RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

OCR Report to Centres

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Oxford Cambridge and RSA Examinations

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This report on the examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the examination.

OCR will not enter into any discussion or correspondence in connection with this report.

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OCR REPORT TO CENTRES

Overview

General Comments

We continue to be impressed by the care and commitment shown by the assessment of coursework and there are many submissions for which the marks submitted have been accepted with little or no difficulty. The purpose of this report is to highlight the areas where the mark submitted is at variance with the external moderator.

Whilst a number of small adjustments to marks were made, there were a number where the adjustments made were significant (4 or more). If this is a result of a misunderstanding of the expectation for each criterion and there is consistency in the marking with internal moderation, then the order of merit will be sound and adjustments can be made quite easily. When the marking is inconsistent and there is no evidence of internal moderation then the order of merit is jeopardised and external moderation becomes impossible. In these circumstances the only way forward is to ask the centre to remark the work. This is inconvenient for both the external moderator and the centre assessor. Centres are urged to ensure consistent marking by individual assessors and that, where there is more than one assessor, a robust internal moderation process is set up.

We would like to make a few general comments applicable to all three units.

It is a requirement that assessors fill in and sign the form CCS160 to authenticate the work. This form can accompany any mark sheets that are sent in advance of the work or with the work of candidates. Failure to produce the form results in a lot of extra work trying to locate it.

There was an increase this series in incorrect work being ticked. While it is not expected that assessors will check every piece of arithmetic there are some pointers that should alert assessors to check. If work is not checked then we ask assessors not to tick it.

Centres are reminded that providing a writing frame for a task is regarded as additional assistance that must be recorded, and the appropriate marks deducted. Not to do so could be regarded as malpractice. In a tiny number of cases moderators raised with the Principal Moderator whether centres should be reported to OCR for suspected malpractice. (See 4.8.3 on page 30 of the specification.)

The oral communication marks have two purposes. It is to enable candidates to discuss their work as a separate skill from writing and it also serves to enable the assessor to authenticate the work as that of the candidate.

Assessors should note two points:

- It is a requirement that a short report is given,
- It is not permissible to award marks in any of the other domains as a result of something that was said by the candidate during the interview or discussion. All work in the first 5 domains must be available for external moderation and therefore must be written within the script.

The comments that follow for each unit indicate points where the demands of criteria are not fulfilled but credit is given. We feel that if assessors were aware of the standards required they would not award marks where they are not justified. Consequently we would urge centres to ensure that all assessors have access to these reports or gain specific training. All the points made have been made before; assessors are therefore either ignoring the advice offered or are not given access to this report.

4751 Introduction to Advanced Mathematics

General Comments

Candidates found the paper generally accessible, though quite a number struggled with questions 3, 10(iii) and 12(iii). Where candidates felt familiar with topics there were often good answers to questions; however candidates in general were weak on those that looked a bit different, for instance question 3.

Some ran out of space for 10(iii) and needed to use additional paper, especially if they rearranged their equations to make *y* the subject, necessitating the use of fractions.

Some candidates did not respond to all of question 12, but examiners felt that this was usually because they did not know how to proceed, rather than the problem being an issue of time.

Comments on Individual Questions

Section A

1 The first part was very well answered on the whole, with the majority scoring full marks. Most inverted first and attempted to square second.

In part (ii) again a high proportion of correct answers was seen. Among the common errors were responses from candidates who either thought that $81^{\frac{1}{4}} = \sqrt{3}$ or that they needed to find $\left(\sqrt[3]{81}\right)^4$. Regrettably, the error $3^3 = 9$ was not rare.

2 Whereas the numerical work with indices is good, as evidenced in the high number of correct answers in question 1, the algebraic work is definitely weaker – as was seen in this question. There were still a pleasing number of correct solutions, but quite a few dropped a mark or two here – often for not cubing the 4 in the numerator - and/or for having x^{10} in the denominator.

3 Many candidates did not know where to start. Having picked up on the keyword 'circle' many just wrote down the general equation of a circle and nothing else, or offered no response at all. For some candidates, lack of real understanding of algebra meant that when confronted with a different style of question they were unable to find an appropriate strategy. Some students did not remember the required circle formulae, eg $A = 2\pi r^2$ was not uncommon. Those starting with the given form $Cd = kA$ and putting in the correct formulae were often most successful. The squaring of $\frac{d}{2}$ was often the downfall, many getting $\frac{d^2}{2}$, leading to $k = 2$. Many had several attempts at this question and solutions were often scrappily presented and difficult to follow.

4 A few made basic mistakes in factorising and finding the end-points. Those who sketched the graph of the quadratic usually reached the correct inequality. Some used the quadratic formula, which often led to unsimplified end points. Those who did not sketch often made an error such as '$(5x - 2) \le 0$ or $(x - 6) \le 0$' as their next step after factorising. Unusually, some candidates offered final answers such as $-0.4 \leqslant 0 \leqslant 6$.

5 Most candidates were able to make a start and substitute 2 and −3 into f(x), although not all used the information given to write the results as equations. Errors in handling $(-3)^2$ or the 35 were common. Having obtained equations, many did not then go on to use standard methods to solve the simultaneous equations, or made errors in doing so. This meant that the full 4 marks available were given less often than examiners had hoped, although many picked up 2 or 3 marks.

6 A large proportion of candidates did not understand what was meant by 'a term which is constant'. A good number still found the term $20\ 2x^{3}\left(\frac{5}{x}\right)^{3}$ but did not recognise it as the term needed to find the constant. Even those who did know what was meant by a constant term usually wrote out the whole expansion rather than identifying which was the relevant term from the start. Brackets were often missing, leading to incorrect evaluations.

7 Simplifying and adding the surds was done correctly by a high proportion of candidates.

Most candidates knew how to rationalise a denominator for the second part but mistakes in implementation were common, the denominator being more frequently correct than the numerator.

8 A good number were successful in the rearrangement, but some very poor work was also seen, revealing fundamental misconceptions about algebraic manipulation. Common errors included dividing some terms by a but not others, and confusion of division and subtraction.

9 Most candidates obtained full marks for sketching the cubic curve, although their cubics were often unshapely, partly due to the incorrect assumption by many that there was a turning point where the graph crossed the y-axis. Most had the cubic the correct way up and realised that it touched the x-axis at −2. A few labelled the y-intersection as 12 rather than −12. A minority sketched parabolas.

Full marks were less common in the second part; a small proportion translated to the right rather than to the left as f(x + 3) required. A larger minority did not know what to do and obtained no marks, often giving the single root $x = -3$.

Section B

10(i) This part was usually done well. Most candidates were confident finding the gradient of AB, although a few failed to show their working. Almost all were then able to find the perpendicular gradient. A minority were unaware that the perpendicular bisector would pass through the midpoint of AB. Most who realised this were able to calculate the midpoint accurately. Once all the information was combined into a straight line equation, a significant minority struggled to rearrange the equation correctly because the arithmetic involved fractions. Pleasingly almost all the candidates managed to work towards the given equation, rather than trying to use the given equation to get back to a common form with their answer. Some wasted time finding the equation of AB first.

(ii) Some wasted time finding the equation of CD, which was given. Many solved the simultaneous equations correctly, but sometimes using less efficient methods, giving themselves complicated fractions to work with. A few who eliminated x struggled with simplifying y = 115/23. A significant minority used the implication in part (iii) that E was the midpoint of CD to obtain a solution, gaining no marks for this.

(iii) Most knew the form for the equation of the circle, although some used r or $\sqrt{r}$ instead of r^2. Some used C or D or the length of CD to calculate the radius, instead of using A or B. Others assumed that AB was a diameter. Very few produced enough to show that CD is a diameter, with many thinking that showing that CD is twice the radius was enough. Some stated that E was the midpoint of CD without any working to support it. This meant that the full 5 marks on this question were rarely awarded, though a significant number obtained 4 marks.

11 Overall, this question was well done, with the vast majority of candidates being able to achieve some measure of success.

(i) The majority are quite confident in the technique of completing the square, although some struggled with the arithmetic since fractions were involved. Some candidates did not complete the question and omitted to state the coordinates of the turning point; some others made sign errors such as (–2.5, 0.25) after a correct completion of the square.

(ii) Apart from the occasional upside down parabola and the odd cubic, most candidates made a good attempt at drawing a sketch of the curve, showing the relevant information about the intersections with the axes. They found the required factorisation straightforward, though a few candidates did resort to using the formula and in several of these cases they failed to recognise that $\sqrt{0.25}$ is equal to 0.5. The quality of the curve was often poor, probably because candidates marked the intersections on the axis first and then tried to draw the curve through them, but it was usually good enough to earn the mark.

(iii) A good number found $x = 2$ correctly. Some candidates chose to eliminate x rather than y and more often than not went wrong. Many candidates realised that a repeated root meant that the line was a tangent to the curve, but quite a few clearly did not, with some omitting the final step of showing that the line was a tangent to the curve. A small number of candidates justified the tangent by using calculus in order to determine the slope of the line and the curve at their point of intersection.

12(i) A large number of candidates successfully used the factor theorem to score the first mark and many went on to find the correct cubic factor - the majority of these choosing to do long division rather than use the inspection method. Some did not use the factor theorem but still showed that $x = 1$ was a root by successful division with no remainder. Those who used inspection without first applying the factor theorem did not in general show enough working for a convincing argument that there was no remainder and therefore that $x = 1$ was a root. A small number did not appear to understand what was meant by 'express f(x) as the product of a linear factor and a quadratic factor' - some of these gained partial credit for the correct division seen in parts (ii) or (iii).

(ii) Many used the correct method but made careless errors in calculations especially when trying negative values of x. Very few realised that they could use the factor theorem on the cubic they had found to obtain another root. Many confused 'root' with 'factor' and lost a mark.

(iii) Only about a third of the candidates found the correct quadratic factor. Those who found the quadratic usually gave sensible arguments based on the discriminant to show that only two real roots existed for the quartic. Some tried to use $b^2 - 4ac$ on the cubic $x^3 + x + 10$. Several candidates went back to square one and attempted to factorise the quartic rather than linking the earlier parts to the problem. Some candidates who had not progressed far in the first two parts sometimes made no attempt at this part.

4752 Concepts for Advanced Mathematics

General Comments

The majority of candidates seemed well prepared for this paper and some excellent work was seen. Solutions were often concise and clearly set out, although in some cases candidates wasted time by adopting convoluted methods: the number of marks available is usually a reliable pointer to the amount of work expected. Nevertheless, some candidates were defeated by routine work: solving the quadratic equation in question 9 (ii) proving to be beyond a surprising number.

Comments on Individual Questions

1 The majority of candidates scored full marks on this question. However, a significant minority omitted "+ *c*" and lost an easy mark. Similarly, some candidates failed to simplify $\frac{30}{5/2}$ correctly or didn't try to, and thus lost a mark. Occasionally $\frac{30^{5/2}}{5/2}$ was seen, which of course scored 0.

2 A considerable number of candidates ignored the request to state a reason, and therefore failed to score. Some simply wrote out the first few terms of each sequence and others made comments which were too vague to be credited, such as "decreasing, so converging". A few lost the mark in one or more parts because there was no statement of "convergent" or "neither" – even if a correct reason had been identified.

