



Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER



Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

- 1 Fig. 1 shows the graphs of $y = |x|$ and $y = a|x + b|$, where a and b are constants. The intercepts of $y = a|x + b|$ with the x - and y -axes are $(-1, 0)$ and $(0, \frac{1}{2})$ respectively.

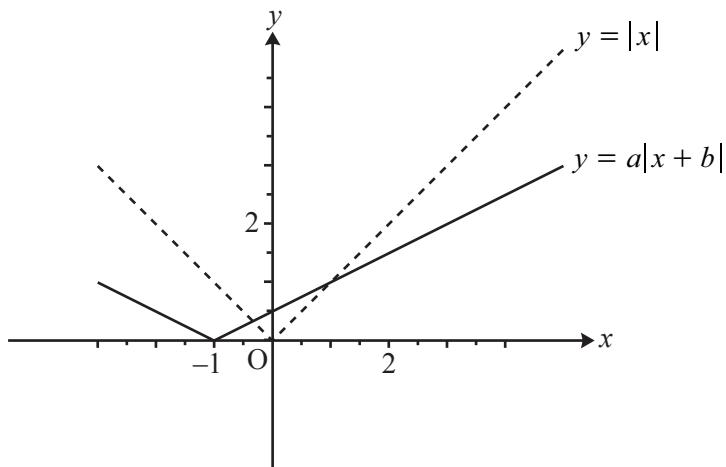


Fig. 1

- (i) Find a and b . [2]
- (ii) Find the coordinates of the two points of intersection of the graphs. [4]
- 2 (i) Factorise fully $n^3 - n$. [2]
- (ii) Hence prove that, if n is an integer, $n^3 - n$ is divisible by 6. [2]

- 3 The function $f(x)$ is defined by $f(x) = 1 - 2 \sin x$ for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$. Fig. 3 shows the curve $y = f(x)$.

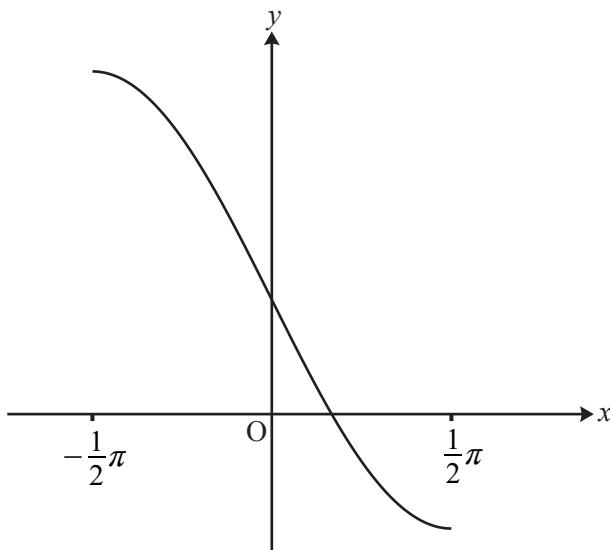


Fig. 3

- (i) Write down the range of the function $f(x)$. [2]
- (ii) Find the inverse function $f^{-1}(x)$. [3]
- (iii) Find $f'(0)$. Hence write down the gradient of $y = f^{-1}(x)$ at the point $(1, 0)$. [3]
- 4 Water flows into a bowl at a constant rate of $10 \text{ cm}^3 \text{ s}^{-1}$ (see Fig. 4).

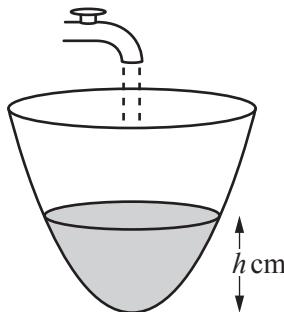


Fig. 4

When the depth of water in the bowl is $h \text{ cm}$, the volume of water is $V \text{ cm}^3$, where $V = \pi h^2$. Find the rate at which the depth is increasing at the instant in time when the depth is 5 cm. [5]

- 5 Given that $y = \ln\left(\sqrt{\frac{2x-1}{2x+1}}\right)$, show that $\frac{dy}{dx} = \frac{1}{2x-1} - \frac{1}{2x+1}$. [4]
- 6 Using a suitable substitution or otherwise, show that $\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{3 + \cos 2x} dx = \frac{1}{2} \ln 2$. [5]

- 7 (i) Show algebraically that the function $f(x) = \frac{2x}{1-x^2}$ is odd. [2]

Fig. 7 shows the curve $y = f(x)$ for $0 \leq x \leq 4$, together with the asymptote $x = 1$.

