

# **Mathematics (MEI)**

Advanced Subsidiary GCE

Unit **4751**: Introduction to Advanced Mathematics

## **Mark Scheme for January 2013**

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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**Annotations**

<b>Annotation</b>	<b>Meaning</b>
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
<b>Other abbreviations in mark scheme</b>	<b>Meaning</b>
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

**Subject-specific Marking Instructions**

- a Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep \*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

## g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.



Question			Answer	Marks	Guidance	
3			obtaining a correct relationship in any 3 of $C$ , $d$ , $r$ and $A$	M2	may substitute into given relationship;	eg M2 for $Cd = 4\pi r^2$ or $\pi d^2 = k\pi r^2$ seen/obtained
			or obtaining a correct relationship in $k$ and no more than 2 other variables		or M1 for at least two of $A = \pi r^2$ , $C = \pi d$ , $C = 2\pi r$ , $d = 2r$ or $r = \frac{d}{2}$ seen or used	condone eg Area $= \pi r^2$ ; allow $A = \pi \left(\frac{d}{2}\right)^2$ to imply $A = \pi r^2$ and $r = \frac{d}{2}$ and so earn M1, if M2 not earned
			convincing argument leading to $k = 4$	A1	must be from general argument, not just substituting values for $r$ or $d$ ; may start from given relationship and derive $k = 4$	eg M1 only for eg $A = \pi r^2$ and $C = \pi d$ and so $k = 4$ with no further evidence
				[3]		
4			$(5x + 2)(x - 6)$	M1	for factors giving at least two out of three terms correct when expanded and collected	or use of formula or completing the square with at most one error (comp square must reach $[5](x - a)^2 \leq b$ oe or $(5x - c)^2 \leq d$ oe stage) if correct: $5(x - 2.8)^2 \leq 51.2$ or $(x - 2.8)^2 \leq 10.24$ or $(5x - 14)^2 \leq 256$
			boundary values $-0.4$ oe and 6 soi	A1	A0 for just $\frac{28 \pm \sqrt{1024}}{10}$	
			$-0.4 \leq x \leq 6$ oe	A2	may be separate inequalities; mark final answer  A1 for one end correct eg $x \leq 6$ or for $-0.4 < x < 6$ oe  or B1 for $a \leq x \leq b$ ft their boundary values	condone unsimplified but correct $\frac{28 - \sqrt{1024}}{10} \leq x \leq \frac{28 + \sqrt{1024}}{10}$ etc allow A1 for $-0.4 \leq 0 \leq 6$
				[4]		condone errors in the inequality signs during working towards final answer



Question			Answer	Marks	Guidance	
5			$4 + 2k + c = 0$ or $2^2 + 2k + c = 0$	B1	may be rearranged	condone $-3^2$ seen if used as 9  M0 for addition of eqns unless also multiplied appropriately  if no errors and no method seen, allow correct answers to imply M1 provided B1B1 has been earned
			$9 - 3k + c = 35$	B1	may be rearranged; the $(-3)^2$ must be evaluated / used as 9	
			correct method to eliminate one variable from their eqns	M1	eg subtraction or substitution for $c$ ; condone one error	
			$k = -6, c = 8$	A1	from fully correct method, allowing recovery from slips	
			<b>or</b> $[x^2 + kx + c =] (x - 2)(x - a)$	<b>or</b> M1	or $(x - 2)(x + b)$	
			$-5 \times (-3 - a) = 35$ oe	M1		
			$a = 4$ $k = -6, c = 8$	A1 A1		
				[4]		

