

OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE
4753/01

MATHEMATICS (MEI)
Methods for Advanced
Mathematics (C3)

WEDNESDAY 23 JANUARY 2013: Morning
DURATION: 1 hour 30 minutes
plus your additional time allowance
MODIFIED ENLARGED 18pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4753/01
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- **The Question Paper will be found in the centre of the Printed Answer Book.**
- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **Answer ALL the questions.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Final answers should be given to a degree of accuracy appropriate to the context.**

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- **The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.**
- **You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.**
- **The total number of marks for this paper is 72.**
- **The Printed Answer Book consists of 16 pages. Any blank pages are indicated.**

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

SECTION A (36 marks)

- 1 (i) Given that $y = e^{-x} \sin 2x$, find $\frac{dy}{dx}$. [3]
- (ii) Hence show that the curve $y = e^{-x} \sin 2x$ has a stationary point when $x = \frac{1}{2} \arctan 2$. [3]
- 2 A curve has equation $x^2 + 2y^2 = 4x$.
- (i) By differentiating implicitly, find $\frac{dy}{dx}$ in terms of x and y . [3]
- (ii) Hence find the exact coordinates of the stationary points of the curve. [You need not determine their nature.] [3]
- 3 Express $1 < x < 3$ in the form $|x - a| < b$, where a and b are to be determined. [2]
- 4 The temperature $\theta^\circ\text{C}$ of water in a container after t minutes is modelled by the equation
- $$\theta = a - be^{-kt},$$
- where a , b and k are positive constants.
- The initial and long-term temperatures of the water are 15°C and 100°C respectively. After 1 minute, the temperature is 30°C .
- (i) Find a , b and k . [6]
- (ii) Find how long it takes for the temperature to reach 80°C . [2]

5 The driving force F newtons and velocity v km s⁻¹ of a car at time t seconds are related by the equation $F = \frac{25}{v}$.

(i) Find $\frac{dF}{dv}$. [2]

(ii) Find $\frac{dF}{dt}$ when $v = 50$ and $\frac{dv}{dt} = 1.5$. [3]

6 Evaluate $\int_0^3 x(x+1)^{-\frac{1}{2}} dx$, giving your answer as an exact fraction. [5]

7 (i) Disprove the following statement:

$3^n + 2$ is prime for all integers $n \geq 0$. [2]

(ii) Prove that no number of the form 3^n (where n is a positive integer) has 5 as its final digit. [2]

SECTION B (36 marks)

- 8 Fig. 8 below shows parts of the curves $y = f(x)$ and $y = g(x)$, where $f(x) = \tan x$ and $g(x) = 1 + f(x - \frac{1}{4}\pi)$.

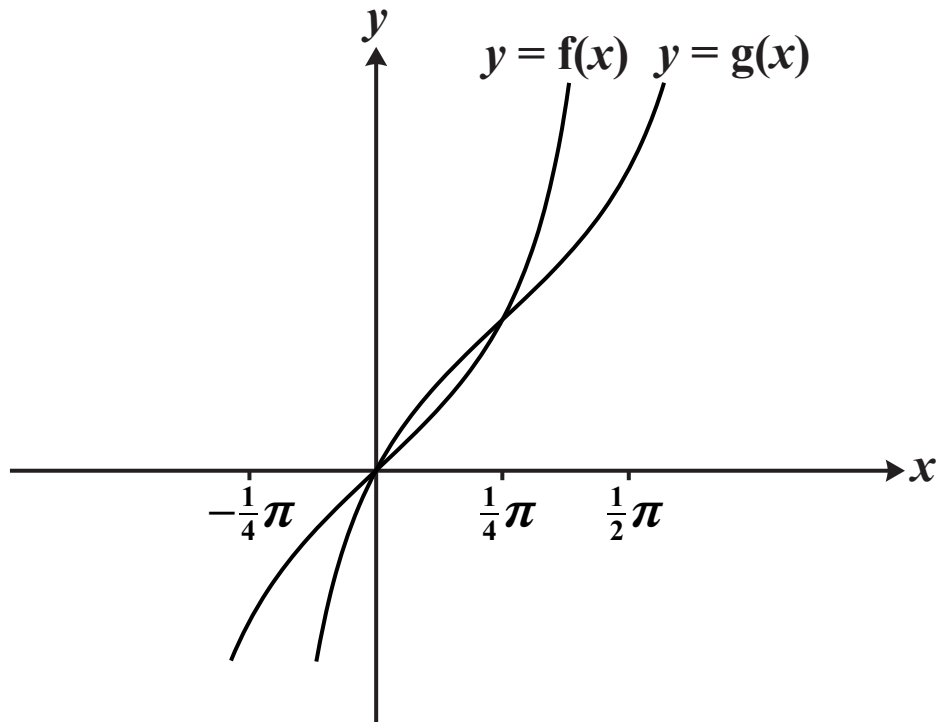


Fig. 8

- (i) Describe a sequence of two transformations which maps the curve $y = f(x)$ to the curve $y = g(x)$. [4]

It can be shown that $g(x) = \frac{2 \sin x}{\sin x + \cos x}$.

