

**Wednesday 23 January 2013 – Morning**

**A2 GCE MATHEMATICS (MEI)**

**4753/01** Methods for Advanced Mathematics (C3)

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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**Section A** (36 marks)

- 1 (i) Given that  $y = e^{-x} \sin 2x$ , find  $\frac{dy}{dx}$ . [3]

(ii) Hence show that the curve  $y = e^{-x} \sin 2x$  has a stationary point when  $x = \frac{1}{2} \arctan 2$ . [3]

- 2 A curve has equation  $x^2 + 2y^2 = 4x$ .

(i) By differentiating implicitly, find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ . [3]

(ii) Hence find the exact coordinates of the stationary points of the curve. [You need not determine their nature.] [3]

- 3 Express  $1 < x < 3$  in the form  $|x - a| < b$ , where  $a$  and  $b$  are to be determined. [2]

- 4 The temperature  $\theta^\circ\text{C}$  of water in a container after  $t$  minutes is modelled by the equation

$$\theta = a - be^{-kt},$$

where  $a$ ,  $b$  and  $k$  are positive constants.

The initial and long-term temperatures of the water are  $15^\circ\text{C}$  and  $100^\circ\text{C}$  respectively. After 1 minute, the temperature is  $30^\circ\text{C}$ .

(i) Find  $a$ ,  $b$  and  $k$ . [6]

(ii) Find how long it takes for the temperature to reach  $80^\circ\text{C}$ . [2]

- 5 The driving force  $F$  newtons and velocity  $v \text{ km s}^{-1}$  of a car at time  $t$  seconds are related by the equation  $F = \frac{25}{v}$ .

(i) Find  $\frac{dF}{dv}$ . [2]

(ii) Find  $\frac{dF}{dt}$  when  $v = 50$  and  $\frac{dv}{dt} = 1.5$ . [3]

- 6 Evaluate  $\int_0^3 x(x+1)^{-\frac{1}{2}} dx$ , giving your answer as an exact fraction. [5]

- 7 (i) Disprove the following statement:

$3^n + 2$  is prime for all integers  $n \geq 0$ . [2]

(ii) Prove that no number of the form  $3^n$  (where  $n$  is a positive integer) has 5 as its final digit. [2]

## Section B (36 marks)

- 8 Fig. 8 shows parts of the curves  $y = f(x)$  and  $y = g(x)$ , where  $f(x) = \tan x$  and  $g(x) = 1 + f(x - \frac{1}{4}\pi)$ .

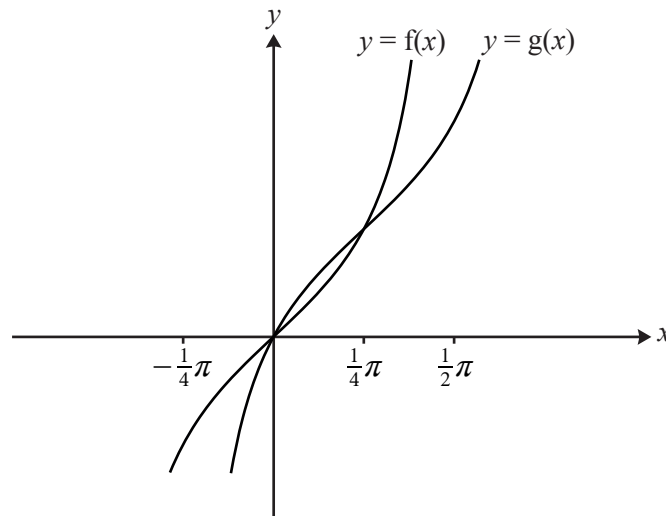


Fig. 8

- (i) Describe a sequence of two transformations which maps the curve  $y = f(x)$  to the curve  $y = g(x)$ . [4]

It can be shown that  $g(x) = \frac{2 \sin x}{\sin x + \cos x}$ .

- (ii) Show that  $g'(x) = \frac{2}{(\sin x + \cos x)^2}$ . Hence verify that the gradient of  $y = g(x)$  at the point  $(\frac{1}{4}\pi, 1)$  is the same as that of  $y = f(x)$  at the origin. [7]

- (iii) By writing  $\tan x = \frac{\sin x}{\cos x}$  and using the substitution  $u = \cos x$ , show that  $\int_0^{\frac{1}{4}\pi} f(x) dx = \int_{\frac{1}{\sqrt{2}}}^1 \frac{1}{u} du$ .  
Evaluate this integral exactly. [4]

- (iv) Hence find the exact area of the region enclosed by the curve  $y = g(x)$ , the  $x$ -axis and the lines  $x = \frac{1}{4}\pi$  and  $x = \frac{1}{2}\pi$ . [2]

- 9 Fig. 9 shows the line  $y = x$  and the curve  $y = f(x)$ , where  $f(x) = \frac{1}{2}(e^x - 1)$ . The line and the curve intersect at the origin and at the point  $P(a, a)$ .

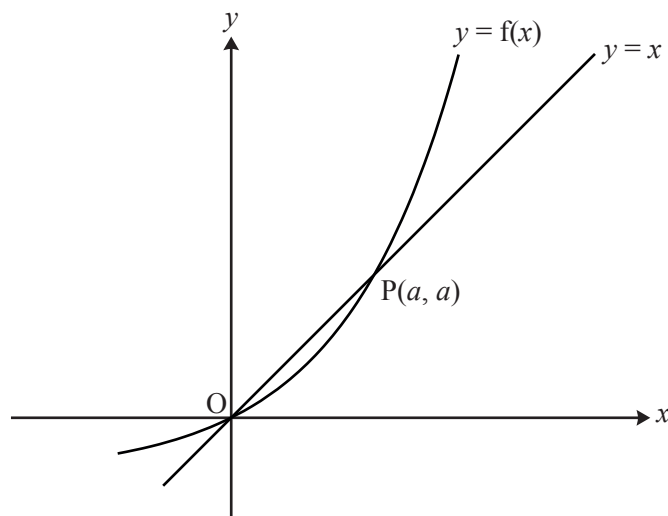


Fig. 9

- (i) Show that  $e^a = 1 + 2a$ . [1]
- (ii) Show that the area of the region enclosed by the curve, the  $x$ -axis and the line  $x = a$  is  $\frac{1}{2}a$ . Hence find, in terms of  $a$ , the area enclosed by the curve and the line  $y = x$ . [6]
- (iii) Show that the inverse function of  $f(x)$  is  $g(x)$ , where  $g(x) = \ln(1 + 2x)$ . Add a sketch of  $y = g(x)$  to the copy of Fig. 9. [5]
- (iv) Find the derivatives of  $f(x)$  and  $g(x)$ . Hence verify that  $g'(a) = \frac{1}{f'(a)}$ . [7]

Give a geometrical interpretation of this result.

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