Question			Answer	Marks	Guidance	
6			identifying term as $20(2x)^3\left(\frac{5}{x}\right)^3$ oe	M3	condone lack of brackets;  M1 for $[k](2x)^3\left(\frac{5}{x}\right)^3$ soi (eg in list or table), condoning lack of brackets  and M1 for $k = 20$ or eg $\frac{6 \times 5 \times 4}{3 \times 2 \times 1}$ or for 1 6 15 20 15 6 1 seen (eg Pascal's triangle seen, even if no attempt at expansion)  and M1 for selecting the appropriate term (eg may be implied by use of only $k = 20$ , but this M1 is not dependent on the correct $k$ used)	xs may be omitted; eg M3 for $20 \times 8 \times 125$  first M1 not earned if elements added not multiplied; otherwise, if in list or table bod intent to multiply  M0 for binomial coefficient if it still has factorial notation
			20 000	A1	or B4 for 20 000 obtained from multiplying out $\left(2x + \frac{5}{x}\right)^6$  allow SC3 for 20000 as part of an expansion	may be gained even if elements added
				[4]		
7	(i)		$9\sqrt{3}$ www oe as final answer	2	M1 for $\sqrt{48} = 4\sqrt{3}$ or $\sqrt{75} = 5\sqrt{3}$ soi	
				[2]		
7	(ii)		$\frac{39 + 7\sqrt{5}}{44}$ www as final answer	3	M1 for attempt to multiply numerator and denominator by $7 - \sqrt{5}$  B1 for each of numerator and denominator correct (must be simplified)	condone $\frac{39}{44} + \frac{7\sqrt{5}}{44}$ for 3 marks  eg M0B1 if denominator correctly rationalised to 44 but numerator not multiplied
				[3]		

Question			Answer	Marks	Guidance	
8			$5c + 9t = 2ac + at$	M1	for correct expansion of brackets	for each M, ft previous errors if their eqn is of similar difficulty;  may be earned before $t$ terms collected  treat as MR if $t$ is the subject, with a penalty of 1 mark from those gained, marking similarly
			$5c - 2ac = at - 9t$ oe	M1	for correct collection of terms, ft eg after M0 for $5c + 9t = 2ac + t$ allow this M1 for $5c - 2ac = -8t$ oe	
			$c(5 - 2a) = at - 9t$ oe	M1	for correctly factorising, ft; must be $c \times$ a two-term factor	
			$[c =] \frac{at - 9t}{5 - 2a}$ or $\frac{t(a - 9)}{5 - 2a}$ oe as final answer	M1	for correct division, ft their two-term factor	
				[4]		
9	(i)		sketch of cubic the right way up, with two tps	B1		No section to be ruled; no curving back; condone some curving out at ends but not approaching another turning point; condone some doubling (eg erased curves may continue to show); ignore position of turning points for this mark
			their graph touching the $x$ -axis at $-2$ and crossing it at 3 and no other places	B1	if intns are not labelled, they must be shown nearby	mark intent if ‘daylight’ between curve and axis at $x = -2$
			intersection of $y$ -axis at $-12$	B1		if no graph but $-12$ marked on $y$ -axis, or in table, allow this 3 <sup>rd</sup> mark
				[3]		
9	(ii)		$-5$ and 0	B2	B1 each; allow B2 for $-5, -5, 0$ ; or B1 for both correct with one extra value or for $(-5, 0)$ and $(0, 0)$	if their graph wrong, allow $-5$ and 0 from starting again with eqn, or ft their graph with two intns with $x$ -axis
					or SC1 for both of 1 and 6	
				[2]		

Question		Answer	Marks	Guidance	
10	(i)	midpt of AB = $\left(\frac{1}{2}, \frac{5}{2}\right)$ oe www	B2	allow unsimplified B1 for one coordinate correct	if working shown, should come from $\left(\frac{3 + -2}{2}, \frac{4 + 1}{2}\right)$ oe  NB B0 for $x$ coord. = $\frac{5}{2}$ , (obtained from subtraction instead of addition)
		grad AB = $\frac{4 - 1}{3 - (-2)}$ oe	M1	must be obtained independently of given line; accept 3 and 5 correctly shown eg in a sketch, followed by 3/5  M1 for rise/run = 3/5 etc  M0 for just 3/5 with no evidence	for those who find eqn of AB first, M0 for just $\frac{y - 4}{1 - 4} = \frac{x - 3}{-2 - 3}$ oe, but M1 for $y - 4 = \frac{1 - 4}{-2 - 3}(x - 3)$ oe  ignore their going on to find the eqn of AB after finding grad AB
		using gradient of AB to obtain grad perp bisector	M1	for use of $m_1 m_2 = -1$ soi or ft their gradient AB  M0 for just $\frac{-5}{3}$ without AB grad found	this second M1 available for starting with given line = $\frac{-5}{3}$ and obtaining grad. of AB from it
		$y - 2.5 = \frac{-5}{3}(x - 0.5)$ oe	M1	eg M1 for $y = \frac{-5}{3}x + c$ and subst of midpt; ft their gradient of perp bisector and midpt;  M0 for just rearranging given equation	no ft for gradient of AB used