3(i) The majority obtained both marks. The usual errors were present: (20, −2), (4, −10) and (0.8, −0.4) being the most common, but (9, −2) and (0, −2) were also seen occasionally.

(ii) Surprisingly few candidates used the word "translate", and opted for their own terminology such as "move to the right" or "shift to the right". Many candidates identified 90 to the right or gave the appropriate vector form. A few gave ambiguous answers or gave the answer "90 to the left".

4(i) Almost all candidates achieved full marks on this question. Some converted to degrees and rounded prematurely, thus losing the accuracy mark for the final answer, and a few used the formula for the area of a sector.

(ii) This straightforward question defeated a surprisingly large number of candidates. Many of these misunderstood the question and used the Cosine Rule to calculate the length AB, or simply answered their own question and calculated the area of the sector or the segment. Many of the successful candidates used convoluted methods, such as finding AB and then using Pythagoras – premature rounding sometimes caused a mark to be lost; forgetting to halve AB cost both marks. The Sine Rule was sometimes used successfully – but this was sometimes spoiled by the use of $\sin\pi$ in conjunction with 3.5. A few candidates found the area of the triangle and then used ½ base × height. Surprisingly few were able to use the expected approach: $d = 3.5\cos 0.6$.

5 This was done very well. Some candidates lost the second mark through premature rounding or simply giving the answer as 0.6. Only a few calculated the reciprocal of the gradient (which didn't score) and nearly all gave an appropriate value for x_c. A few candidates differentiated and substituted values in the derivative.

6 Most candidates differentiated correctly and identified the correct values of x. The final mark was often lost, either due to a misunderstanding of what had been found – answer given as $-4 < x < 1$ or poor notation – answer given as $-4 > x > 1$. Those who used a graphical approach with the derivative generally scored full marks. A few candidates missed the last term out, converted the first plus sign to a minus sign or failed to multiply 2 by 3 correctly, and lost the first mark.

7 Nearly all candidates adopted the expected approach successfully and achieved full marks. A few rounded the angle prematurely and lost the final mark. Some lost the last two marks by using "cos" instead of "sin" in the area formula, and similarly a very few candidates used "sin" instead of "cos" in the Cosine Rule. Most candidates went on to use the correct sides with the angle that had been found. After using the Cosine Rule successfully a few candidates opted for ½ base × height and about half of these did so successfully. A tiny minority of candidates used Hero's formula successfully. Only a small number treated the triangle as right angled and failed to score.

8(i) This was tackled successfully by most. Most sketches were correct in both quadrants, and (0, 1) was often identified. A small number of candidates only sketched the curve in the first quadrant.

(ii) This was very well done. A correct initial step of $\log_3 500\,000$ or $^{\log 500\,000}/_{\log 3}$ was almost always present. The most common error was to then subtract 1 from each side. Occasionally only 1 term was divided by 5, and again some candidates rounded prematurely and lost the final mark.

9(i) Many candidates answered this question well, although there were a number of attempted fudges using $\tan\theta = {}^{\cos\theta}/_{\sin\theta}$. Some adopted a scattergun approach and it was not always possible to follow their method.

(ii) This defeated a significant minority of candidates. However, many obtained the correct quadratic equation. Most then went on to attempt factorisation, going wrong and failing to score. A minority successfully completed the square or used the formula. Many of these went on to score full marks, but some candidates missed the last mark because they presented extra values in the range, or because they didn't realise that further work was needed after obtaining the roots of the quadratic.

10(i) This was done extremely well, with the majority of even the weakest candidates scoring full marks. A few wrote $2x - 4 = 0$ to incorrectly obtain $m = 2$ and made no further progress, and a very small minority tried to answer the question without using calculus and working backwards.

(ii) Nearly all candidates identified the coordinates of B correctly. However, most – as if by rote – subtracted the equation of the line from the equation of the curve and then integrated. Some candidates integrated the equation of the curve correctly, but used the wrong limits (usually 3 to 16) and made no further progress, and of those that did adopt the correct approach, a large number were unable to find the area of the triangle correctly (½ × 12 × 4 was common).

11(i)(A) Most candidates formed the correct equations and went on to solve them successfully.

(i)(B) Many achieved full marks here. Of those who didn't, most candidates scored two marks for $S_{50} - S_{20}$ with their A and D. A few used S_{21} and just scored 1. Other candidates earned the first mark for u_{21} and about half then earned the second mark for a correct formula with $n = 30$. Fortunately hardly any candidates tried to sum all 30 terms individually.

(ii) Most earned the first mark, but then there was much toil for the second mark, which was often not earned due to wrong working or to leaving too much to the marker's imagination. Faced with solving the given statement, most opted for multiplying by $r^2 - 1$ and were then stumped by the quartic. Careless work led to $r^2 = 10$ or 11. A good number of candidates who successfully found *r* neglected to find *a*. A small number of candidates produced elegant work for full marks.

12(i) The correct equation was often seen, but in many cases it stemmed from wrong working and didn't score. Some candidates stopped at $\log p = \log a + kt\log 10$. $\log p = \log a \times kt$ was a common error; occasionally $\log p = \log a + k\log t$ or $\log p = \log a + \log kt$ were seen.

(ii) This was done very well indeed, with just a few candidates making slips with the plots (usually the middle point), and a few joining each point with a ruler or drawing a curve of best fit to lose the last mark. Only a few candidates lost an easy mark by drawing their line of best fit freehand.

(iii) Most were able to obtain values for the gradient and the *y*-intercept within the acceptable range, but not all knew what to do with these. For example, log 1.66 or $10^{1.66}$ were often seen in the equation for $\log p$. A surprising number of candidates neglected to include an equation for $\log p$ at all, and went straight to an equation for *p*. This was sometimes correct, even if the equation for $\log p$ was incorrect. However, a common error was (for example) $p = 45.7 + 10^{0.012t}$.

(iv) Although many candidates correctly identified the value of $\log a$ as crucial in their response, many of them neglected to include the word "million" and lost an easy mark.

(v) Most candidates had the sense to revert to their graph. Accurate plotting and a good line of best fit often rewarded them with full marks. However, most candidates used their answer to part (iii) and sometimes lost the final mark due to rounding. A few used 200 000 000 instead of 200 in one of their equations and failed to score.

4753 Core C3

Over the years we have seen an increased use of software packages. This is to be encouraged, but it results in an increasing responsibility on the candidate to demonstrate an understanding of the method. For instance, it is possible to achieve the correct accuracy for the roots of an equation in domain 2 including a graph and production of tangents using some software packages. Such use does no more than demonstrate an understanding of how to use the package. It certainly does not indicate an understanding of the method. Candidates need to demonstrate an understanding by using the method, finding the derivative, choosing an initial value that is known to be “close to the root” (usually one of the end points of the integer interval within which the root lies) and showing how the iterates are produced.

Sometimes marks were given for hypothetical cases where the illustration does not match the arithmetical work. Other cases include comments such as “the method would fail if I tried on a graph like this” with no equation being given and no iterates shown.

Assessors are also reminded that the graph of an equation is not in itself an illustration of the method.

Domain 1

Occasionally we have observed that the root was quoted to an accuracy not justified by the working in the tables. It is expected that candidates will find, for instance f(1.234) > 0 and f(1.235) < 0 leading to the root being given as 1.2345 ±0.0005. It is not correct to give, without further working, the root as either end of the interval. Please note also that a value for the root is expected; giving the interval [1.234, 1.235] is not a value for the root.

Domain 2

Roots are not always given to 5 significant figures, and the error bounds are often not justified by change of sign.

The illustration should include two clearly drawn tangents. Often the point at which the line is a tangent is not shown or the second tangent is not obvious – both of these can be resolved by a change in scale on the axes.

Domain 3

The most significant problem in this domain was a justification for convergence. It is acceptable (though not a requirement) for candidates to differentiate their function and to find its value close to the root (the initial value should not be used) and compare with the criterion for convergence. Alternatively a geometric argument may be employed, commenting on the gradient in comparison with that of the line $y = x$ (which has a gradient of 1) at the point of intersection. A significant number of candidates do not complete either method completely but are given credit.

Domain 4

The first criterion is clear – one of the equations should be used to find the same root that has already been found in one of the domains by the other two methods. We have seen credit given where a different root has been found or the same root to a different level of accuracy. Additionally, in order to compare speed of convergence it is necessary to start the iterative methods at the same initial value. Failure to do so renders any discussion of speed meaningless.

Candidates do not often state and discuss the value of the software they have used but are given credit. If they have used an Excel spreadsheet, for instance, then they should say so and describe how it has made the task easier.

Domain 5

The purpose of the task is to investigate the solution of equations. A significant number of candidates do not write any equations! We see many times a comment of the cover sheet saying "fine" and then turn to the first domain and see the candidate writing "I am going to solve the equation $x^3 + 3x - 5$." Candidates who persist in writing functions or expressions instead of equations, or refer to finding roots of a function should be penalised in this domain.

4758: Differential Equations

This unit is not entered by many candidates in the winter series so it is difficult to make many generalisations. However, one point worth making is that, where it is appropriate, a force diagram should be included when deriving the model. Its omission should affect the marking in Domain 1.

Whilst most students investigating 'Aeroplane Landing' did so correctly, scripts are still being received where the initial model does not take into account the braking force. That is, a decision is made to reject the initial model based on only the first 9 seconds. This should be penalised in Domains 2 and 3.

Finally, when modifying the initial model a justification must be given for the revised model. In particular, in many cases, there is a tendency to simply state that eg resistance is proportional to v^2 with no justification as to why that might be a possibility.

4776: Numerical Methods

The vast majority of candidates chose to do a numerical integration and so these comments refer only to this task, though some general points may be transferred to other tasks.

Domain 1

It is expected that candidates will state the investigation to be undertaken in precise mathematical terms. If an area is to be found numerically then a correctly written integral (with limits and "d*x*" included!) is required.

It is acceptable for candidates to assert that the integral they have chosen cannot be solved analytically. They do not have to "try" standard methods known to them which lead nowhere.

There are some integrals which are solvable analytically within the A level specification, but it is possible that candidates will have embarked on this further Mathematics Unit before completing the A level course. Such integrals are therefore acceptable. What is not acceptable is that candidates, having asserted that they cannot do the integral by analytical methods, then proceed to give such a solution. This leads them down a route that prevents them from gaining marks in that there is then a tendency to compare the values they obtain through the numerical process with a "known" value.

Domain 2

For numerical integration there are three standard methods. Any one of the three can be used to find a value to the required minimum accuracy. Therefore, if candidates are going to use more than one such method (or all three) then the reason for doing so (and there are many good reasons) should be stated. It is not necessary to describe how they work but why they are being used. This is a common error in this domain that is frequently rewarded.

Domain 3

For numerical integration it is expected that for a “substantial application”, candidates will work their chosen method(s) to at least 64 strips.

Domain 4

The technology used is usually an Excel spreadsheet; what is important for the second mark is that the formulae inserted into the cells are explained. This means more than just giving a printout showing what they are.

Domain 5

The problem of candidates who know the answer before they undertake the work has been mentioned above. If they then carry out some sort of analysis of differences between their values and the known values then the criteria of this domain have not been satisfied. The expectation is that the analysis of errors should be worked from within the process and conclusions achieved as a result of that analysis. Failure to do so means that the marks in this domain and in the next are not available and so credit should not be given.