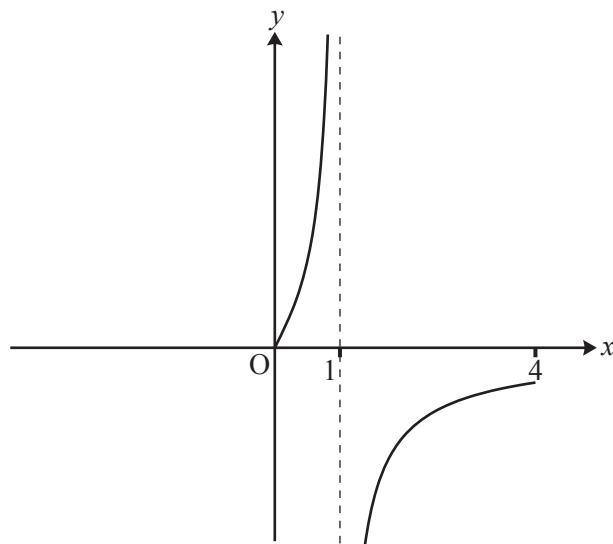


Fig. 7

- (ii) Use the copy of Fig. 7 to complete the curve for $-4 \leq x \leq 4$. [2]

Section B (36 marks)

- 8 Fig. 8 shows the curve $y = f(x)$, where $f(x) = (1 - x)e^{2x}$, with its turning point P.

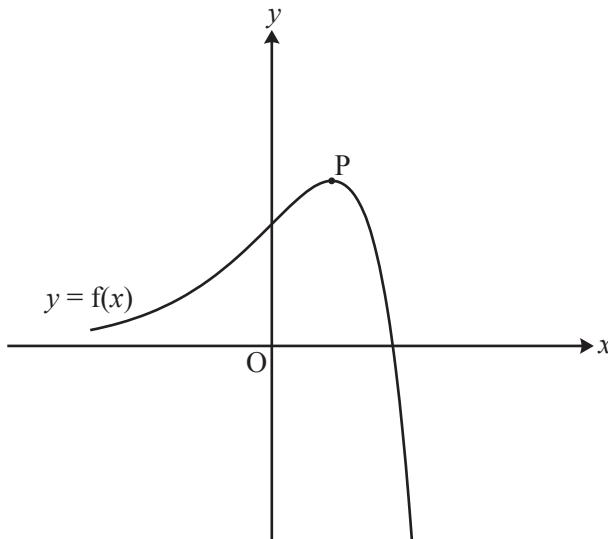


Fig. 8

- (i) Write down the coordinates of the intercepts of $y = f(x)$ with the x - and y -axes. [2]

- (ii) Find the exact coordinates of the turning point P. [6]

- (iii) Show that the exact area of the region enclosed by the curve and the x - and y -axes is $\frac{1}{4}(e^2 - 3)$. [5]

The function $g(x)$ is defined by $g(x) = 3f\left(\frac{1}{2}x\right)$.

- (iv) Express $g(x)$ in terms of x .

Sketch the curve $y = g(x)$ on the copy of Fig. 8, indicating the coordinates of its intercepts with the x - and y -axes and of its turning point. [4]

- (v) Write down the exact area of the region enclosed by the curve $y = g(x)$ and the x - and y -axes. [1]

- 9 Fig. 9 shows the curve with equation $y^3 = \frac{x^3}{2x-1}$. It has an asymptote $x = a$ and turning point P.

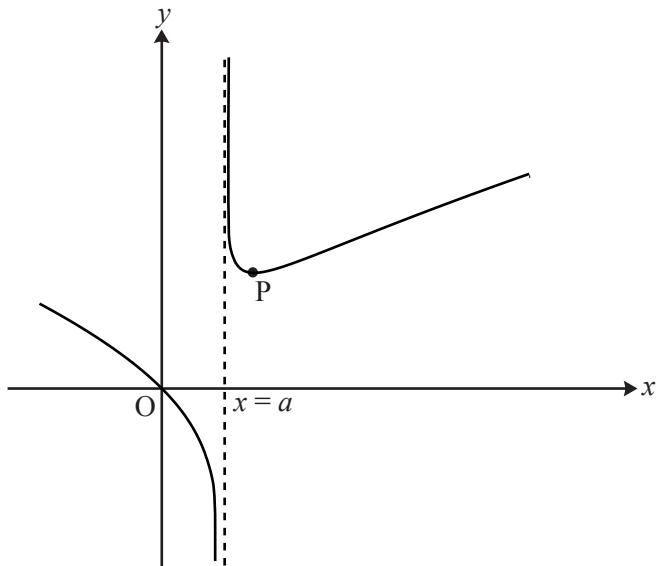


Fig. 9

- (i) Write down the value of a .

[1]

- (ii) Show that $\frac{dy}{dx} = \frac{4x^3 - 3x^2}{3y^2(2x-1)^2}$.

Hence find the coordinates of the turning point P, giving the y -coordinate to 3 significant figures. [9]

- (iii) Show that the substitution $u = 2x - 1$ transforms $\int \frac{x}{3\sqrt[3]{2x-1}} dx$ to $\frac{1}{4} \int (u^{\frac{2}{3}} + u^{-\frac{1}{3}}) du$.

Hence find the exact area of the region enclosed by the curve $y^3 = \frac{x^3}{2x-1}$, the x -axis and the lines $x = 1$ and $x = 4.5$. [8]

BLANK PAGE



RECOGNISING ACHIEVEMENT

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.