

Wednesday 23 January 2013 – Morning

AS GCE MATHEMATICS (MEI)

4755/01 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

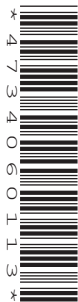
OCR supplied materials:

- Printed Answer Book 4755/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

- 1 Transformation A is represented by matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and transformation B is represented by matrix $\mathbf{B} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.
- (i) Describe transformations A and B. [3]
- (ii) Find the matrix for the composite transformation A followed by B. [2]
- 2 Given that $z = a + bj$, find $\operatorname{Re}\left(\frac{z}{z^*}\right)$ and $\operatorname{Im}\left(\frac{z}{z^*}\right)$. [4]
- 3 You are given that $z = 2 + j$ is a root of the cubic equation $2z^3 + pz^2 + 22z - 15 = 0$, where p is real. Find the other roots and the value of p . [6]
- 4 (i) Show that $x^2 - x + 2 > 0$ for all real x . [2]
- (ii) Solve the inequality $\frac{2x}{x^2 - x + 2} > x$. [5]
- 5 You are given that $\frac{3}{(5 + 3x)(2 + 3x)} \equiv \frac{1}{2 + 3x} - \frac{1}{5 + 3x}$.
- (i) Use this result to find $\sum_{r=1}^{100} \frac{1}{(5 + 3r)(2 + 3r)}$, giving your answer as an exact fraction. [5]
- (ii) Write down the limit to which $\sum_{r=1}^n \frac{1}{(5 + 3r)(2 + 3r)}$ converges as n tends to infinity. [1]
- 6 Prove by induction that $1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2}$. [8]

Section B (36 marks)

- 7 Fig. 7 shows a sketch of $y = \frac{x-4}{(x-5)(x-8)}$.

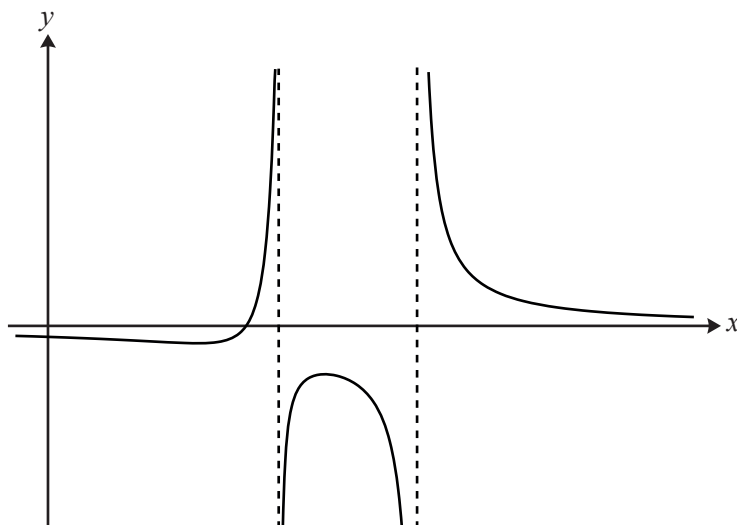


Fig. 7

- (i) Write down the equations of the three asymptotes and the coordinates of the points where the curve crosses the axes. Hence write down the solution of the inequality $\frac{x-4}{(x-5)(x-8)} > 0$. [6]

- (ii) The equation $\frac{x-4}{(x-5)(x-8)} = k$ has no real solutions. Show that $-1 < k < -\frac{1}{9}$. Relate this result to the graph of $y = \frac{x-4}{(x-5)(x-8)}$. [7]

- 8 (i) Indicate on an Argand diagram the set of points z for which $|z - (-8 + 15j)| < 10$. [4]

- (ii) Using the diagram, show that $7 < |z| < 27$. [3]

- (iii) Mark on your Argand diagram the point, P , at which $|z - (-8 + 15j)| = 10$ and $\arg z$ takes its maximum value. Find the modulus and argument of z at P . [4]

9 You are given that $\mathbf{A} = \begin{pmatrix} 8 & -7 & -12 \\ -10 & 5 & 15 \\ -9 & 6 & 6 \end{pmatrix}$ and $\mathbf{A}^{-1} = k \begin{pmatrix} 4 & 2 & 3 \\ 5 & 4 & 0 \\ 1 & -1 & 2 \end{pmatrix}$.

(i) Find the exact value of k .

[2]

(ii) Using your answer to part (i), solve the following simultaneous equations.

$$\begin{aligned} 8x - 7y - 12z &= 14 \\ -10x + 5y + 15z &= -25 \\ -9x + 6y + 6z &= 3 \end{aligned}$$

[4]

You are also given that $\mathbf{B} = \begin{pmatrix} -7 & 5 & 15 \\ a & -8 & -21 \\ 2 & -1 & -3 \end{pmatrix}$ and $\mathbf{B}^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 0 & 5 \\ -4 & -3 & 1 \\ 2 & 1 & b \end{pmatrix}$.

(iii) Find the values of a and b .

[2]

(iv) Write down an expression for $(\mathbf{AB})^{-1}$ in terms of \mathbf{A}^{-1} and \mathbf{B}^{-1} . Hence find $(\mathbf{AB})^{-1}$.

[4]

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