OCR
RECOGNISING ACHIEVEMENT

Monday 14 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Turn over

Section A (54 marks)

Answer all the questions

1 **(a)** **(i)** Differentiate with respect to x the equation $a\tan y = x$ (where a is a constant), and hence show that the derivative of $\arctan\frac{x}{a}$ is $\frac{a}{a^2+x^2}$. **[3]**

(ii) By first expressing $x^2 - 4x + 8$ in completed square form, evaluate the integral $\int_0^4 \frac{1}{x^2-4x+8}\,dx$, giving your answer exactly. **[4]**

(iii) Use integration by parts to find $\int \arctan x \, dx$. **[4]**

(b) **(i)** A curve has polar equation $r = 2\cos\theta$, for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$. Show, by considering its cartesian equation, that the curve is a circle. State the centre and radius of the circle. **[5]**

(ii) Another circle has radius 2 and its centre, in cartesian coordinates, is (0, 2). Find the polar equation of this circle. **[2]**

2 **(a)** **(i)** Show that

$$1 + e^{j2\theta} = 2\cos\theta(\cos\theta + j\sin\theta).$$ **[2]**

(ii) The series C and S are defined as follows.

$$C = 1 + \binom{n}{1}\cos 2\theta + \binom{n}{2}\cos 4\theta + \ldots + \cos 2n\theta$$

$$S = \binom{n}{1}\sin 2\theta + \binom{n}{2}\sin 4\theta + \ldots + \sin 2n\theta$$

By considering $C + jS$, show that

$$C = 2^n\cos^n\theta\cos n\theta,$$

and find a corresponding expression for S. **[7]**

(b) **(i)** Express $e^{j2\pi/3}$ in the form $x + jy$, where the real numbers x and y should be given exactly. **[1]**

(ii) An equilateral triangle in the Argand diagram has its centre at the origin. One vertex of the triangle is at the point representing $2 + 4j$. Obtain the complex numbers representing the other two vertices, giving your answers in the form $x + jy$, where the real numbers x and y should be given exactly. **[6]**

(iii) Show that the length of a side of the triangle is $2\sqrt{15}$. **[2]**

3 You are given the matrix $\mathbf{M} = \begin{pmatrix} 1 & 3 & 0 \\ 3 & -2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$.

(i) Show that the characteristic equation of $\mathbf{M}$ is

$$\lambda^3 - 13\lambda + 12 = 0.$$ **[3]**

(ii) Find the eigenvalues and corresponding eigenvectors of $\mathbf{M}$. **[12]**

(iii) Write down a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that

$$\mathbf{M}^n = \mathbf{PDP}^{-1}.$$

(You are not required to calculate $\mathbf{P}^{-1}$.) **[3]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Show that the curve with equation

$$y = 3\sinh x - 2\cosh x$$

has no turning points.

Show that the curve crosses the x-axis at $x = \frac{1}{2}\ln 5$. Show that this is also the point at which the gradient of the curve has a stationary value. **[7]**

(ii) Sketch the curve. **[2]**

(iii) Express $(3\sinh x - 2\cosh x)^2$ in terms of $\sinh 2x$ and $\cosh 2x$.

Hence or otherwise, show that the volume of the solid of revolution formed by rotating the region bounded by the curve and the axes through 360° about the x-axis is

$$\pi\left(3 - \tfrac{5}{4}\ln 5\right).$$ **[9]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 This question concerns the curves with polar equation

$$r = \sec\theta + a\cos\theta, \qquad (*)$$

where a is a constant which may take any real value, and $0 \leqslant \theta \leqslant 2\pi$.

(i) On a single diagram, sketch the curves for $a = 0$, $a = 1$, $a = 2$. **[3]**

(ii) On a single diagram, sketch the curves for $a = 0$, $a = -1$, $a = -2$. **[2]**

(iii) Identify a feature that the curves for $a = 1$, $a = 2$, $a = -1$, $a = -2$ share. **[1]**

(iv) Name a distinctive feature of the curve for $a = -1$, and a different distinctive feature of the curve for $a = -2$. **[2]**

(v) Show that, in cartesian coordinates, equation (*) may be written

$$y^2 = \frac{ax^2}{x-1} - x^2.$$

Hence comment further on the feature you identified in part **(iii)**. **[5]**

(vi) Show algebraically that, when $a > 0$, the curve exists for $1 < x < 1 + a$.

Find the set of values of x for which the curve exists when $a < 0$. **[5]**