Question		Answer	Marks	Guidance	
10	(iii)	$(x - a)^2 + (y - b)^2 = r^2$ seen or used  $1^2 + 4^2$ oe (may be unsimplified), from clear use of A or B  $(x + 1)^2 + (y - 5)^2 = 17$  showing midpt of CD = $(-1, 5)$  showing CE or DE = $\sqrt{17}$ oe or showing one of C and D on circle	M1  M1  A1  M1  M1	or for $(x + 1)^2 + (y - 5)^2 = k$ , or ft their E, where $k > 0$  for calculating AE or BE or their squares, or for subst coords of A or B into circle eqn to find $r$ or $r^2$ , ft their E;  for eqn of circle centre E, through A and B;  allow A1 for $r^2 = 17$ found after $(x + 1)^2 + (y - 5)^2 = r^2$ stated and second M1 clearly earned  if $(x + 1)^2 + (y - 5)^2 = 17$ appears without clear evidence of using A or B, allow the first M1 then M0 SC1  alt M1 for showing $CD^2 = 68$ oe  allow to be earned earlier as an invalid attempt to find $r$	this M not earned for use of CE or DE or $\frac{1}{2}$ CD  NB some cand's finding $AB^2 = 34$ then obtaining 17 erroneously so M0  SC also earned if circle comes from C or D and E, but may recover and earn the second M1 later by using A or B



Question			Answer	Marks	Guidance	
11	(ii)		(2, 0) and (3, 0)	B2	B1 each  or B1 for both correct plus an extra  or M1 for $(x - 2)(x - 3)$ or correct use of formula or for <i>their a</i> $\pm \sqrt{\text{their b}}$ ft from (i)	condone not expressed as coordinates, for both $x$ and $y$ values;  accept eg in table or marked on graph
			(0, 6)	B1		
			graph of quadratic the correct way up and crossing both axes	B1	ignore label of their tp; condone stopping at $y$ -axis	condone ‘U’ shape or slight curving back in/out; condone some doubling / feathering – deleted work sometimes still shows up in scoris; must not be ruled; condone fairly straight with clear attempt at curve at minimum; be reasonably generous on attempt at symmetry
				[4]		
11	(iii)		$x^2 - 5x + 6 = 2 - x$	M1	for attempt to equate or subtract eqns or attempt at rearrangement and elimination of $x$	accept calculus approach: $y' = 2x - 5$
			$x^2 - 4x + 4 [= 0]$	M1	for rearrangement to zero ft and collection of terms; condone one error; if using completing the square, need to get as far as $(x - k)^2 = c$ , with at most one error $[(x - 2)^2 = 0 \text{ if correct}]$	use of $y' = -1$ M1



Question			Answer	Marks	Guidance
			$x = 2, [y = 0]$  ‘double root at $x = 2$ so tangent’ oe; www;	A1  A1  <b>[4]</b>	condone omission of $y = 0$ since already found in (ii)  if they have eliminated $x$ , $y = 0$ is not sufft for A1 – need to get $x = 2$  A0 for $x = 2$ and another root  eg ‘only one point of contact, so tangent’;  or showing $b^2 - 4ac = 0$ , and concluding ‘so tangent’; www  $x = 2$ A1  tgt is $y[-0] = -(x - 2)$ and obtaining given line A1
12	(i)		$f(1) = 1 - 1 + 1 + 9 - 10 [= 0]$  attempt at division by $(x - 1)$ as far as $x^4 - x^3$ in working  correctly obtaining $x^3 + x + 10$	B1  M1  A1  <b>[3]</b>	allow for correct division of $f(x)$ by $(x - 1)$ showing there is no remainder,  or for $(x - 1)(x^3 + x + 10)$ found, showing it ‘works’ by multiplying it out  allow equiv for $(x + 2)$ as far as $x^4 + 2x^3$ in working  or for inspection with at least two terms of cubic factor correct  or $x^3 - 3x^2 + 7x - 5$  condone $1^4 - 1^3 + 1^2 + 9 - 10$  eg for inspection, M1 for two terms right and two wrong  if M0 and this division / factorising is done in part (ii) or (iii), allow SC1 if correct cubic obtained there; attach the relevant part to (i) with a formal chain link if not already seen in the image zone for (i)