Some candidates choose to find an integral on a function that is not well behaved. The most common problems are if $y = \mathrm{f}(x)$ is not defined at an end point or the gradient is infinite at an end point. In these cases the ratio of differences will not converge in the way that theory suggests. Far from avoiding such cases however, a candidate would find such a case rich in opportunities to discuss validity and limitations. It is crucial, therefore, to establish the value to which the ratios converge and the work must be done to find them. Assuming the theoretical value is not credit worthy and may also lead to an incorrect solution.

Domain 6

The criterion for accuracy is not 6 significant figures. This is a minimum to be expected. The solution should be expressed to an accuracy that is justified in the work. Usually this will be more than 6 significant figures and part of the task is to be able to discern the level of accuracy that is valid from the error analysis.

Limitations could, as described above, relate to the value that is used for the ratio of differences to extrapolate to a best solution including the fact that rarely has the value subsequently used been achieved. It should be noted that the number of significant figures used by Excel means that the software will rarely limit the solution.

4753 Methods for Advanced Mathematics (Written Examination)

General Comments

This paper proved to be of an equivalent standard to recent C3 examinations. There were many excellent scripts, with over a quarter of the candidates scoring over 60 marks. There were very few really weak scripts, with less than 10% of candidates scoring fewer than 25 marks. This suggests than most candidates are well prepared for the examination. Few had time problems in completing the paper in the allotted time.

In general, the topics which were answered best on this paper were differentiation techniques, exponential growth and decay, and transformations of functions, Weaker topics were integration by parts and/or substitution, and calculus applied to ex and ln x functions.

The presentation of scripts was generally good. However, in more extended questions, such as 9(iv), the notion of presenting proofs in a coherent and logical manner often proved to be lacking, with candidates casting about and writing statements in random order and fashion. In particular, there is a tendency for candidates to 'argue backwards': for example, by starting from f'(a) = 1/g'(a), and arriving at ea = 1 + 2a! While we generally condone this, we hope that this practice is discouraged in the classroom.

Comments on Individual Questions

1(i) This proved to be a straightforward start to the paper, with the large majority of candidates getting full marks. Of those who did not, the most common errors were in the derivative of sin 2x (getting cos 2x or ½ cos 2x) or e−x (omitting the negative sign).

(ii) This part was somewhat less successful. Quite a few candidates just substituted the given answer into the derivative and claimed that this was zero.

2(i) This relatively simple implicit differentiation was very well done by almost all candidates.

(ii) Most candidates scored two out of three for the point (2, √2), but missed the y = −√2 solution. In a few cases, the denominator was set to zero, giving y = 0.

3 The non-standard nature of the question made this one of the harder section A questions. Some candidates were able to write the answer down while others used an algebraic approach.

4(i) In general, this is a well-known topic which is done successfully. Candidates who managed to deduce that a = 100 using $e^{-kt} \to 0$ as $t \to \infty$ usually gained full marks; those who did not often wasted time trying to solve simultaneous equations using $a - b = 15$ and $30 = a - b\,e^{-k}$.

(ii) There was an easy method mark to be gained from following through their values of *a, b* and *k*. Almost all who got these correct in part (i) scored both marks here, though very occasionally a premature rounding of k produced an insufficiently accurate answer.

5(i) This was almost invariably correctly done. No candidates seemed to be put off by the rather excessive speed of the car. Occasionally, the quotient rule was seen, with errors in differentiating the '25'.

(ii) Again, this was very well answered, provided part (i) was correct. Almost all candidates scored an M1 for the chain rule.

6 Most candidates used integration by substitution, though a significant minority used integration by parts. In general, the former were more successful, with the main difficulty being in expanding $(u – 1)u^{-1/2}$ as $u^{1/2} – u^{-1/2}$. Some proceeded from here using integration by parts, with mixed success. When parts were used, the most common error was in deriving $v = 2(1 + x)1/2$ from $v' = (1 + x)^{-1/2}$.

7(i) This proved to be very straightforward, with nearly everyone quoting the first correct counter-example of 245 (though a few came up with some much larger numbers).

(ii) This was not quite so easy as part (i). Most candidates who got full marks spotted the cyclic pattern in the units digits of 3^n as n increases. However, a significant minority evaluated 3^n for n = 0 to 9 and then cited 'proof by exhaustion'. The second approach, less commonly used, was to use the fact that numbers ending in '5' must be multiples of 5, and 3^n contains no factors of 5. However, many candidates who used this approach were unable to express the argument clearly enough and made incorrect statements.

8(i) We usually insist on the word 'translation' here, but in this case allowed 'move', 'shift', etc. A vector on its own does not in our view imply a translation. Occasionally, candidates clearly knew what the transformations were, but wrote the vectors incorrectly, for example the wrong way up. Nevertheless, this topic is usually well known and done well.

(ii) The quotient rule is generally well known, and errors here usually stemmed from faulty derivatives or poor algebra. Brackets are not optional in an expression like this, and their removal was not always successfully achieved. We also needed evidence of the use of $\cos^2x + \sin^2x = 1$, either by its direct quotation or by factoring out the '2' in the numerator. The evaluation of g'(x) was usually correct. With f'(x), some used a quotient rule on sin x/cos x rather than quoting the derivative of tan $x = \sec^2x$; we also got some occasional 'translation' arguments here which misunderstood the nature of the verification.

(iii) This was a case where giving the transformed integral proved to be of doubtful value, as many candidates 'lost' the negative sign in their $\int -1/u \, du$, and placed the limits the wrong way round. It appears that the idea of swapping limits making the integral negative was not generally understood. The evaluation of the given integral with respect to u was more successfully done, though quite a few candidates approximated their final answer.

(iv) These marks were gained by candidates who managed to spot the rectangle of area added by the translation upwards of the graph of f(x).

9(i) This mark was usually earned.

(ii) Virtually everyone scored M1 for writing down the correct integral and limits, but many candidates made a meal of trying to integrate ½ $(e^x – 1)$, with ¼ $(e^x – 1)2$ not an uncommon wrong answer. Having successfully negotiated this hurdle, using part (i) to derive ½ a was spotted by about 50% of the candidates. Quite a few candidates managed to recover to earn the final 2 marks for ½ $(a^2 – a)$ (without incorrectly simplifying this to ½ a!).

(iii) Finding the inverse function proved to be an easy 3 marks for most candidates – candidates are clearly well practiced in this. The graphs were usually recognisable reflections in $y = x$, but only well drawn examples – without unnecessary maxima or inflections – were awarded the 'A' mark.

(iv) This proved to be more difficult, as intended for the final question in the paper. As with the integral, many candidates struggled to differentiate ½ (e^x – 1) correctly, and equally many omitted the '2' in the numerator of the derivative of ln(1 + 2x). Once these were established correctly the substitution of $x = a$ and establishing of f'(a) = 1/g'(a) was generally done well, though sometimes the arguments using the result in part (i) were either inconclusive or done 'backwards'. The final mark proved to be elusive for most, as we needed the word 'tangent' used here to provide a geometric interpretation of the reciprocal gradients.

4754 Applications of Advanced Mathematics

General Comments

This paper was of a similar standard to previous papers. The questions were accessible to all candidates but there were sufficient questions for the more able candidates to show their skills. Some candidates scored full marks and other high scores were seen. There were very few low scores as is the norm in the January series.

On Paper A questions 6, 8(ii) and 8(iii) scored least well.

The Comprehension was well understood and good marks were scored.

Candidates made similar errors as in previous papers. These included:

- Sign errors such as $-(x+1)= -x+1$ in Question 1
- Failure to include constants of integration in Question 8
- Poor anti-logging and rules of logarithms in Question 8
- Failure to read questions carefully as in Question 7(ii)
- Failure to give clear descriptions as in Comprehension Question 7
- Inappropriate accuracy, for example in Question 4b(i) giving say,7dp answers following working with 4dp values.
- Failure to give exact answers when required as in Questions3(i) and 4(a)
- Failure to give sufficient detail when verifying given results as in Question 8(ii).

Centres should be reminded that as Papers A and B are now marked separately it is essential that any additional sheets should be attached to the appropriate Paper to ensure that it is marked.

Comments on Individual Questions

Paper A

1 Most candidates understood the method for solving the equation involving algebraic fractions. The main errors were sign errors, especially $-(x+1) = -x+1$.

2 The method for finding the binomial expansion was understood by almost all candidates. Many candidates scored full marks here. The most common errors were sign errors, the omission of the validity or the use of $2x$ throughout instead of $(-2x)$.

3(i) This question was successfully answered by most candidates. Some failed to give their answers in exact form.

(ii) Most candidates used $y=\sin2\theta=2\sin\theta\cos\theta$ and many squared this but not all candidates subsequently used $\cos^2\theta=1-\sin^2\theta$ to continue to the required result.

4(a) The method for finding the volume of revolution was usually correct. Many scored full marks in this part. There were a few errors in the integration (commonly either $\int 1\mathrm{d}x = 0$ or $\int e^{2x}\mathrm{d}x = 2e^{2x}$) but the main errors were failing to substitute the lower limit or giving the answer in inexact form.

(b)(i) Most candidates were able to use the trapezium rule, but many candidates gave their answers to an excessive degree of accuracy given that they were using values that were only given to 4dp.

(b)(ii) The explanations here were often excellent although a few were incomplete. Some said the trapezium rule was always an over-estimate whilst others failed to say that it was originally an over-estimate (or equivalent) in this case. Some used diagrams to illustrate their explanations with success.

5 Candidates seemed equally to choose the two approaches in the mark scheme to solve the trigonometric equation. Both were equally successful and few offered extra unnecessary solutions. The main error was to give insufficient accuracy in the final solutions.

Where solving tan θ=2 in degrees leads to θ=63.4º to 3sf,
giving θ= 1.11 radians = 63.598º (63.6º) and θ=1.1radians=63.0º were insufficiently accurate so we needed θ=1.107radians to achieve the same accuracy as 63.4º.

6(i) This question was answered well by the most able but many others could not cope with the fractions in part (i). AC was generally correct but often AD=cosφ/sinθ or sinθ/cosφ was the given answer, whilst others left their answer as a fraction within a fraction.

(ii) Good candidates were able to answer this with ease. Quite a few candidates made no response. Much depended upon their answers in part (i) which were followed through for the method marks. Those who then did not obtain the given answer should have realised that they ought to have reconsidered their answer to part (i).

7(i) The majority of candidates correctly found the length of the ridge of the tent.

Most candidates attempted to find the required angle using scalar products-often using an incorrect vector, particularly the vertical.

(ii) This part was well answered. Those who found two vectors in the plane and showed that they were both perpendicular to **i**–4**j**+5**k** and then proceeded to use x–4y+5z=d to find d and then a usually obtained full marks, although a few gave the equation as

i–4j+5z=16 and lost a mark.

Candidates should however be careful that they read the question carefully. They were asked to show that **i**–4**j**+5**k** was normal to the plane and **then**,… ‘ *find the equation of the plane’*. Some candidates started by trying to find the equation of the plane and did not attempt to show that the vector was normal. Other candidates decided to substitute the coordinates of points in the plane- in many cases without ever writing down the equation of the plane (for example, 1x0–4x–4+5x0=16) or mentioning a normal. These candidates are not showing that the vector is normal unless they provide a clear argument to support their calculations, nor are they finding the equation of the plane if they do not write it down in Cartesian form (or equivalent), 16 three times is not enough on its own.

