OCR
RECOGNISING ACHIEVEMENT

Friday 18 January 2013 – Afternoon

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 **(i)** You are given that the equation

$$x^4 - 4x + 1 = 0$$

has exactly two real roots. Show that these roots both lie in the interval $[0, 2]$. **[2]**

(ii) Use the Newton-Raphson method to find the larger of these roots correct to 4 decimal places. **[5]**

2 The table shows the first few values of the Fibonacci sequence, F_0, F_1, F_2, … .

F_0	F_1	F_2	F_3	F_4	F_5	F_6
0	1	1	2	3	5	8

Note that $F_{r+1} = F_r + F_{r-1}$ for $r > 0$.

An approximate formula for F_r is as follows.

$$F_r \approx \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^r \qquad (*)$$

(i) Find the absolute and relative errors in (*) when $r = 1$ and when $r = 6$. **[4]**

(ii) Given that the absolute error in (*) decreases in magnitude as r increases, use (*) to find F_{20} and F_{21}. **[3]**

3 The values of a function $f(x)$ are shown in the table.

x	1.0	1.1	1.2
$f(x)$	1.464	1.516	1.569

(i) Use the forward difference method to find two estimates of $f'(1)$.

Comment on the numbers of significant figures in your answers. **[4]**

(ii) In the forward difference method, the error is approximately halved as h is halved. Use this fact to obtain a better estimate of $f'(1)$, explaining your reasoning. **[3]**

4 The table below shows a trapezium rule estimate, T, and two mid-point rule estimates, M, of an integral, I.

h	T	M
1	1.332 375	1.377 495
0.5		1.366 179

(i) Find a further trapezium rule estimate of I. **[2]**

(ii) Find two Simpson's rule estimates of I. **[3]**

(iii) Give the value of I to the accuracy that appears justified. Explain your reasoning. **[2]**

5 The function $g(x)$ is known to be a cubic. Some values of $g(x)$ are given in the table below. The value of $g(3)$ is unknown and it is shown as k.

x	1	2	3	4	5
$g(x)$	–15	–14	k	54	145

(i) Use a difference table to find k. **[4]**

(ii) Extend the difference table to find $g(0)$. **[2]**

(iii) Use linear interpolation to estimate a value of x for which $g(x) = 0$. **[2]**

Section B (36 marks)

6 The following values of a function, $f(x)$, have been obtained experimentally.

x	–1	2	4
$f(x)$	7.5	9.0	2.2

(i) Use Lagrange's method to find a quadratic approximation to $f(x)$.

Hence estimate $f(0)$ and the positive value of x for which $f(x) = 0$. Comment on the likely reliability of these estimates. **[11]**

Now let $I = \int_{-1}^{4} f(x)\,dx$.

(ii) Estimate the value of I using the trapezium rule. You should use all the data in the table. **[2]**

(iii) Explain why it is not possible to use Simpson's rule on the data in the table.

Find a suitable value of $f(x)$ and hence obtain an estimate of I using Simpson's rule. **[5]**

[Question 7 is printed overleaf.]

7 **(i)** Sketch, on the same axes, the graphs of $y = \frac{1}{x}$ and $y = 1 + \sin x$ for $0 < x < 2\pi$, where x is in radians.

Hence show that the equation

$$\frac{1}{x} = 1 + \sin x$$

has three roots in the interval $[0, 2\pi]$. **[4]**

These roots are denoted by α, β, γ, where $\alpha < \beta < \gamma$.

(ii) Use the iterative formula $x_{r+1} = \frac{1}{1 + \sin x_r}$ to find α correct to 3 decimal places. **[3]**

(iii) Show that $3.9 < \beta < 4.1$.

Show that the iterative formula used in part **(ii)** does not converge to β.

Use the bisection method to find an estimate of β with maximum possible error 0.025. **[7]**

(iv) Use the secant method with $x_0 = 5.2$ and $x_1 = 5.4$ to find γ correct to 3 significant figures. **[4]**