- (ii) Show that $g'(x) = \frac{2}{(\sin x + \cos x)^2}$. Hence verify that the gradient of $y = g(x)$ at the point $(\frac{1}{4}\pi, 1)$ is the same as that of $y = f(x)$ at the origin. [7]

(iii) By writing $\tan x = \frac{\sin x}{\cos x}$ and using the substitution

$$u = \cos x, \text{ show that } \int_0^{\frac{1}{4}\pi} f(x) dx = \int_{\frac{1}{\sqrt{2}}}^1 \frac{1}{u} du.$$

Evaluate this integral exactly. [4]

(iv) Hence find the exact area of the region enclosed by the curve $y = g(x)$, the x -axis and the lines $x = \frac{1}{4}\pi$ and $x = \frac{1}{2}\pi$. [2]

- 9 Fig. 9 below shows the line $y = x$ and the curve $y = f(x)$, where $f(x) = \frac{1}{2}(e^x - 1)$. The line and the curve intersect at the origin and at the point $P(a, a)$.

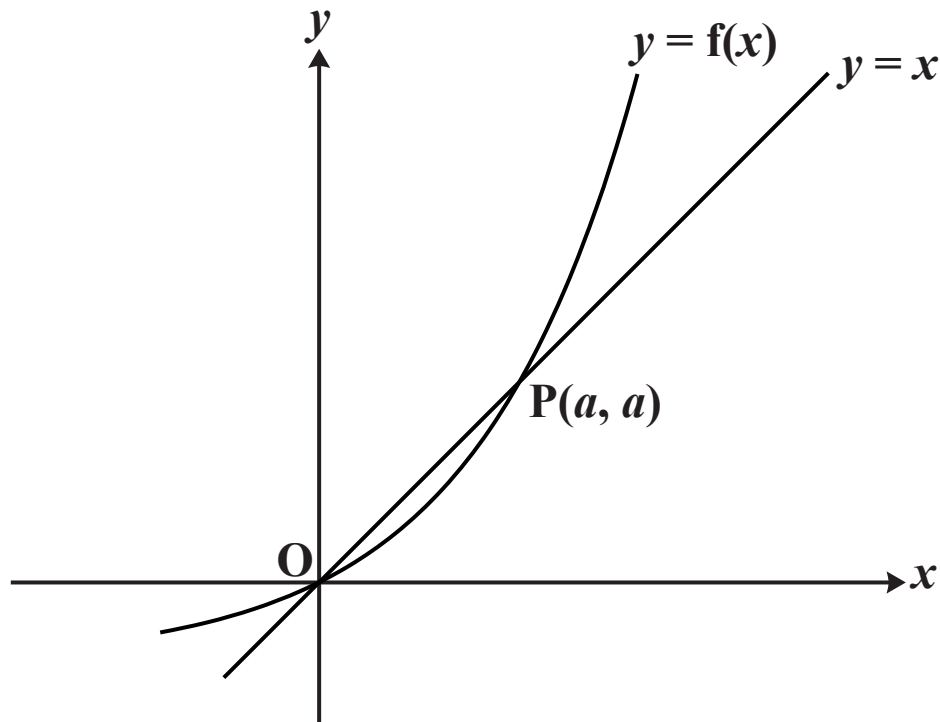


Fig. 9

- (i) Show that $e^a = 1 + 2a$. [1]
- (ii) Show that the area of the region enclosed by the curve, the x -axis and the line $x = a$ is $\frac{1}{2}a$. Hence find, in terms of a , the area enclosed by the curve and the line $y = x$. [6]
- (iii) Show that the inverse function of $f(x)$ is $g(x)$, where $g(x) = \ln(1 + 2x)$. Add a sketch of $y = g(x)$ to the copy of Fig. 9. [5]
- (iv) Find the derivatives of $f(x)$ and $g(x)$. Hence verify that $g'(a) = \frac{1}{f'(a)}$.

Give a geometrical interpretation of this result. [7]

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