

**Tuesday 18 June 2013 – Morning**

**A2 GCE MATHEMATICS (MEI)**

**4753/01** Methods for Advanced Mathematics (C3)

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

## Section A (36 marks)

- 1 Fig. 1 shows the graphs of  $y = |x|$  and  $y = a|x + b|$ , where  $a$  and  $b$  are constants. The intercepts of  $y = a|x + b|$  with the  $x$ - and  $y$ -axes are  $(-1, 0)$  and  $(0, \frac{1}{2})$  respectively.

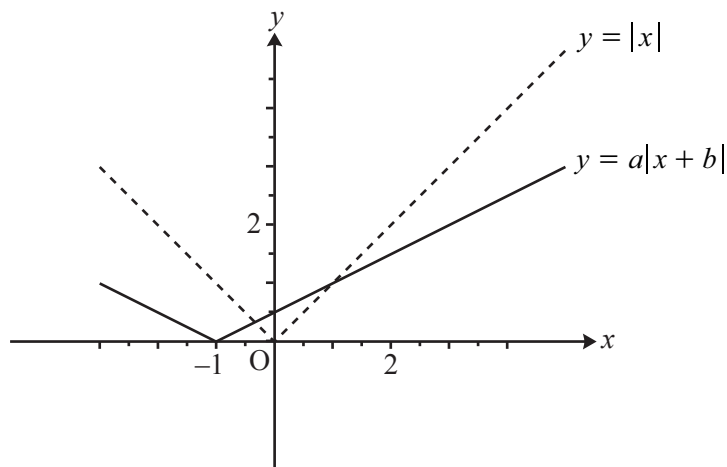


Fig. 1

- (i) Find  $a$  and  $b$ . [2]
- (ii) Find the coordinates of the two points of intersection of the graphs. [4]
- 2 (i) Factorise fully  $n^3 - n$ . [2]
- (ii) Hence prove that, if  $n$  is an integer,  $n^3 - n$  is divisible by 6. [2]

- 3 The function  $f(x)$  is defined by  $f(x) = 1 - 2 \sin x$  for  $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$ . Fig. 3 shows the curve  $y = f(x)$ .

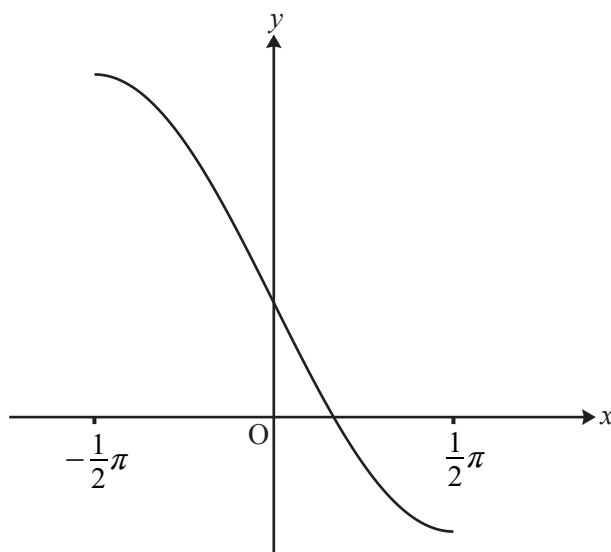


Fig. 3

- (i) Write down the range of the function  $f(x)$ . [2]
- (ii) Find the inverse function  $f^{-1}(x)$ . [3]
- (iii) Find  $f'(0)$ . Hence write down the gradient of  $y = f^{-1}(x)$  at the point  $(1, 0)$ . [3]
- 4 Water flows into a bowl at a constant rate of  $10 \text{ cm}^3 \text{ s}^{-1}$  (see Fig. 4).

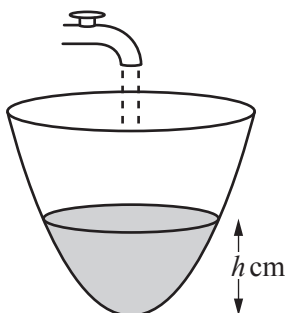


Fig. 4

When the depth of water in the bowl is  $h$  cm, the volume of water is  $V \text{ cm}^3$ , where  $V = \pi h^2$ . Find the rate at which the depth is increasing at the instant in time when the depth is 5 cm. [5]

- 5 Given that  $y = \ln\left(\sqrt{\frac{2x-1}{2x+1}}\right)$ , show that  $\frac{dy}{dx} = \frac{1}{2x-1} - \frac{1}{2x+1}$ . [4]
- 6 Using a suitable substitution or otherwise, show that  $\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{3 + \cos 2x} dx = \frac{1}{2} \ln 2$ . [5]

- 7 (i) Show algebraically that the function  $f(x) = \frac{2x}{1-x^2}$  is odd. [2]

Fig. 7 shows the curve  $y = f(x)$  for  $0 \leq x \leq 4$ , together with the asymptote  $x = 1$ .

