RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4754A:** Applications of Advanced Mathematics: Paper A

Mark Scheme for January 2012

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he / she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1			$\frac{x+1}{x^2(2x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1} = \frac{Ax(2x-1)+B(2x-1)+Cx^2}{x^2(2x-1)}$	B1	correct partial fractions
			$\Rightarrow \quad x+1 = Ax(2x-1)+B(2x-1)+Cx^2$	M1	Using a correct method to find a coefficient (equating numerators and substituting oe or using cover-up) Condone omission of brackets only if brackets are implied by subsequent work. Must go as far as finding a coefficient. Not dependent on B1
			$x = 0,\ 1 = -B \Rightarrow B = -1$	A1	$B = -1$ www
			$x = ½\ ,\ 1½ = C/4 \Rightarrow C = 6$	A1	$C = 6$ www
			x^2 coeffs: $0 = 2A + C \Rightarrow A = -3$	A1	$A = -3$ www
			$\Rightarrow \quad \frac{x+1}{x^2(2x-1)} = -\frac{3}{x} - \frac{1}{x^2} + \frac{6}{2x-1}$		isw for incorrect assembly of partial fractions following correct A,B,C
					SC $\frac{A}{x^2} + \frac{B}{2x-1}$ can get 2/5 max from B0 M1 A1 (for B=6) SC $\frac{Ax+B}{x^2} + \frac{C}{2x-1}$ can get B1 M1 A1 (C=6) and can continue for full marks if the first fraction is then split. SC $\frac{A}{x} + \frac{B}{x^2} + \frac{C+Dx}{2x-1}$ can get B1 M1 A1 A1 A1 (C=6, D=0)
				[5]	

Question			Answer	Marks	Guidance
2			$\cot 2\theta = 3$ $\Rightarrow \tan 2\theta = 1/3$ $\Rightarrow 2\theta = 18.43°$ $\theta = 9.22°$ $2\theta = 198.43°$ $\theta = 99.22°$ or $(2\tan\theta)/(1-\tan^2\theta) = 1/3$ $\Rightarrow 6\tan\theta = 1-\tan^2\theta$ $\Rightarrow \tan^2\theta + 6\tan\theta - 1 = 0$ $\Rightarrow \tan\theta = [-6 \pm \sqrt{(36+4)}]/2 = 0.1623$ or -6.1623 $\Rightarrow \theta = 9.22°, 99.22°$	M1 A1 M1 A1 M1 M1 A1 A1 **[4]**	tan=1/ cot **used** soi for first correct solution (9.22 or better eg 9.217) for method for second solution for θ. for second correct solution and no others in range (99.22 or better) or SC ft A1 for 90 +their first solution use of correct double angle formula rearranged to a quadratic = 0 and attempt to solve by formula oe first correct solution second correct solution and no others in the range (9.22, 99.22 or better) or SC ft A1 for 90 + their first solution -1 MR if radians used (0.16, 1.73 or better)
3			$3\sin x + 2\cos x = R\sin(x+\alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$ $\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 2$ $\Rightarrow R^2 = 3^2 + 2^2 = 13,\ R = \sqrt{13}$ $\tan\alpha = 2/3$, $\alpha = 0.588$ $\Rightarrow 3\sin x + 2\cos x = \sqrt{13}\sin(x+0.588)$ maximum when $x + 0.588 = \pi/2$ $\Rightarrow x = \pi/2 - 0.588 = 0.98$ rads $\Rightarrow y = \sqrt{13} = 3.61$ So coords of max point are (0.98, 3.61)	M1 B1 M1 A1 M1 A1 B1 **[7]**	Correct pairs. Condone omission of R if used correctly. Condone sign error. or 3.6 or better, not $\pm\sqrt{13}$ unless $+\sqrt{13}$ chosen ft from first M1 0.588 or better (accept 0.59), with no errors seen in method for angle (allow 33.7° or better) any valid method eg differentiating 0.98 only. Do not accept degrees or multiples of π. condone $\sqrt{13}$, ft their R if ,say $=\sqrt{14}$

