2 and 4 correctly marked	
			$BC^2 = (3-(-1))^2 + (-1-1)^2$	M1	oe, or square root of this; condone poor notation re roots; condone $(3+1)^2$ instead of $(3-(-1))^2$ oe allow M1 for vector BC = $\begin{pmatrix}4\\-2\end{pmatrix}$, condoning poor notation, or triangle with hyp BC and lengths 4 and 2 correctly marked	
			shown equal eg $AB^2 = 2^2 + 4^2$ [=20] and $BC^2 = 4^2 + 2^2$ [=20] with correct notation for final comparison	A1 **[3]**	or statement that AB and BC are each the hypotenuse of a right-angled triangle with sides 2 and 4 so are equal SC2 for just $AB^2 = 2^2 + 4^2$ and $BC^2 = 4^2 + 2^2$ (or roots of these) with no clearer earlier working; condone poor notation	eg A0 for AB = 20 etc

Question			Answer	Marks	Guidance	
10	(ii)		[grad. of AC =] $\frac{5-(-1)}{1-3}$ or $\frac{6}{-2}$ oe	M1	award at first step shown even if errors after	
			[grad. of BD =] $\frac{5-1}{11-(-1)}$ or $\frac{4}{12}$ oe	M1		if one or both of grad AC = -3 and grad BD = 1/3 seen without better working for both gradients, award one M1 only. For M1M1 it must be clear that they are obtained independently
			showing or stating product of gradients = –1 or that one gradient is the negative reciprocal of the other oe	B1	eg accept $m_1 \times m_2 = -1$ or 'one gradient is negative reciprocal of the other' B0 for 'opposite' used instead of 'negative' or 'reciprocal'	may be earned independently of correct gradients, but for all 3 marks to be earned the work must be fully correct
				[3]		

Question			Answer	Marks	Guidance	
10	(iii)		midpoint E of AC = (2, 2) www	B1	condone missing brackets for both B1s	0 for ((5+ –1)/2, (1+3)/2) = (2, 2)
			eqn BD is $y = \frac{1}{3}x + \frac{4}{3}$ oe	M1	accept any correct form isw or correct ft their gradients or their midpt F of BD this mark will often be gained on the first line of their working for BD	may be earned using (2, 2) but then must independently show that B or D or (5, 3) is on this line to be eligible for A1
			eqn AC is $y = –3x + 8$ oe	M1	accept any correct form isw or correct ft their gradients or their midpt E of AC this mark will often be gained on the first line of their working for AC [see appendix for alternative methods instead showing E is on BD for this M1]	if equation(s) of lines are seen in part ii, allow the M1s if seen/used in this part
			using both lines and obtaining intersection E is (2, 2) (NB must be independently obtained from midpt of AC)	A1		[see appendix for alternative ways of gaining these last two marks in different methods]
			midpoint F of BD = (5,3)	B1	this mark is often earned earlier see the appendix for some common alternative methods for this question; for all methods, for A1 to be earned, all work for the 5 marks must be correct	for all methods show annotations M1 B1 etc then omission mark or A0 if that mark has not been earned
				[5]		

Question			Answer	Marks	Guidance	
11	(i)		$(2x + 1)(x + 2)(x – 5)$	M1	or $(x + 1/2)(x + 2)(x – 5)$; need not be written as product	throughout, ignore '=0'
			correct expansion of two linear factors of their product of three linear factors	M1		for all Ms in this part condone missing brackets if used correctly
			expansion of their linear and quadratic factors	M1	dep on first M1; ft one error in previous expansion; condone one error in this expansion	
					or for direct expansion of all three factors, allow M2 for $2x^3 – 10x^2 + 4x^2 + x^2 – 20x – 5x + 2x – 10$ [or half all these], or M1 if one or two errors,	dep on first M1
			$[y =] 2x^3 – 5x^2 – 23x – 10$ or $a = –5$, $b = – 23$ and $c = – 10$	A1		condone poor notation when 'doubling' to reach expression with $2x^3$...
					for an attempt at setting up three simultaneous equations in a, b, and c: M1 for at least two of the three equations	$250 + 25a + 5b + c = 0$ $–16 + 4a –2b + c = 0$ $–¼ + ¼ a – ½ b + c = 0$ oe
					then M2 for correctly eliminating any two variables or M1 for correctly eliminating one variable to get two equations in two unknowns	
					and then A1 for values.	
				[4]		