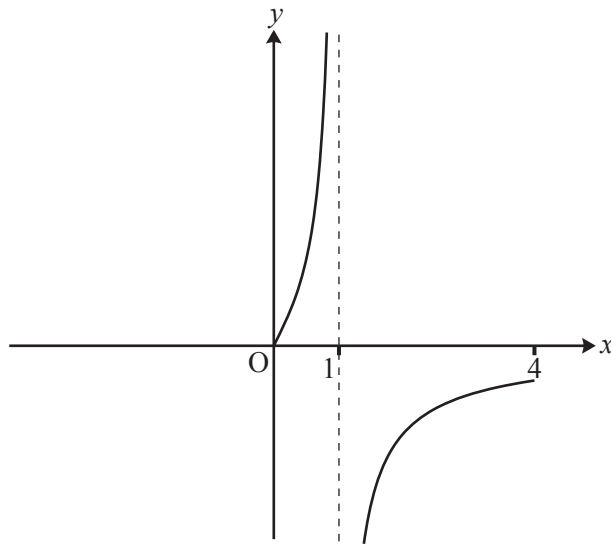


Fig. 7

- (ii) Use the copy of Fig. 7 to complete the curve for  $-4 \leq x \leq 4$ . [2]

## Section B (36 marks)

- 8 Fig. 8 shows the curve  $y = f(x)$ , where  $f(x) = (1 - x)e^{2x}$ , with its turning point P.

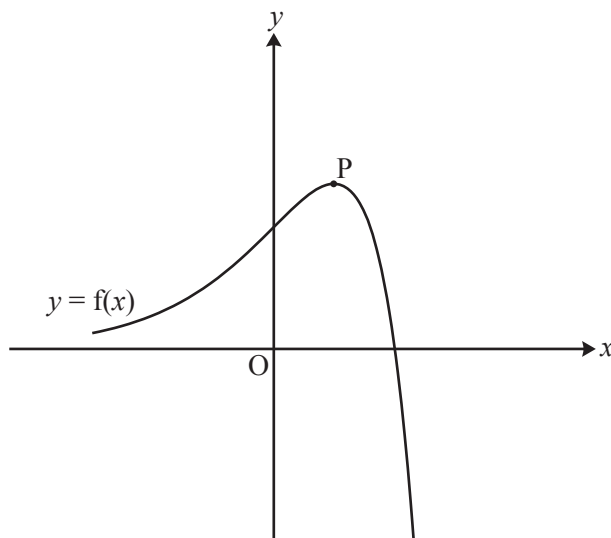


Fig. 8

- (i) Write down the coordinates of the intercepts of  $y = f(x)$  with the  $x$ - and  $y$ -axes. [2]
- (ii) Find the exact coordinates of the turning point P. [6]
- (iii) Show that the exact area of the region enclosed by the curve and the  $x$ - and  $y$ -axes is  $\frac{1}{4}(e^2 - 3)$ . [5]

The function  $g(x)$  is defined by  $g(x) = 3f\left(\frac{1}{2}x\right)$ .

- (iv) Express  $g(x)$  in terms of  $x$ .

Sketch the curve  $y = g(x)$  on the copy of Fig. 8, indicating the coordinates of its intercepts with the  $x$ - and  $y$ -axes and of its turning point. [4]

- (v) Write down the exact area of the region enclosed by the curve  $y = g(x)$  and the  $x$ - and  $y$ -axes. [1]

- 9 Fig. 9 shows the curve with equation  $y^3 = \frac{x^3}{2x-1}$ . It has an asymptote  $x = a$  and turning point P.

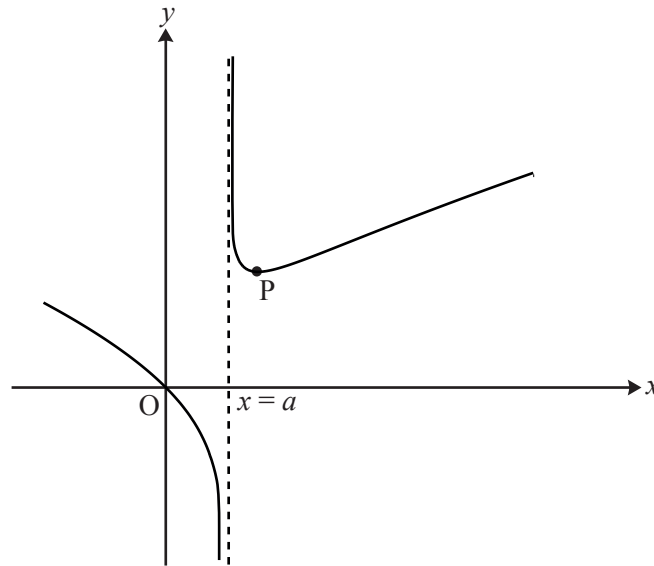


Fig. 9

- (i) Write down the value of  $a$ . [1]

(ii) Show that  $\frac{dy}{dx} = \frac{4x^3 - 3x^2}{3y^2(2x-1)^2}$ .

Hence find the coordinates of the turning point P, giving the  $y$ -coordinate to 3 significant figures. [9]

- (iii) Show that the substitution  $u = 2x - 1$  transforms  $\int \frac{x}{\sqrt[3]{2x-1}} dx$  to  $\frac{1}{4} \int (u^{\frac{2}{3}} + u^{-\frac{1}{3}}) du$ .

Hence find the exact area of the region enclosed by the curve  $y^3 = \frac{x^3}{2x-1}$ , the  $x$ -axis and the lines  $x = 1$  and  $x = 4.5$ . [8]

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