Most candidates knew how to find a, either by substitution in the plane or by using scalar products, but some careless errors such as 4a=8, a=4 were seen. Some substituted +a instead of –a.

(iii) Those who substituted points were usually successful in verifying that the equation of the plane was $x+z=8$. Some derived the equation. Others showed that **i**+**k** was perpendicular to two vectors in the plane but not all of these then went on to establish that the equation of the plane was $x+z=8$. Most scored the marks for finding the acute angle between the planes.

8(i) Most candidates correctly wrote down the value of h but quite a number failed to give the interpretation that the tree stopped growing when its height was 20m.

(ii) Those who approached the verification by integration were quite successful. The common errors were:-

- omitting the negative sign when integrating $1/(20–h)$
- omitting the constant of integration
- giving $\ln(h–20)$ in their answers (without modulus signs) despite having usually given $h=20$ as a maximum value in (i)
- incorrect anti-logging.

Those who approached from differentiation usually obtained some marks, particularly the mark for checking the initial conditions but many gave insufficient detail when verifying the given result.

(iii) There were a few completely correct solutions to this part. However, many different errors were seen from the majority of candidates. There was also a lot of confused work.

Those who started with the correct partial fractions, from $200/(20–h)(20+h)$ or $1/(20–h)(20+h)$, usually obtained the first three marks and then integrated having scored M1A1A1M1 thus far. Common errors then included omitting the negative sign when integrating $5/(20–h)$ (ie giving $5\ln(20–h)$ and hence A0) or failing to state and then evaluate a constant. Those who had no constant were unable to score further marks. Those who did score the first 5 or 6 marks (dependent upon when the constant was evaluated) often used the laws of logarithms correctly and anti-logged although some fiddled the signs when subsequently making h the subject.

Some candidates thought that $1/(400–h^2)=1/(h–20)(h+20)$. Marks were scored for using partial fractions on $1/(h–20)(h+20)$ but logarithms such as $\ln(h–20)$ for $h<20$ and constants such as $\ln(–1)$ could not obtain accuracy marks although the marks for anti-logging and making h the subject were still available.

There were also a number who felt that $1/(400–h^2)=1/(200–h)(200+h)$.

The use of modulus signs was rarely seen.

(iv) Usually correct.

(v) Most candidates scored all three marks.

Paper B

1 Most candidates scored all three marks.

2 This was generally well answered although some just gave one point. Some got the sloping part right but had vertical, not horizontal, lines. Others had the right horizontals but the sloping line was flatter.

3(i) This was usually correct although some stated t(P,A)=t(P,B)=3 which was wrong.

(ii) t(P,A)=3 was usually given.

4 Many obtained both marks although some left their answer as $n(3,4)+n(4,3)$ without evaluation.

5 This was less successful. Many gave the answer 10 as $n(3,2)$ is 10. Others gave 10+2=12.

6 There were some wrong answers with unclear methods in part (i) although there were also many correct solutions. Part (ii) was usually right.

7 There were many correct solutions using mathematical terms such as the x-axis, y-axis, $x=0, y=0, y=x$ and $y=-x$. Some others gave appropriate descriptions such as all points $(p,0)$ for all real values p or said all points vertically above and below and horizontally from A or equivalent whilst others used the points of the compass in their descriptions. Some only gave a list of points which was insufficient without a general statement to include all points. Some only gave integer values. In several cases one felt that the candidates probably did know the correct answer but were unable to explain it clearly.

4755 Further Concepts for Advanced Mathematics

General Comments

Some of the questions in this paper were unusual, and presented a challenge to which not all managed to rise. Many candidates demonstrated a lack of basic algebraic skills, which marred their progress. There were often different approaches to the questions and candidates sometimes confused themselves by changing between methods, and if not confused, still using up time. There was evidence that some found the paper too long. Many were however successful in showing not only knowledge but also excellent understanding.

Comments on Individual Questions

1 This straightforward question was well answered by most. The transformation A was sometimes described as a rotation while B, more often, was written down as enlargement, even though the different scale factors were correctly described. A few candidates favoured “shear”. In (ii) there were very few multiplications performed in the wrong sequence.

2 This was the first question to reveal poor algebraic skill. Most candidates knew that the complex number fraction had to be multiplied by $(a+b\text{j})/(a+b\text{j})$ but could not accurately carry out the work in the numerator. The term in j was frequently seen as $2b$j and the term in j^2 became $-b$. The candidates who successfully multiplied and defined Re(z/z^*) then in many cases wrote down Im(z/z^*) to include j. Many candidates did not know how to start, and tried to cancel terms within the fraction.

3 A well answered question that was tackled in a variety of ways. Probably the swiftest solution was through the root relationships, but finding the quadratic and linear factors was also a useful route to take. Those candidates that chose a “pick and mix” approach took up time. There were a number of candidates who were confused between a “factor” and a “root”.

4 Part (i) was surprisingly badly done. Trials of different values for x were popular, as well as sweeping generalisations about x^2 being greater than x. Partial explanations were frequently seen. Solutions by completing the square or by elementary calculus were usually well explained. Part (ii) was not well done. Many candidates failed to factorise their cubic expression, prematurely dividing by x or x^2 thereby losing essential roots. Careless algebraic work initially failed to obtain the simple cubic expression and a few candidates became enmeshed in multiplying both sides of the initial inequality by $(x^2-x+2)^2$ which frequently led to errors.

5 Part (i) was well done, most candidates scored full marks but the factor 1/3 was fairly commonly forgotten and sometimes seen misapplied as multiplication by 3. In (ii) very many made the mistake of believing the limit was zero.

6 Most candidates knew what to do and were meticulous in presenting their argument. The factor $(-1)^k$ was successfully dealt with by many although it caused a problem for some. The precise wording needed to round off the argument was in the main well expressed.

7 In part (i) few candidates encountered any difficulty, but it would be good to see coordinates presented, as requested. Some candidates used one or more inclusive inequalities, which lost a mark. Part (ii) was not well answered, apart from a few who not only saw a clear method to follow but also possessed the algebraic precision required and a good understanding of the relationship between the initial analysis and the graph, which they expressed with clarity. The bulk of the work did not need calculus (which second guessed the final answer), although a calculus method was of course given full credit where the argument covered all the detail satisfactorily. The straightforward method of finding a negative discriminant was most often attempted but was bedevilled by careless work with signs and brackets. Substitution of the given values –1/9 and –1 for *k* was chosen as a method by some but then needed a thorough explanation of the nature of the resulting points to earn all the marks.

8 Very few fully correct solutions to this question were seen. In part (i) there were many very poor circles. In an Argand diagram a sketch should show the nature of the object first and any scales shown on the axes need little beyond what might be used to establish the centre and radius. Candidates on the whole placed the object in roughly the right place. A key remains the best way of explaining both region and boundary as there is no accepted convention on how to show this. Part (ii) was not well explained in many cases, $|z|$ was frequently used to refer to the centre of the circle, not to a general point. The diagram was often ignored. In part (iii) P was often placed wrongly and even when roughly correctly placed (quality of diagram permitting) it was given assumed co-ordinates which were incorrect.

9 This question was often very well answered, although there was also evidence that some candidates lacked time by this point. In part (i) the correct *k* was usually found, but some were confused between *k* and 1/*k*. Part (ii) was usually correctly done, and the right value of *k* frequently recovered. Algebraic error was common in part (iii), which could be avoided by writing down complete equations in a row by column calculation. In part (iv) many candidates did not know that $(AB)^{-1}$ was equal to $B^{-1}A^{-1}$ and the subsequent calculation had to show this.

4756 Further Methods for Advanced Mathematics

General Comments

The level of performance in this paper was comparable with previous January series. The distribution of marks showed a strong negative skew, with over one-quarter of candidates scoring 60 marks or more and only about 4% scoring 20 marks or fewer. Question 3 was the best-scoring question, followed in order by Questions 1, 4 and 2. Only one candidate attempted Question 5, which was making its last appearance in this unit. There was no evidence of time trouble, but once again many candidates found it necessary to use supplementary sheets, often because they wished to replace an incorrect answer and had already filled the space. Centres should not issue candidates with graph paper for rough working. Presentation varied between the exemplary and the almost illegible: perhaps there were more badly-presented scripts in this series than before, and it was particularly difficult to follow some of the solutions to Q3(i).

“Standard” questions, such as the integrals in Q1(a), the derivation of the characteristic equation in Q3(i) and finding the eigenvectors in Q3(ii), were confidently and accurately handled by the vast majority of candidates. On the other hand, many candidates struggled with the geometry of complex numbers in Q2(b), where about half of all candidates scored zero in parts (ii) and (iii).

Candidates could have improved their performance if they had

- been more careful with their elementary algebra, errors included writing $-4\ ^{n}$ as -4^{n} in Q3(iii); $y-2\ ^{2}=y^{2}-2y+4$ in Q1(b)(ii) (and similar in Q4(iii)) and $\dfrac{1}{x-2\ ^{2}+4}=\dfrac{1}{x-2\ ^{2}}+\dfrac{1}{4}$ in Q1(a)(iii).
- made better use of the structure of a question, eg the use of arctan in the integral in Q1(a)(ii) is suggested by Q1(a)(i), Q2(b)(i) assists with Q2(b)(ii), and Q4(i) assists with Q4(ii).
- shown sufficient steps or sufficient clarity when establishing a given answer, eg in Q2(a)(ii), Q4(i) and Q4(iii).
- used simple methods, eg in Q4(i), a number of candidates reached $e^{2x}=5$, to which they applied the quadratic formula: $e^{x}\ ^{2}=5$ so $e^{x}=\dfrac{-0\pm\sqrt{0-4\times1\times\ -5}}{2\times1}$ etc.
- appreciated what is meant by “exact form”, eg leaving $\cos\dfrac{2\pi}{3}+\mathrm{j}\sin\dfrac{2\pi}{3}$ as the final answer in Q2(b)(i), and conversely giving the complex numbers as decimals in Q2(b)(ii).
- read the questions more carefully, eg in Q3(iii).

Comments on Individual Questions

1 Calculus with inverse trigonometric functions; polar co-ordinates

Part (b) was generally found more challenging than part (a).

(a)(i) Most candidates found this an attractive start, although poor algebra or a lack of sufficient detail prevented many from scoring full marks. Some candidates mixed up *y* and *x* at an early stage. Some looked up or recalled the derivative of arcsin *x* and just applied the Chain Rule, which attracted no credit.

(ii) The vast majority could complete the square and then most took the hint provided by part (i). The most common error was to evaluate incorrectly at the lower limit, which often produced an answer of $\frac{\pi}{8}$. The argument that $\frac{1}{(x-2)^2+4} = \frac{1}{(x-2)^2} + \frac{1}{4}$ which gave, after integration, $\ln(x-2)^2 + \ln 4$ or similar, was seen more often than expected. There were some elegant and fully correct answers by explicit substitution.

(iii) This integration by parts was handled very confidently. The vast majority produced a line containing $\int \frac{x}{1+x^2}\,dx$ without trouble: the integral of this caused more difficulty, with many forgetting the $\frac{1}{2}$ and others writing $\int \frac{x}{1+x^2}\,dx = x\int \frac{1}{1+x^2}\,dx = x\arctan x$.