Question			Answer	Marks	Guidance
4	(i)		1, 0.6186, 0 $A \approx (\pi/16)\{1 + 0 + 2(0.9612 + 0.8409 + 0.6186)\}$ $= 1.147$ (3 dp)	B1 M1 A1 **[3]**	4dp (or more) ft their table. Need to see trapezium rule. cao
4	(ii)		The estimate will increase, because the trapezia will be below but closer to the curve, reducing the error.	B1 **[1]**	o.e., or an illustration using the curve full answer required
5			$\overrightarrow{AB} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} -2 \\ -4 \\ 0 \end{pmatrix}$	M1	scalar product with any two directions in the plane (BC= $\begin{pmatrix} -1 \\ -6 \\ -1 \end{pmatrix}$)
			$\mathbf{n}.\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} . \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = 2\times(-1) + (-1)\times 2 + 4\times 1 = 0$	B1	evaluation needed
			$\mathbf{n}.\overrightarrow{AC} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} . \begin{pmatrix} -2 \\ -4 \\ 0 \end{pmatrix} = 2\times(-2) + (-1)\times(-4) + 4\times 0 = 0$ $\Rightarrow$ **n** is perpendicular to plane.	B1	evaluation needed thus finding the scalar product with only one direction vector is M0 B1 B0. No marks for scalar product with position vectors. or SC finding direction of normal vector by using vector cross product, M1A1 eg $4i-2j+8k$ and showing this is a multiple of $2i-j+4k$, A1
			Equation of plane is **r**.**n** = **a**.**n** $\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} . \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$	M1	For any complete method leading to the cartesian equation of the plane eg from vector form and eliminating parameters (there are many possibilities eg $r = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -4 \\ 0 \end{pmatrix}$ $x=2-\mu-2\lambda, y=2\mu-4\lambda,$ $z=1+\mu$, $2x-y = 4-4\mu = 4-4(z-1) = 8-4z$, $2x-y+4z=8$ gets M1 once the parameters have been eliminated.
			$\Rightarrow \quad 2x - y + 4z = 8$	A1 **[5]**	oe **<u>SC1</u>** If they say the plane is of the form $2x-y+4z=c$ and then show all points satisfy $2x-y+4z=8$ they can have M1 A1 for the first point and B2 for both the others. **<u>SC2</u>** If they omit verification and find equation from vector form without using normal as above and then state $2i-j+4k$ is perpendicular they can get M1A1B2

Question			Answer	Marks	Guidance
6			$(1 + qx)^p = 1 + pqx + ½\, p(p-1)q^2x^2 + \ldots$	B1 B1	$(1)\ldots + pqx$ $\ldots + ½\, p(p-1)q^2x^2$
			$\Rightarrow\ pq = -1,\ q = -1/p$ $½\, p(p-1)q^2 = 2$ $\Rightarrow\ p(p-1)/2p^2 = (p-1)/2p = 2$	M1	eliminating q (or p) from simultaneous equations involving both variables oe $\frac{1}{2}\left(\frac{-1}{q}\right)\left(\frac{-1}{q}-1\right)q^2 = 2$, $-1(-1-q)=4$, $q=3$
			$\Rightarrow\ p - 1 = 4p,\ p = -1/3$	A1	$p = -1/3$ www (or $q = 3$)
			$\Rightarrow\ q = 3$	A1ft	$q = 3$ (or $p = -1/3$) for second value, ft their p or q eg -1/the other , provided only a single computational error in the method and correct initial equations
			Valid for $-1 < 3x < 1 \Rightarrow -1/3 < x < 1/3$	B1 **[6]**	or $\lvert x\rvert < 1/3$ www, allow $-1/3<\lvert x\rvert<1/3$ but not say, $x<1/3$ (actually $-1/3<x\le 1/3$ is correct)
7			$\begin{pmatrix} 4+3\lambda \\ 2 \\ 4+\lambda \end{pmatrix} = \begin{pmatrix} -1-\mu \\ 4+\mu \\ 9+3\mu \end{pmatrix}$		
			$\Rightarrow\ 4 + 3\lambda = -1 - \mu$ (1) $2 = 4 + \mu$ (2) $4 + \lambda = 9 + 3\mu$ (3)	M1	equating components
			(2) $\Rightarrow \mu = -2$	B1	$\mu = -2$
			(1) $\Rightarrow 4 + 3\lambda = -1 + 2 \Rightarrow \lambda = -1$	A1	$\lambda = -1$
			(3) $\Rightarrow 4 + (-1) = 9 + 3 \times (-2)$ so consistent	A1	checking third component
			Point of intersection is (1, 2, 3)	A1 **[5]**	dependent on all previous marks being obtained
8	(i)		$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4}{4t} = \frac{1}{t}$	M1 A1	their $dy/dt\,/dx/dt$ accept $4/4t$ here
			But gradient of tangent = tan θ * $\Rightarrow$ tan $\theta = 1/t$	A1 **[3]**	**ag** -need reference to gradient is tan θ