Question			Answer	Marks	Guidance	
11	(ii)		graph of cubic correct way up	B1		must not be ruled; no curving back; condone slight 'flicking out' at ends; allow min on y axis or in 3rd or 4th quadrants; condone some 'doubling' or 'feathering' (deleted work still may show in scans)
			crossing x axis at -2, $-1/2$ and 5	B1	B0 if stops at x-axis on graph or nearby in this part mark intent for intersections with both axes	allow if no graph, but marked on x-axis
			crossing y axis at -10 or ft their cubic in (i)	B1 **[3]**	or $x = 0$, $y = -10$ or ft in this part if consistent with graph drawn;	allow if no graph, but eg B0 for graph nowhere near their indicated -10 or ft
11	(iii)		(0, -18); accept -18 or ft their constant $-$ 8	1 **[1]**	or ft their intn on y-axis $-$ 8	
11	(iv)		roots at 2.5, 1, 8	M1	or attempt to substitute $(x - 3)$ in $(2x + 1)(x + 2)(x - 5)$ or in $(x + 1/2)(x + 2)(x - 5)$ or in their unfactorised form of f(x)– attempt need not be simplified	
			$(2x - 5)(x - 1)(x - 8)$	A1	accept $2(x - 2.5)$ oe instead of $(2x - 5)$	M0 for use of $(x + 3)$ or roots -3.5, -5, 2 but then allow SC1 for $(2x + 7)(x + 5)(x - 2)$
			(0, -40); accept -40	B2 **[4]**	M1 for $-5 \times -1 \times -8$ or ft or for f(-3) attempted or g(0) attempted or for their answer ft from their factorised form	eg M1 for (0, -70) or -70 after $(2x + 7)(x + 5)(x - 2)$ after M0, allow SC1 for f(3) = -70

Question			Answer	Marks	Guidance	
12	(i)		(–1, 6) (0,1) (1,–2) (2,–3) (3,–2) (4, 1) (5,6) seen plotted	B2	or for a curve within 2 mm of these points; B1 for 3 correct plots or for at least 3 of the pairs of values seen eg in table	use overlay; scroll down to spare copy of graph to see if used [or click 'fit height' also allow B1 for ($2 \pm \sqrt{3}$, 0) and (2, –3) seen or plotted and curve not through other correct points
			smooth curve through all 7 points	B1 dep	dep on correct points; tolerance 2 mm;	condone some feathering/ doubling (deleted work still may show in scans); curve should not be flat-bottomed or go to a point at min. or curve back in at top;
			(0.3 to 0.5, –0.3 to –0.5) and (2.5 to 2.7, –2.5 to –2.7) and (4, 1)	B2 **[5]**	may be given in form $x = ..., y = ...$ B1 for two intersections correct or for all the x values given correctly	
12	(ii)		$\frac{1}{x-3} = x^2 - 4x + 1$	M1		
			$1 = (x - 3)(x^2 - 4x + 1)$	M1	condone omission of brackets only if used correctly afterwards, with at most one error;	condone omission of '=1' for this M1 only if it reappears allow for terms expanded correctly with at most one error
			at least one further correct interim step with '=1' or '=0' ,as appropriate, leading to given answer, which must be stated correctly	A1 **[3]**	there may also be a previous step of expansion of terms without an equation, eg in grid if M0, allow SC1 for correct division of given cubic by quadratic to gain $(x - 3)$ with remainder –1, or vice-versa	NB mark method not answer - given answer is $x^3 - 7x^2 + 13x - 4 = 0$

Question			Answer	Marks	Guidance	
12	(iii)		quadratic factor is $x^2 - 3x + 1$	B2	found by division or inspection; allow M1 for division by $x - 4$ as far as $x^3 - 4x^2$ in the working, or for inspection with two terms correct	
			substitution into quadratic formula or for completing the square used as far as $\left(x - \frac{3}{2}\right)^2 = \frac{5}{4}$	M1	condone one error	no ft from a wrong 'factor';
			$\frac{3 \pm \sqrt{5}}{2}$ oe	A2	A1 if one error in final numerical expression, but only if roots are real	isw factors
				[5]		

Appendix: alternative methods for 10(iii) [details of equations etc are in main scheme]
for a mixture of methods, look for the method which gives most benefit to candidate, but take care not to award the second M1 twice
the final A1 is not earned if there is wrong work leading to the required statements
ignore wrong working which has not been used for the required statements
for full marks to be earned in this part, there must be enough to show both the required statements

find midpt E of AC	B1	find midpt E of AC	B1	find midpt E of AC	B1	find midpt E of AC	B1
find eqn BD	M1	find eqn BD	M1	find eqn BD	M1	use gradients or vectors to show E is on BD eg grad BE = $\frac{2-1}{2--1} = \frac{1}{3}$ and grad ED = $\frac{5-2}{11-2} = \frac{1}{3}$ [condone poor vector notation]	M2
show E on BD	M1	show E on BD	M1	show E on BD	M1		
find midpt F of BD	B1	find midpt F of BD	B1	show $BE^2 = 10$ and $DE^2 = 90$ oe	B1	find midpt F of BD	B1
state so not E	A1	find eqn of AC and correctly show F not on AC (the correct eqn for AC earns the second M1 as per the main scheme, if not already earned)	A1	showing $BE^2 = 10$ and $DE^2 = 90$ oe earns this A mark as well as the B1 if there are no errors elsewhere	A1	state so not E or show F not on AC	A1
	[5]						**5]**

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