(b)(i) Although many candidates produced concise, efficient solutions, many confused *r* (as in the polar equation) with *r* (as in the radius of the circle) and, having obtained $x^2 + y^2 = 4\cos^2\theta$, asserted that this was a circle centre (0, 0), radius $2\cos\theta$. Others appeared to be solving equations in *θ*. The correct centre and radius often appeared independently of a correct cartesian equation.

(ii) The usual approach was to obtain the cartesian equation as $x^2 + (y-2)^2 = 4$, and then multiply out and obtain $x^2 + y^2 = 4y$ and hence the correct polar equation. Unfortunately very many candidates obtained 2*y* rather than 4*y*, or started with a cartesian equation with 2 or even $\sqrt{2}$ on the right hand side.

2 Complex numbers and geometry

This question produced the lowest mean mark by some margin, and a fairly even distribution of marks between 0 and 18.

(a)(i) This was mainly handled very efficiently although the clarity of proofs often left something to be desired, with some candidates making liberal use of "invisible brackets". All three alternatives in the markscheme were seen regularly.

(ii) There were many admirably clear and concise solutions, although not all the required steps were always present to establish the given expression for *C*. Others were determined that *C* + j*S* should be a geometric series, and many even omitted the binomial coefficients to achieve this aim. The link with (a)(i) was not always recognised, especially among those who took the geometric route.

(b)(i) This was often, but not always, correct. Those who wrote $e^{j\frac{2\pi}{3}} = \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}$ and then evaluated the trigonometric functions were much more frequently correct than those who drew diagrams, which often resulted in the minus sign being omitted from the real part.

(ii) Candidates found this part-question challenging. The structure of the question was, as always, intended to assist candidates and those who realised that they could multiply $2+4j$ by the complex number in (b)(i) twice usually achieved the correct exact answers with little difficulty. Many related the situation to the three cube roots of $(2+4j)^3 = -88-16j$, but this rarely led to a correct answer in an acceptable exact form.

A very common approach was to find the modulus and argument of 2 + 4j, rotate through $\pm\frac{2\pi}{3}$ (which often appeared as $\frac{\pi}{3}$) and work with expressions of the form $\sqrt{20}\left(\cos\left(\arctan 2 \pm \frac{2\pi}{3}\right) + \mathrm{j}\sin\left(\arctan 2 \pm \frac{2\pi}{3}\right)\right)$ which rarely produced anything acceptably exact, although a little credit was given for answers such as −4.46 − 0.27j. One or two candidates battled away at this with compound angle formulae and deserve warm praise for perseverance, if not for elegance. There were a couple of successful solutions using matrices.

(iii) Success here really depended on something useful being produced in (b)(ii). Successful candidates used a variety of approaches, such as the cosine rule and the formula for the distance between two points. Many attempted to use the results of right-angle trigonometry in triangles without right angles.

3 Matrices: eigenvalues and eigenvectors

This question provided a good source of marks for almost all candidates, with the majority scoring 17 or 18 out of 18.

(i) This was often fully correct. Most expanded by the first row, although Sarrus’ method was also popular. The algebra employed in obtaining the given result was usually correct and sufficiently clear, although very poor handwriting made some solutions difficult to decode and “invisible brackets” were commonly employed. A worrying assertion, seen frequently, was that $\det \mathbf{M} - \lambda\mathbf{I} = -\lambda^3 + 13\lambda - 12 = \lambda^3 - 13\lambda + 12$.

(ii) Solving the characteristic equation presented few problems, and was usually accomplished by spotting one root (usually $\lambda = 1$) and then obtaining the corresponding quadratic factor, although quite a few started with an expression of the form $\lambda + a \quad \lambda + b \quad \lambda + c$, multiplied out, and worked with equations such as $a + b + c = 0$ and $abc = 12$. This seems inefficient although it did sometimes work. Then the eigenvectors were usually produced without trouble, although $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ was frequently seen and equations such as $y = 0$ and $3x - z = 0$ were held to lead to $\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ or similar as an eigenvector.

(iii) Most identified **P** correctly as a matrix of eigenvectors but many just gave **D** as $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{pmatrix}$, without taking into account the power of *n*. Those who did often defied BIDMAS and gave -4^n as one of the elements.

4 Hyperbolic functions

This question was an effective discriminator, with the full range of marks being awarded in each part.

(i) $\frac{dy}{dx}$ was usually found correctly and a variety of correct explanations were offered as why this could never be zero, the most common probably being that it leads to $e^{2x} = -5$ (having reached this, it was not necessary to consider the discriminant of a quadratic equation) or $\tanh x = \frac{3}{2}$ (although mention of $|x| < 1$ in this context was not credited). Then $y = 0$ was often solved efficiently, by exponentials (perhaps more common) or by obtaining $\tanh x = \frac{2}{3}$ and using the logarithmic form of artanh. Those who tried substituting the given value of x often needed to give much more detail in their verification: merely stating that $3\sinh\left(\frac{1}{2}\ln 5\right) - 2\cosh\left(\frac{1}{2}\ln 5\right) = 0$ attracted no credit. The final part of the question was frequently interpreted as "find the value of $\frac{dy}{dx}$ at $\left(\frac{1}{2}\ln 5, 0\right)$" and much effort was wasted in evaluating $\frac{dy}{dx}$ exactly. Those who just stated that $3\sinh\left(\frac{1}{2}\ln 5\right) - 2\cosh\left(\frac{1}{2}\ln 5\right) = 0$ without linking it with y or attempting an evaluation were not rewarded.

(ii) There were many fully correct curves, although some candidates clearly did not connect their sketch with information they had obtained in (i). Many did not mark (0,−2), had a curve bending the wrong way, had a stationary point (often on the x-axis), or had multiple points of inflection.

(iii) Multiplying out $(3\sinh x - 2\cosh x)^2$ caused a little trouble for some candidates, with $-6\sinh x\cosh x$ being a fairly common middle term. Then this expression had to be expressed in terms of cosh 2x and sinh 2x. The correct answer was seen fairly frequently, but there were many factor and sign errors, particularly involving cosh 2x. Many stopped at this point, but most knew that this expression had to be integrated to produce the required volume of revolution. Some were determined to use $\int \frac{1}{2} r^2 d\theta$, thereby introducing a spurious $\frac{1}{2}$, and/or limits 0 and 2π. The integration itself was often done well but the lower limit of 0 was often neglected, and obtaining the given answer was not always done with the required clarity and transparency.

5 Investigations of curves

Only one candidate attempted this question in its final appearance in this unit.

4758 Differential Equations (Written Examination)

General Comments

Candidates showed a sound understanding of the basic methods of solution of the types of differential equations covered by this specification. The presentation of solutions was usually of a good quality with clear explanations. As in previous series, Questions 1 and 4, on second and higher order differential equations, were chosen by almost all candidates. There was evidence that Question 3 was the next question of preference, but many candidates struggled with the first two parts and then opted for Question 2 instead. Others appeared to answer any parts of Questions 2 and 3 that they could, leaving the examiner to determine which of the two questions counted towards their total.

There were fewer fully correct solutions to questions than in previous series. There were more arithmetical errors than usual, particularly in the first parts of Questions 1 and 4. Candidates should be encouraged to work accurately even with work that is familiar. Although incorrect work is followed through and given credit, it is inevitable that errors early in a solution will lead to some loss of marks. In the case of second order linear differential equations an incorrect auxiliary equation, or the incorrect solution of an auxiliary equation, can lead to a different form of general solution and this may impact on the ease or feasibility of the requests in the later parts of a question.

Comments on Individual Questions

1 Third order linear differential equation

(i) Almost all candidates were able to use the information in the question to make a good attempt at solving the given third order differential equation. The complementary function was usually found accurately, but there were a surprising number of numerical errors when finding the particular integral.

(ii) The majority of candidates were successful in using the initial conditions to find a particular solution; it was pleasing to see that the less familiar form of condition, that *y* was bounded for very large values of *x*, was interpreted correctly.

(iii) Candidates seemed less confident in answering this part. A common error was to assume that the coefficient of either the sine or the cosine term was equal to the amplitude of the motion.

(iv) The majority of candidates were able to use the condition that *y* was bounded for very large negative values of *x* to deduce that two of the unknown coefficients in their particular solution from part (i) had to be zero. Many candidates then went on to apply the other two initial conditions to obtain two different values for the remaining coefficient, indicating that there was no consistent solution. A common error was to apply just one of the initial conditions to find the unknown coefficient and then state that this was a solution.

2 First order differential equations

(i) Almost all candidates applied Newton's second law to establish the given differential equation. They then proceeded to find the general solution either by the integrating factor method or by separating the variables or by finding the complementary function and particular integral. The three methods were equally popular and successful. A significant minority of candidates did not seem to realise that they needed to apply the initial condition, that the ball fell from rest, to find the value of their constant of integration.

(ii) This was almost always correct.

(iii) There were some excellent solutions to this part of the question with very clear accurate work and gaining full marks. However, the majority of candidates struggled to solve the differential equation. Separating the variables leads to an integral of the form $\int \frac{1}{a^2 - x^2}\,\mathrm{d}x$ and evaluating this proved to be a stumbling block for the majority. The most common attempts resulted in inverse tangents or in incorrect logarithmic terms of the form $\ln(a^2 - x^2)$. A denominator that is the difference of two squares is a common occurrence in problems of this type and candidates should be encouraged to be vigilant.

(iv) Euler's method is familiar territory for candidates and the majority of candidates were successful, presenting their working clearly.

(v) This was almost always correct.

3 First order differential equation

A significant number of candidates were not able to complete their attempts at the first two parts of this question. They chose to omit the remainder of the question and attempt another question instead.

(a) Almost all candidates recognised the method of solution as the integrating factor method and made a good start. Most candidates evaluated the integrating factor correctly as cos *x*, but they did not recognise a method of evaluating $\int \sin x \cos x \mathrm{d}x$, the integral that appeared on the right hand side of the differential equation. There are many valid approaches, either by direct integration or by using a double angle formula or using a substitution and it was surprisingly that none of these came to mind for many candidates.

(b) Only a handful of candidates made any meaningful progress with this request and the majority offered very little as an attempt at a solution. A common error was to substitute $y = \mathrm{p}(x)$ into equation (1) only partially, with $\frac{\mathrm{d}y}{\mathrm{d}x}$ retained. A similar procedure for equation (2) led to inevitable confusion between *y*, p and c.

(c)(i) Many candidates missed the obvious approach here, expressed clearly in the instruction to *verify*. All that was necessary was to differentiate the given function and substitute into equation (3). A surprising number of candidates attempted to solve equation (3), not realising that this was not within their scope, and that the structure of the question was to lead them through a method of solution.

(ii) Many candidates had abandoned their attempts at this question by this stage, but to those who proceeded, this was familiar territory.

(iii) This part brought together parts (b) and (c) but only a few candidates grasped the structure of the question and pursued it to the end.

4 Simultaneous linear differential equations

(i) Almost all candidates gained the majority of the marks in this part. The method of approach was understood and any marks lost were due to arithmetic slips in manipulating the functions or in solving the auxiliary equation.

(ii) Again, candidates knew what they had to do. Answers to part (i) were followed through for the majority of the marks.

