

Monday 28 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

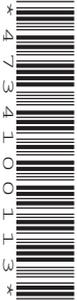
OCR supplied materials:

- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The differential equation

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = \sin x$$

is to be solved.

- (i) Show that 2 is a root of the auxiliary equation. Find the other two roots and hence find the general solution of the differential equation. [10]

When $x = 0$, $y = 1$ and $\frac{dy}{dx} = 0$. Also, y is bounded as $x \rightarrow \infty$.

- (ii) Find the particular solution. [6]

- (iii) Write down an approximate solution for large positive values of x . Calculate the amplitude of this approximate solution and sketch the solution curve for large positive x . [4]

Suppose instead that a solution is required that is bounded as $x \rightarrow -\infty$.

- (iv) Determine whether there is a solution for which $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$. [4]

2 A ball of mass m kg falls vertically from rest through a liquid. At time t s, the velocity of the ball is v m s⁻¹ and the ball has fallen a distance x m. The forces on the ball are its weight and a total upwards force of R N. A student investigates three models for R .

In the first model $R = mkv$, where k is a positive constant.

- (i) Show that $\frac{dv}{dt} = 9.8 - kv$ and hence find v in terms of t and k . [7]

The terminal velocity of the ball is observed to be 7 m s⁻¹.

- (ii) Find k . [1]

In the second model, $R = 0.2mv^2$.

- (iii) Find v in terms of t . Show that your solution is consistent with a terminal velocity of 7 m s⁻¹. [10]

In the third model, $R = 0.529mv^{\frac{3}{2}}$. Euler's method is to be used to solve for v numerically.

The algorithm is given by $t_{r+1} = t_r + h$, $v_{r+1} = v_r + hv_r$ with $(t_0, v_0) = (0, 0)$.

- (iv) Show that $\frac{dv}{dt} = 9.8 - 0.529v^{\frac{3}{2}}$ and find v when $t = 0.2$ using Euler's method with a step length of 0.1 . [5]

- (v) Show that this model is consistent with a terminal velocity of approximately 7 m s⁻¹. [1]

- 3 (a) Solve the differential equation

$$\frac{dy}{dx} - y \tan x = \sin x$$

to find y in terms of x subject to the condition $y = 1$ when $x = 0$.

[9]

- (b) Consider the differential equations

$$\frac{dy}{dx} + f(x)y = g(x), \quad (1)$$

$$\frac{dy}{dx} + f(x)y = 0. \quad (2)$$

Show that if $y = p(x)$ satisfies (1) and $y = c(x)$ satisfies (2), then $y = p(x) + Ac(x)$ satisfies (1), where A is an arbitrary constant. [5]

- (c) The differential equation

$$\frac{dy}{dx} + \frac{2y}{x} = 2e^{x^2} \left(\frac{x^2 + 1}{x} \right) \quad (3)$$

is to be solved.

- (i) Verify that $y = e^{x^2}$ satisfies (3). [3]

- (ii) Find the general solution of $\frac{dy}{dx} + \frac{2y}{x} = 0$, giving y in terms of x . [4]

- (iii) Use the result of part (b) to find a solution of (3) for which $y = 1$ when $x = 1$. [3]

- 4 The simultaneous differential equations

$$\frac{dx}{dt} = -\frac{1}{2}x - \frac{3}{2}y + t$$

$$\frac{dy}{dt} = \frac{3}{2}x - \frac{1}{2}y + 2t$$

are to be solved.

- (i) Eliminate y to obtain a second order differential equation for x in terms of t . Hence find the general solution for x . [13]

- (ii) Find the corresponding general solution for y . [4]

When $t = 0$, $x = 1$ and $y = 0$.

- (iii) Find the particular solutions. [3]

- (iv) Show that in this case $x + y$ tends to a finite limit as $t \rightarrow \infty$ and state its value. Determine whether $x + y$ is equal to this limit for any values of t . [4]

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.



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