Question			Answer	Marks	Guidance
8	(ii)		Gradient of QP = $\frac{4t}{2t^2-2}=\frac{2t}{t^2-1}$ $=\frac{2\frac{1}{\tan\theta}}{\frac{1}{\tan^2\theta}-1}$ $=\frac{2\tan\theta}{1-\tan^2\theta}=\tan 2\theta$ $\Rightarrow \tan\phi=\tan 2\theta$ $\Rightarrow \phi=2\theta$ * $\Rightarrow$ Angle QPR = 180 – 2θ $\Rightarrow \angle$TPQ + 180 – 2θ + θ = 180 $\Rightarrow \angle$TPQ = θ *	M1 A1 M1 A1 A1 M1 M1 A1 **[8]**	correct method for subtracting co-ordinates correct (does not need to be cancelled) either substituting $t=1/\tan\theta$ in above expression or substituting $\tan\theta=1/t$ in double angle formula for $\tan 2\theta$. ($\tan 2\theta=2\tan\theta/(1-\tan^2\theta)=2/t/(1-1/t^2)=2t/(t^2-1)$) showing expressions are equal **ag** supplementary angles oe angles on a straight line oe **ag**
8	(iii)		$t = y/4$ $\Rightarrow\quad x = 2\,y^2/16 = y^2/8$ $\Rightarrow\quad y^2 = 8x$* When $t=\sqrt{2}$, $x = 2\times(\sqrt{2})^2 = 4$ So $V=\int_0^4 \pi y^2\,\mathrm{d}x=\int_0^4 8\pi x\,\mathrm{d}x$ $=\left[4\pi x^2\right]_0^4$ $= 64\pi$	M1 A1 B1 M1 A1 B1 A1 **[7]**	eliminating t from parametric equation **ag** for M1 allow no limits or their limits need correct limits but they may appear later for $4\pi x^2$ (ignore incorrect or missing limits) in terms of π only allow SC B1 for omission of π throughout integral but otherwise correct

Question			Answer	Marks	Guidance
9	(i)		$\mathrm{d}V/\mathrm{d}x = \pi(20x - x^2)$	B1	
			$\Rightarrow \quad \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}x}.\frac{\mathrm{d}x}{\mathrm{d}t}$	M1	
			$= \pi x(20-x).\frac{\mathrm{d}x}{\mathrm{d}t} = k(20-x)$	A1	oe
			$\Rightarrow \quad \pi x\frac{\mathrm{d}x}{\mathrm{d}t} = k$ *	A1	**ag**
				[4]	
9	(ii)		$\int \pi x\,\mathrm{d}x = \int k\,\mathrm{d}t$	M1	separate variables and attempt integration of both sides
			$\Rightarrow \quad ½\ \pi x^2 = kt + c$	A1	condone absence of c
			When $t = 0$, $x = 0 \Rightarrow c = 0$	B1	c=0 www
			$\Rightarrow \quad ½\ \pi x^2 = kt$		
			Full when $x = 10$, $t = T$		
			$\Rightarrow \quad 50\pi = kT$	M1	substitute t or T=50 π/k or x=10 and rearranging for the other (dependent on first M1) oe
			$\Rightarrow \quad T = 50\pi/k$ *	A1	**ag,** need to have c=0
				[5]	
9	(iii)		$\mathrm{d}V/\mathrm{d}t = -kx$	B1	correct
			$\Rightarrow \pi x(20-x).\frac{\mathrm{d}x}{\mathrm{d}t} = -kx$	M1	$\mathrm{d}V/\mathrm{d}x.\mathrm{d}x/\mathrm{d}t = \pm kx$ ft
			$\Rightarrow \pi(20-x)\frac{\mathrm{d}x}{\mathrm{d}t} = -k$ *	A1	**ag**
				[3]	

Question			Answer	Marks	Guidance
9	(iv)		$\int \pi(20-x)\,\mathrm{d}x = \int -k\,\mathrm{d}t$	M1	separate variables and intend to integrate both sides
			$\pi(20x - ½x^2) = -kt + c$	B1 A1	LHS (not dependent on M1) RHS ie $-kt + c$ (condone absence of c)
			When $t = 0$, $x = 10$ $\Rightarrow$ $\pi(200 - 50) = c$ $\Rightarrow$ $c = 150\pi$	A1	evaluation of c cao oe (x=10, t=0)
			$\Rightarrow$ $\pi(20x - ½x^2) = 150\pi - kt$ $x = 0$ when $150\pi - kt = 0$	M1	substitute x=0 and rearrange for t -dependent on first M1 and non-zero c ,oe
			$\Rightarrow$ $t = 150\pi/k = 3T$*	A1 **[6]**	**ag**

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

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Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

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Facsimile: 01223 552553