(iii) The method was applied successfully, although by this stage most candidates had made at least one arithmetical slip. Fully correct expressions for x and y were rarely seen.

(iv) Candidates who had made errors in the previous parts of this question were at a disadvantage because their expression for $x + y$ did not usually tend to a finite limit.

Credit was given for the correct interpretation of the behaviour of the candidate's incorrect expression.

4761 Mechanics 1

General Comments

Most candidates were able to answer many of the questions on this paper well and to demonstrate their knowledge of the specification. However, the final parts of two questions proved to be quite challenging and as a result very high marks were rare.

Comments on Individual Questions

1 This question, about a block moving on a slope, was well answered. Most candidates scored full marks on the force diagram in part (i), an improvement on similar questions in previous papers. In part (ii) a few candidates did not resolve the weight correctly, or in some cases at all, to find the normal reaction. Part (iii) considered forces parallel to the slope and this was well answered.

2 This question was about motion in two dimensions using column vectors. It was well answered. Such marks as were lost were usually as a result of candidates not fully answering the questions, omitting the velocity at time t and the speed in part (i) and the distance travelled in part (ii).

3 The first two parts of this question, about two people pushing a car, were well answered. In part (i) almost all candidates found the car's acceleration correctly and in part (ii) many resolved the forces correctly to find the resistance to the car's motion. However, there were few good answers to part (iii); candidates were expected to comment on the sideways resistance to motion (acting on the car's tyres).

4 This question involved a particle travelling under constant acceleration along a straight line. In part (i) two constant acceleration formulae were required and most candidates obtained the right answers but there were some careless mistakes, and some mistakes in quoting the standard results. In part (ii) candidates were asked to **prove** that the particle was never at a certain point and it was a pleasure to see how well this was answered, usually either by setting up a quadratic equation and showing it had a negative discriminant, or by finding the turning point in the motion. A handful of candidates just tested some particular cases and no credit was given for this.

5 In this question a model was presented for the familiar game of "ducks and drakes", skimming a stone along the surface of some water. The stone's initial velocity was in the horizontal direction and this presented a difficulty for the many candidates who did not infer that the vertical component of the initial velocity was zero; it was common to give it the value of the horizontal component (20 m s-1) instead. Consequently, although there were many fully correct answers to this question, there were also many that were worth few marks, if any.

6 This was the first of the two Section B questions. It involved two models for the speed of a runner covering 100 metres. It was well answered. In part (i) candidates worked from a given speed-time graph and most were successful in doing so; however some did not realise that the second request needed some calculation and could not be obtained just from reading off the graph. The question then presented the second model as an equation for v in terms of t; most candidates realised that the questions on this involved the use of calculus and answered them correctly. The last two parts of the question involved comparing results from the two models and there were many correct answers, well presented with clear statements as to which model was being considered.

7 The final question was about different ways of suspending a block from two fixed points on a beam using a string of a given length. In the first three parts it was done using a smooth pulley. Initially candidates were expected to use mechanics and geometry to establish the symmetry of the situation; many lost marks on this but they were frequently able to recover to find the tension in the string correctly in part (iii). Parts (iv) and (v) dealt with a different situation in which the string was cut into two unequal parts, and this proved rather more familiar to candidates. In part (v), they were asked to find the tensions in the strings and, although most knew what they were trying to do, there were many careless mistakes with cos-sin interchanges and sign errors common. Most candidates resolved in the horizontal and vertical directions and tried to solve the resulting simultaneous equations; however, some used one of two other available methods (resolving in the directions of the strings, and a triangle of forces); because the strings were at right angles either of these required less work. In the last part of the question the situation was considered in which the string was cut at a different point; only a few candidates saw that with the given lengths all the weight would be carried by one of the strings and the other would be slack.

4762 Mechanics 2

General Comments

The quality of the responses of candidates on many topics was again of a pleasingly high standard. Candidates seem confident when they are on familiar territory, with questions on centres of mass, conservation of momentum, Newton's experimental law and basic resolution of forces and moments. On this paper, there were two more unusual requests in Question 3(iv) and Question 4(ii)(*A*) and candidates struggled to apply their knowledge to unfamiliar situations. In the case of Question 4, many candidates put themselves at great disadvantage by attempting to proceed without the use of a diagram. It cannot be emphasised enough that a diagram is crucial when tackling a question involving the equilibrium of forces. Having drawn a diagram, candidates are then advised to take a moment to think about ALL of the forces that are involved, and the direction in which each acts.

Comments on Individual Questions

1 Momentum, impulse and collisions

(a) A significant number of candidates did not understand the concept of momentum as a vector quantity and worked only with the 'horizontal' component. Of those candidates who did use vectors, only a minority went on to find the magnitude of the impulse. A common error by many candidates was to treat the mass of the tanker as 120 000 kg rather than 120 000 tonnes.

(b)(i) The majority of candidates understood that the directions of motion after the collision had to be opposite, in order that momentum was conserved. There seemed to be confusion, however, between speeds and velocities.

(ii) Candidates offered a large variety of different solutions, often very convoluted, but still successful. Those candidates who did not consider the distances travelled by P and Q before their collision rarely earned any marks.

(iii) There were many concise and accurate solutions. Any loss of marks was usually due to an incorrect calculation for the frictional force.

(iv) Candidates were on very familiar territory here with a routine application involving the principle of conservation of linear momentum and Newton's experimental law. A few candidates made arithmetical and sign errors or used incorrect velocities, but the majority produced neat accurate solutions.

2 Work and energy

(i) The majority of candidates produced solutions that indicated a good understanding of the relationship between forces and work done. Others seemed to confuse themselves and gave a force rather than the work done by the force as the final answer.

(ii) Again, there were many good solutions, demonstrating good understanding of energy and work. Any errors were usually due to the omission of the gravitational potential term in the energy equation.

(iii) A significant number of candidates omitted either the weight component or part or all of the resistive term in their application of Newton's second law. Almost all, having found a force, used the formula for power as force times distance and thereby gained follow through marks. The modal mark for this question was 4/7.

3 Centres of mass and stability

(i) The vast majority of candidates scored full marks with solutions displaying very clear systematic approaches to the problem.

(ii) Again, a high proportion of candidates scored well on this part of the question. The minority of candidates who fared less well usually attempted to solve the problem without the aid of a diagram. As always in requests of this type, a clear diagram with relevant lengths marked would have been invaluable.

(iii) Solutions to this more searching request were of a pleasingly good standard, with many candidates displaying a sound understanding of the principles involved. A surprising number of candidates did, however, repeat unnecessarily their calculations from part (i). The range of the final answers given by candidates varied considerably, largely because of some premature or incorrect rounding errors in the value of the *y*-coordinate the initial centre of mass. Candidates should be encouraged to work with exact values wherever possible.

(iv) This was an unusual question that seemed to throw the vast majority of candidates. Few were able to visualise the physical situation being described and the shading indicating the region of stability of the folded object often appeared to be either vague and random or non-existent. A significant number of candidates did seem to appreciate that the *y*-coordinate of the centre of mass was unchanged, although this was often deduced from yet another recalculation of work already done in part (i). However, few candidates plotted the position of the centre of mass and concluded that the object was stable because the point was within their shaded region. A few candidates worked out equations for the lines bordering the region and argued algebraically that their centre of mass was in their region.

4 Forces and equilibrium

(i) Most candidates earned full marks on this simple application of moments. A minority of candidates found only the tension in the string.

(ii) Candidates seemed to struggle with the two parts of this question. A major reason for this was the lack of a diagram. It is difficult to imagine how a candidate might hope to resolve and take moments for an equilibrium situation when they do not have a diagram with all the relevant forces marked on it. The evidence suggests very strongly that a diagram was absolutely key to any meaningful progress.

(ii)(a) Only a very small minority of candidates scored well. The majority of candidates, many working without a diagram, filled the page with a selection of equations, resulting from resolving and taking moments with largely unidentified forces. Of those candidates who drew a diagram, the normal reaction between the rod and the floor at A was often omitted or assumed to be perpendicular to the rod rather than the floor. Many solutions suggested that the candidate was confusing the given situation with the more familiar one of a block resting on an inclined plane.

(ii)(b) Again, a diagram with all the forces labelled was crucial. There seemed to be a lot of confusion about the directions of the friction and the normal reaction at A and the normal reaction at P. Candidates are also advised to think more carefully about the most appropriate directions in which to resolve and the points about which to take moments, before embarking on filling the page with equations. An apparently trivial point is to note that not all normal reactions have to be referred to as R. This assumption led to some simpler but totally erroneous attempts at solutions.

4766 Statistics 1

General Comments

The level of difficulty of the paper appeared to be appropriate for the candidates and indeed they performed slightly better than last January. Even most of those candidates who used long-winded methods were able to complete the paper in the allocated time. The majority of candidates were generally well prepared and handled the standard parts of questions very well. Most candidates supported their numerical answers with appropriate explanations and working. There was relatively little use of additional answer sheets and very few candidates attempted parts of questions in answer sections intended for a different question/part.

It is pleasing to report that once again the statement of the hypotheses in question 7 was generally well answered, with most candidates again not only giving their hypotheses in terms of p but also defining p. However, because this was a two-tailed test candidates were less successful in carrying out the test, particularly those who used a critical region approach. Rather fewer candidates lost marks due to over specification than last year, but some still lost two or three marks altogether. It should be noted that although answers should not usually be given to more than 4 significant figures, when an answer is an exact whole number such as a combination, it may be appropriate to give it to full accuracy. Candidates should be advised that it is the number of significant figures that is important, not the number of decimal places. For instance in question 6(v), a number of candidates gave the full answer of 146.875 then thought that they had rounded appropriately by giving an answer of 146.88, often stating 'to 2 dp'. Unfortunately this lost them a mark for over-specification.

Comments on Individual Questions

1(i) Approximately 95% of candidates scored this mark.

(ii) There were many fully correct answers. Most used the relevant formulas rather than using the built in functions on their calculators. A few candidates found the variance or the rmsd, and these gained a method mark. The most common error was not to use the key and thus get answers ten times too high. This error was severely penalised, but full marks were allowed in part (iii) for a follow through.

(iii) The limits for outliers were widely known and correctly used by most candidates. Even those with incorrect mean and standard deviation were able to gain 3 or all 4 marks if they followed through correctly. Some candidates used the quartiles method, despite often having got part (ii) correct, but some of these made errors, losing some if not all of the marks.

2(i) Nearly all candidates were able to calculate the correct coefficients of k and sum the terms to get $50k$. Work was generally neatly presented and well structured. Only a very few candidates failed to get the correct answer of 1/50. A small number of candidates did not show the probabilities in a table, thus losing 1 mark.

(ii) Once again, a substantial majority of candidates scored full marks. The most common errors were just calculating $E(X^2)$ believing that to be the variance, finding $E(X^2) - E(X)$ or dividing their answers by 4 or some other factor. Those whose probabilities did not sum to 1 were only able to gained two marks out of the 5 available.

3(i) This question was well answered, with about 80% of candidates scoring both marks.

(ii) Almost all candidates gained the first mark for two labelled intersecting circles. Many candidates put their answer from part (i) into the intersection but then did not subtract their value from P(*W*) so put 0.07 instead of (0.07 – their answer to (i)) in the other part of the circle labelled *W*. However, a reasonable number of candidates gained full credit, either having the correct 0.01 and 0.92 in the other parts or by following through correctly.