Question			Answer	Marks	Guidance	
12	(ii)		$[g(-2) =] -8 - 2 + 10$ or $f(-2) = 16 + 8 + 4 - 18 - 10$	M1	$[in\ this\ scheme\ g(x) = x^3 + x + 10]$ allow M1 for correct trials with at least two values of $x$ (other than 1) using $g(x)$ or $f(x)$ or $x^3 - 3x^2 + 7x - 5$ (may allow similar correct trials using division or inspection)	eg $f(2) = 16 - 8 + 4 + 18 - 10$ or 20 $f(3) = 81 - 27 + 9 + 27 - 10$ or 80 $f(0) = -10$ $f(-1) = 1 + 1 + 1 - 9 - 10$ or $-16$  No ft from wrong cubic ‘factors’ from (i)
			$x = -2$ isw	A1	allow these marks if already earned in (i)	NB factorising of $x^3 + x + 10$ or $x^3 - 3x^2 + 7x - 5$ in (ii) earns credit for (iii) [annotate with a yellow line in both parts to alert you – the image zone for (iii) includes part (ii)]
				[2]		

Question		Answer	Marks	Guidance	
12	(iii)	<p>attempted division of <math>x^3 + x + 10</math> by <math>(x + 2)</math> as far as <math>x^3 + 2x^2</math> in working</p> <p>correctly obtaining <math>x^2 - 2x + 5</math></p> <p>use of <math>b^2 - 4ac</math> with <math>x^2 - 2x + 5</math></p> <p><math>b^2 - 4ac = 4 - 20 [= -16]</math></p> <p>so only two real roots[ of <math>f(x)</math>] [and hence no more linear factors]</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[5]</p>	<p>or <math>x^3 - 3x^2 + 7x - 5</math> by <math>(x - 1)</math> as far as <math>x^3 - x^2</math> in working</p> <p>or inspection with at least two terms of quadratic factor correct</p> <p>allow these first 2 marks if this has been done in (ii), even if not used here</p> <p>may be in attempt at formula (ignore rest of formula)</p> <p>may be in formula;</p> <p>or no real roots of <math>x^2 - 2x + 5 = 0</math>; allow this last mark if clear use of <math>x^2 - 2x + 5 = 0</math>, even if error in <math>b^2 - 4ac</math>, provided result negative, but no ft from wrong factor</p> <p>if last M1 not earned, allow SC1 for stating that the only factors of 5 are 1 and 5 and reasoning eg that <math>(x - 1)(x - 5)</math> and <math>(x + 1)(x + 5)</math> do not give <math>x^2 - 2x + 5</math> [hence <math>x^2 - 2x + 5</math> does not factorise]</p>	<p>alt method: allow M1 for attempted division of quartic by <math>x^2 + x - 2</math> as far as <math>x^4 + x^3 - 2x^2</math> in working, or inspection etc</p> <p>or completing square form attempted</p> <p>or attempt at calculus or symmetry to find min pt</p> <p>NB M0 for use of <math>b^2 - 4ac</math> with cubic factor etc</p> <p>or <math>(x - 1)^2 + 4</math></p> <p>or min = (1, 4)</p> <p>or <math>(x - 1)^2 + 4</math> is always positive so no real roots [of <math>(x - 1)^2 + 4 = 0</math>] [ and hence no linear factors]</p> <p>or similar conclusion from min pt</p>

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