(iii) The vast majority of candidates tried to show non-independence by comparing P(*L*) x P(*W*) with P(*L* intersect *W*). However most of these did not have the correct value of P(*L*) and many had P(*W*) wrong, despite its value being given in the question. A small number of candidates compared P(*L*I*W*) with P(*L*) and these were more often successful.

4(i) This part was usually correctly answered, although a few candidates found $^{11}P_3$ instead of $^{11}C_3$.

(ii) There was a mixed response to this question. Most candidates used the fractions rather than the combinations method but many of these omitted the multiplier of 3 whether finding P(2) + P(3) or finding 1 – (P(0) + P(1)). Roughly 25% of candidates gained just 1 mark only for adding P(2) and P(3) but using a binomial distribution.

5(i) This was well answered with the most candidates gaining all 3 marks. Some calculated $(1/6)^3$ and so scored 0 and some just scored 1 for $^5/_6$.

(ii) Only about 25% of candidates gained marks in this part – most either misread or misinterpreted the question and calculated the probability of needing exactly 10 attempts. Some candidates read the question correctly but spent time calculating all the probabilities from 1 to 10 and summing them, usually successfully. Very few used the method stated in the mark scheme.

6(i) On the whole, this question was answered well. The most common incorrect answer was 22, which was seen fairly frequently. A small minority of candidates wrote 9 + 4, but then calculated incorrectly (both 11 and 12 seen).

(ii) Only about 10% of candidates produced totally correct answers. Many scored SC2 for finding the 50th, rather than 50.5th, value. Those that did state that they were looking for 50.5th value often just gave the mid-value, rather than using interpolation. Many candidates lost a mark due to over-specification.

(iii) The histogram was generally completed rather better than in previous years. Most candidates were able to calculate frequency densities correctly, and they also usually labelled the axes correctly. A fairly common error was to round the first frequency density down to 1.6 rather than to 1.7. Some made errors with careless drawing of bars, making slips with incorrect heights.

(iv) Roughly 90% of candidates scored full marks here.

(v) Many candidates did everything correctly but gave the final answer as 146.875 or 146.88 thus losing the final mark for over-specifying. The scheme allowed for a slip in both the frequencies and the mid-points and candidates were still able to gain 4 marks. The most common error was giving the final mid-point as 165 rather than 167.5.

7(i)(*A*) Most candidates successfully used the formula, a relatively small number using tables. A few who used the binomial formula omitted the coefficient, but on the whole this question was answered better than in previous years.

(i)(*B*) Again many fully correct responses were seen. Candidates usually used the correct table but a common wrong answer was 1 – $P(X\leq 5)$ rather than 1 – $P(X\leq 4)$. Some candidates used the lengthy method of finding the individual probabilities of 5 or more and then adding, sometimes successfully but in many cases with errors.

(i)(*C*) Almost all candidates multiplied 0.35 by 10, but about 20% of them either rounded to 4 or truncated to 3, thus losing the second mark.

(ii) Most candidates were able to identify that this was a two-tailed test and were able to correctly state the null and alternative hypotheses. However, some candidates failed to define p and others failed to explain why it was two-tailed. Some of the weaker candidates used poor notation when defining their hypotheses. Rather more candidates used the critical region method than finding $P(X\geq 10)$. However, those who used the probability method were generally more successful. Those who tried to find the critical region often included either 3 or 11 and so lost the final three marks. Unfortunately, a significant number of candidates made comparisons with 5% instead of 2.5%, or omitted the comparison altogether and so again lost the last three marks. A disappointing number found $P(X=10)$ thus losing all of the final 5 marks. It was pleasing to see that the majority of candidates did however realise that justification, with probabilities, is needed whichever method they employ. Conclusions, for those who get this far, were usually correct. However care should be taken to explain *in words* their findings including an *element of doubt* in their conclusion. Those answering by the critical region method should be aware that '10 is not in CR' is not enough, they also need to add ‘insufficient evidence to reject the null hypothesis’ and then go on to give an answer in context.

(iii) Under half of the candidature scored either mark in this question. Many did not attempt it. A disappointing proportion compared with 5% even though they had correctly compared with 2.5% in part (ii). A further significant proportion failed to correctly state their conclusion within the context of the question.

4763 Mechanics 3

General Comments

Candidates generally presented their answers well and demonstrated a very sound understanding of most of the topics being examined. The notable exception was Q.3 on elastic strings, where the motion was often wrongly assumed to be simple harmonic. Candidates did not appear to have any difficulty completing the paper in the time allowed.

Comments on Individual Questions

1 The simple harmonic motion problem in part (a) was well understood and was usually answered correctly. Having found the parameters of the motion, some candidates omitted the calculation of the period.

In part (b)(i) the dimensions of *G* were almost always found correctly. The method for finding the powers in part (b)(ii) was also very well understood, although a significant number started with the wrong dimensions for angular speed, usually LT^{-1}. Most candidates then used their formula correctly in part (b)(iii) to find the new angular speed.

2 In part (a)(i) the tangential acceleration was usually found correctly, although there was some sine/cosine confusion. Many attempts at the radial acceleration started with an equation of motion, but most recovered from this and then considered energy.

In part (a)(ii) most candidates produced a radial equation of motion with zero normal reaction, together with an energy equation. However, there were some difficulties with the potential energy terms and the subsequent manipulation to find the speed.

In part (b) almost all candidates obtained two equations from the vertical equilibrium and radial acceleration. There were some careless slips in the trigonometry, such as taking RC to be 2.5 m instead of 3.2 m, and several made algebraic errors when solving the simultaneous equations.

3 This was found to be by far the most difficult question, with entirely inappropriate methods often being selected for parts (iii) and (v).

Parts (i) and (ii) on the equilibrium position were usually answered correctly.

In part (iii) it was necessary to apply Newton's second law in the vertical direction. Many of those who did this were successful, although some did not give the direction of the acceleration. Many used energy to find the speed of P, but this is not the first step in any valid method for finding its acceleration. Some tried to use formulae which only apply to simple harmonic motion.

In part (iv) about half the candidates gave a satisfactory explanation, usually by stating that the acceleration is zero in the equilibrium position. There were also very many references to simple harmonic motion here.

In part (v) the expected approach was to use energy; when doing this, common errors were omission of the initial elastic energy and using the elastic energy in just one string instead of the two. However, very many attempts did not consider energy at all, usually treating the motion as if it were simple harmonic.

4 In part (a) the method for finding the centre of mass of a solid of revolution was well understood and usually carried out accurately.

In part (b)(i) the given x-coordinate of the centre of mass of the lamina was almost always found legitimately. The y-coordinate was also usually found correctly, although some omitted the factor ½ and some made errors in the expansion and integration of $(x + \sqrt{x})^2$.

In part (b)(ii) the centre of mass of a composite body was well understood, and this part was usually answered correctly.

4767 Statistics 2

General Comments

Once again, the overall level of ability shown by candidates taking this paper was high. Responses to questions requiring statistical interpretation were, on the whole, good. The majority of candidates coped well with the questions involving probability calculations. Question 4, involving hypothesis tests, was particularly well done; it was pleasing to see many suitably non-assertive conclusions. Over-specification of answers was seen, though most worked to appropriate levels of accuracy. Few candidates were penalised for under-specification.

Comments on Individual Questions

1(i) Most candidates produced an accurately drawn scatter diagram with suitably labelled axes. Few candidates neglected to label their axes. Those using unusual scales on the vertical axis often incorrectly plotted the second or third point.

(ii) This was well answered. The majority of candidates identified “thickness” as the independent variable and provided a suitable reason for their choice; this often involved describing an element of control over the thickness of tile used.

(iii) Most candidates obtained the correct equation for *h* on *t* (though many used variables *y* & *x* instead of *h* & *t*). A small number of candidates estimated the gradient of the line using points on the graph rather than the least squares regression formula provided in the Examination Formulae booklet. Few arithmetic slips were seen. Most candidates opted to give their gradient and intercept values correct to 3 significant figures. Candidates calculating the least squares regression line for *t* on *h* were few in number.

(ivA&) Well answered. Most candidates showed awareness of interpolation and extrapolation, and provided suitable comments

(v) This question was generally well answered. Many candidates scored full marks. A few candidates calculated the residual as “predicted value – observed value”.

(vi) After calculating the predicted height for a tile of thickness 200 mm, most candidates realised that the linear relationship was reliable for thicknesses within the range of values of the data provided, and that the relationship appeared to break down for larger thicknesses. Most of these candidates communicated this idea well, but those candidates simply stating that the (overall) relationship was non-linear did not earn the final mark. A small proportion of candidates managed to make a suitable comment without showing that they had calculated the prediction for the 200 mm tile. Many candidates seemed unaware of the differences between linear relationships and relationships where one variable is proportional to another.

2(iA) This was well answered with most candidates obtaining both marks.

(iB) Also well answered though occasional mistakes using tables, such as looking up the value of P($X \leq 1$) using $\lambda = 2.0$, were seen.

(iC) Most candidates realised that the new mean, $\lambda = 10.5$, was to be used. Many correct answers were seen through a variety of incorrect methods for finding $P(5 \le X \le 10)$ followed; of which "$P(X \le 10) – P(X \le 5)$" and "$P(X \ge 5) – P(X \le 10)$" were typical.

(ii) Many candidates obtained full marks here though some failed to apply the required continuity correction. A few candidates lost the final accuracy mark through premature rounding of their z-value prior to using Normal tables.

(iii) Most candidates provided a suitable comment here, with remarks about "independence" being the most popular.

(iv) This proved to be one of the most challenging parts of the paper. Despite answering part (iii) correctly many candidates reverted back to the inappropriate model by combining the means rather than considering the different combinations of "pairs" and "singles". Of those attempting to consider combinations of pairs and singles only a small proportion obtained a fully correct solution; a variety of approaches was seen and those working systematically were the most successful.

3(iA) Well answered, though inappropriate "continuity corrections" were seen on occasion.

(iB) Well answered, though arithmetic errors were quite common. In several cases, –1.667 was used rather than –1.1667 often as a result of candidates misreading their own figures. A few candidates lost accuracy by prematurely rounding their z-value before using the Normal tables.

(ii) Very well answered. Most candidates scored both marks.

(iii) On the whole, this was well answered. Many candidates provided clear, accurate methods leading to correct final answers. Some candidates started out with one of the required equations containing a sign error which was not picked up, even when the error led to a negative value for σ. Most candidates identified the correct z-values. In the poorest answers, continuity corrections were attempted and z-values were changed to absurd values, such as "1 – 0.8416", before substitution into equations. Over-specification of final answers was seen, on occasion, here.

(iv) Though one of the more challenging parts, many candidates scored full marks here. A variety of correct, "non-symmetrical" solutions were seen though most opted to use z-values of ± 1.96.

4(a) This question was well answered. Many candidates scored full marks. Marks lost typically for over-assertive conclusions, typically containing words such as "not enough evidence to prove that…". The small number of candidates referring to correlation in their hypotheses often lost the first and last marks. Most candidates managed to accurately calculate the test statistic though some did not show all working as required. Most candidates stated the correct number of degrees of freedom and identified the correct critical value, though some thought that this was a 2-tailed test.

(b) Well answered. Most candidates accurately calculated the sample mean and provided hypotheses in terms of μ. Note that candidates should be discouraged from referring to the "sample population mean" when defining μ. In carrying out the test, the test statistic method proved the most popular; those who "reversed their numerator" needed to be very careful how they used their test statistic. Many appropriate, non-assertive conclusions were seen though some failed to include context in the final comments.

4768 Statistics 3

General Comments

As might be expected on a paper at this level, the scripts indicated that most candidates knew what they were doing most of the time. In addition, there were very few scripts which showed evidence of candidates running out of time. That being the case, it is disappointing to report that a large number of scripts suffered from a lack of precision that manifested itself in a number of ways across the paper. Examples include final answers being given to more than 5 significant figures, not enough accuracy being used in calculations, hypotheses and conclusions being given without context, conclusions to hypothesis tests being too assertive, and other examples which will be commented on below. The cumulative effect of these errors was significant for many candidates.

Comments on Individual Questions

1 Water pressure - *t* test

1(i)(ii) Well understood by most candidates, but a lack of precision meant that some candidates did not state that it was the *population* variance that was unknown, and others stated that the data had to be Normally distributed.

(iii) The hypotheses were usually well stated. A few candidates did not define μ, and a few gave a description without context. Of the small proportion who gave their hypotheses in words, the majority used mean rather than population mean. It was pleasing to see the overwhelming majority of candidates correctly opted to use s_{n-1} rather than s_n, but a number of candidates used a truncated value for $\bar{x}$ in the calculation and lost accuracy as a result. Virtual all candidates correctly calculated the test statistic. The correct point of t_8 was usually used, although some candidates opted for t_9. It was not always possible to see if this was through a misunderstanding or a misreading of the table. Most candidates correctly rejected the null hypothesis, but too many gave conclusions which were too assertive or lacked context.

(iv)(v) A majority of candidates knew the meaning of a 95% confidence interval, although some definitions were clearer than others. A minority of candidates gave definitions in terms of just one interval and others had a definition which included the capture of sample means. The calculation of the confidence interval was well done – a few candidates used 1.96 or 2.326 and some changed to t_9.

2 Reed beds - probability density function

(i) A wide range of sketches was seen. A fully correct sketch was the most common outcome, but some sketches extended the parabola well beyond the defined range, others clearly did not have zero gradient at $x = 2$. Some sketches were unlabelled, and some had an increasing, rather than a decreasing gradient. A few sketches reached the maximum point at $x = 2$ and then continued with a horizontal line.

(ii) This was extremely well done by almost all candidates. Apart from a few arithmetic slips, the only errors which occurred were presenting $\mathrm{E}(X^2)$ as the variance, and forgetting to take the square root for the final answer.

(iii) A large number of candidates did not know the meaning of the term *standard error.* Many gave an interval as their response and a significant number did not respond at all.

(iv) This was well done by most candidates, but occasionally the wrong limits were seen and there were some attempts at Normal approximations.

(v) Only a minority of candidates gained full marks here. Defining the reed beds as clusters was rarely seen and many candidates talked about clusters of reeds. Many responses lacked context.

3 Child car seat - linear combinations of Normal distributions

(i) This part was almost invariably correct. Virtually all of those who did not score full marks here had selected the wrong tail.

(ii) This part was again almost invariably correct.

(iii) Most candidates were able to calculate the mean and variance of the distribution correctly. Most of these candidates then correctly identified the correct *z* value of –1.645. The most common error was the omission of the minus sign, but 1.96 was occasionally seen. Many candidates gave answers to 6 or more significant figures.

(iv) Virtually all candidates correctly calculated the mean cost. Many candidates also knew how to calculate the variance, but a few used multipliers of 1.2, 1.3 and 0.8 instead of their squares. Many variances were given to 6 significant figures, and even 8 significant figures were regularly seen.

3(v) Many candidates were able to find the mean and standard deviation, but often not very efficiently. The correct value of 2.576 was usually used, but various other *z* values were also seen. A surprising number of candidates were unable to multiply both sides of their equation by $\sqrt{50}$ correctly.

4 Wilcoxon paired test and goodness of fit test

(i) Some candidates simply gave a definition of a random sample and others described other sampling methods, but most candidates gave a correct description. It was surprising how many candidates wanted to pick numbers out of a hat as a method of random selection. Surely, at this level, a random number generator is a better choice.

(ii) This question was done extremely well by most candidates. Very few errors were made in calculating the differences and the ranks. The Wilcoxon statistic was almost always correctly calculated. Virtually all candidates gave the correct critical value of 17. A small minority of candidates decided that the result was significant, but more common faults were conclusions either not in context or too assertive.

(b)(iii) This question was well done by most candidates. Most candidates gave acceptable hypotheses and were able to calculate X^2 correctly. A very few candidates confused expected and observed values or combined groups. The correct point of the chi squared distribution was usually quoted, although 8, 11 and 12 degrees of freedom were all seen. A small minority of candidates decided that the result was significant, but more common faults were conclusions either not in context or too assertive.

4771 Decision Maths 1

General Comments

Most candidates were able to make good progress on this paper. Few were able to succeed with all of it. Candidates did well with most of the algorithmic aspects.

Comments on Individual Questions

1 Most candidates did well with the straightforward application of Dijkstra in part (i).
Part (ii) was less well done. Candidates were asked to show their working, and many did not do so.
Many candidates gave the total length of their minimum connector as their answer to the question's final demand. From the structure of the question, candidates might have been expecting an answer which was slightly longer than their 51 from part (i), but most candidates did not make the connection between parts (i) and (ii).

2 There were many interesting variations seen in part (i) on the word "bipartite".
Most candidates were able to draw the graph accurately.
Unsurprisingly many more were correct with the first computation than were correct with the second.

3 Working through the given algorithm required precision. Most candidates made progress, but did not make it to the end.
Part (ii) of the question was challenging. Many could see that x needed to be –0.44, but very few could follow it through to a correct estimate.

4 Parts (i) and (ii) if the CPA question were very well done.
Not very many candidates were successful with the scheduling in part (iii).
Very few collected both marks from part (iv). It was relatively easy to see that 2 extra helpers were needed, but a full explanation had to point out that they were needed to help not only with F, G, H and I, but also with K and L.

5 The simulation question had a carefully detailed structure which helped candidates to make progress, but which may have led to some losing sight of the overall scenario. Thus they were often good at specifying simulation rules, but many did not apply the rules well. For instance, in parts (iv) and (viii) many applied their "child/adult" rule to all of the tabulated cells, instead of only to cells representing their simulated occupants of chairs.
The question was attempting to address the loading of the chairlift, and to do so all 80 "up" chairs need to be simulated. The scaling up from 10 chairs in part (v) and from 5 chairs in part (viii) were necessary compromises given the requirements of an examination question, but it revealed a very surprising weakness. Most candidates were simply unable to scale up their results from 10 chairs to 80, or from 5 chairs to 80.
The final part of the question, part (ix), required candidates to realise that to simulate 10 chairs was a better compromise that to simulate 5. It was expected that many would make a routine reference to "accuracy", when in fact nothing was being estimated, and some duly did so. But it was gratifying to note that many gave more relevant answers, using terminology such as "representative" or "reliable".

6 The LP question was generally done reasonably well.

The summer 2012 report on 4771 contained the following quotation: "Far too many candidates, if they remembered to define their variables, neglected that essential phrase "the number of ...". The issue remains live! Again, in this examination, far too many candidates failed explicitly to define their variables. The phrase "*x* is hats", and variants of it, scored zero.

The insistence on the phrase "number of ..." secures the definition of units in the case of continuous variables, eg "Let *x* be the number of litres of ...", and points to the need for integer values in other cases. In this question most candidates failed adequately to deal with that integer requirement in part (ii). The majority of candidates were happy to round the LP solution to (13, 18). Few looked at nearby lattice points, and only a handful found the optimal integer point, (12, 19).

4776 Numerical Methods (Written Examination)

General Comments

The purely computational parts of this paper were found straightforward by most candidates. Some of the algebra was found more challenging, and interpreting results was difficult for some candidates.

The standard of presentation of work, and in particular the systematic setting out of numerical algorithms, continues to be better than it was a few years ago. However some candidates still resort to scattering calculations haphazardly on the page, making it difficult for examiners to detect and reward any correct work.

Comments on Individual Questions

1 Solution of an equation, Newton-Raphson method
This was a very straightforward question for the majority of candidates. In part (i), there were some who thought they had established the existence of two roots in [0, 2] by showing that the signs are the same at the two ends of the interval. The computations in part (ii) were done well.

2 Absolute and relative errors
The calculations in part (i) were usually done well. As usual, credit was give for the variety of conventions used to define absolute error. The two relative errors, however, were required to be of opposite sign for full marks. In part (ii) there were some candidates who gave the answers to several decimal places – rather missing the point of the question, and losing a mark.

3 Numerical differentiation
Once again, the numerical work in part (i) was done well. A majority of candidates said something sensible about the loss of precision as h is reduced. In part (ii) some credit was given to those who extrapolated by a single step to the likely answer with $h = 0.05$, but full marks required full extrapolation. This part proved difficult for many.

4 Numerical integration
Parts (i) and (ii) were very straightforward with only a small number of candidates making arithmetical errors. In part (iii), full marks could be obtained by giving the answer as 1.3624 or as 1.36243, provided the figure was supported by a sensible explanation. Some credit was given for an argument leading to 1.362. Answers with more or fewer decimal places were seen occasionally, but gained no credit.

5 Forward difference table and interpolation
Though almost all candidates appeared to know what to do in this question, algebraic errors were very common. In particular, subtracting terms like $(54 - k)$ seemed to be a skill too far for many. The linear interpolation in part (iii) defeated some, and was dealt with very laboriously by others.

6 Lagrange interpolation; numerical integration
Part (i) was generally done well; the algebra here seemed less of a challenge than that in question 5. The comment required for the final mark was simply that interpolation (f(0)) was likely to be more accurate than extrapolation ($f(x) = 0$). Part (ii) required the use of the trapezium rule on two sub-intervals of different widths. Some took this in their stride, but others tried to 'modify' the trapezium rule formula (for example by averaging the two values of h), or made no attempt at all. In part (iii) candidates were required to observe that f(1.5) is required in order to use Simpson's rule.

7 Solution of an equation, fixed point iteration, secant method, bisections

There were many good sketches of the two graphs in part (i), with clear indications of the two roots. However, a substantial minority got one or other of the sketches wrong: for example, plotting 2sin*x* for 1 + sin*x.* In part (ii) the fixed point iteration was done well by most. In part (iii) the change of sign was straightforward, but demonstrations that the iteration diverges were often unconvincing. Beginning with x_0 equal to one endpoint of the interval containing the root is not convincing; and checking both endpoints is only a little better. Knowing that the root lies in the interval (3.9, 4.1), the natural choice for x_0 is, of course, 4. (Strictly, divergence from 4 is not watertight either, but in this area of practical computational mathematics it is regarded as sufficient.) The bisection method that followed was usually well done. The only mistake seen more than a few times was to omit the maximum possible error, and consequentially to stop at the wrong step. The secant method in part (iv), though routine, was not handled as well as the other computational parts of the paper.

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