OCR
RECOGNISING ACHIEVEMENT

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Fig. 1 shows the graphs of $y = |x|$ and $y = a|x + b|$, where a and b are constants. The intercepts of $y = a|x + b|$ with the x- and y-axes are $(-1, 0)$ and $\left(0, \frac{1}{2}\right)$ respectively.

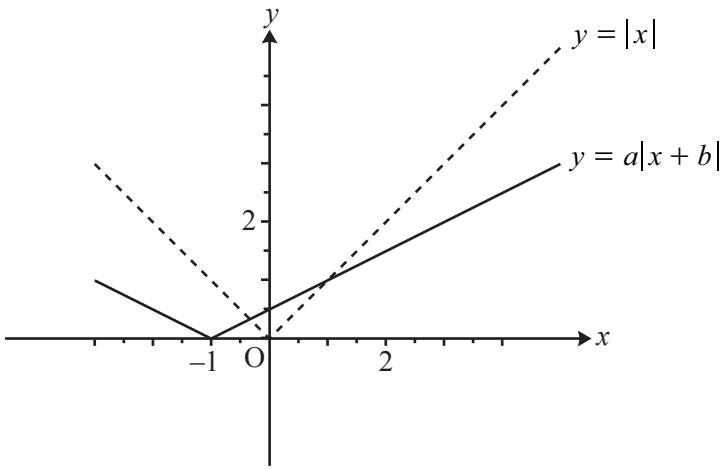

Fig. 1

(i) Find a and b. **[2]**

(ii) Find the coordinates of the two points of intersection of the graphs. **[4]**

2 **(i)** Factorise fully $n^3 - n$. **[2]**

(ii) Hence prove that, if n is an integer, $n^3 - n$ is divisible by 6. **[2]**

3 The function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 1 - 2\sin x$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$. Fig. 3 shows the curve $y = \mathrm{f}(x)$.

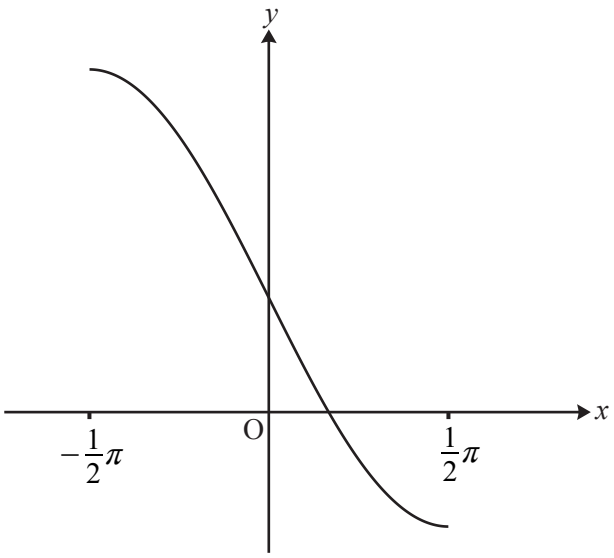

Fig. 3

(i) Write down the range of the function $\mathrm{f}(x)$. **[2]**

(ii) Find the inverse function $\mathrm{f}^{-1}(x)$. **[3]**

(iii) Find $\mathrm{f}'(0)$. Hence write down the gradient of $y = \mathrm{f}^{-1}(x)$ at the point $(1, 0)$. **[3]**

4 Water flows into a bowl at a constant rate of $10\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ (see Fig. 4).

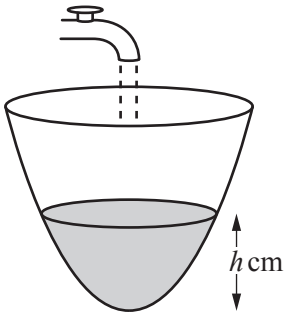

Fig. 4

When the depth of water in the bowl is h cm, the volume of water is $V\,\mathrm{cm}^3$, where $V = \pi h^2$. Find the rate at which the depth is increasing at the instant in time when the depth is 5 cm. **[5]**

5 Given that $y = \ln\left(\sqrt{\dfrac{2x-1}{2x+1}}\right)$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2x-1} - \dfrac{1}{2x+1}$. **[4]**

6 Using a suitable substitution or otherwise, show that $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{3 + \cos 2x}\,\mathrm{d}x = \frac{1}{2}\ln 2$. **[5]**

7 **(i)** Show algebraically that the function $f(x) = \dfrac{2x}{1 - x^2}$ is odd. **[2]**

Fig. 7 shows the curve $y = f(x)$ for $0 \leqslant x \leqslant 4$, together with the asymptote $x = 1$.

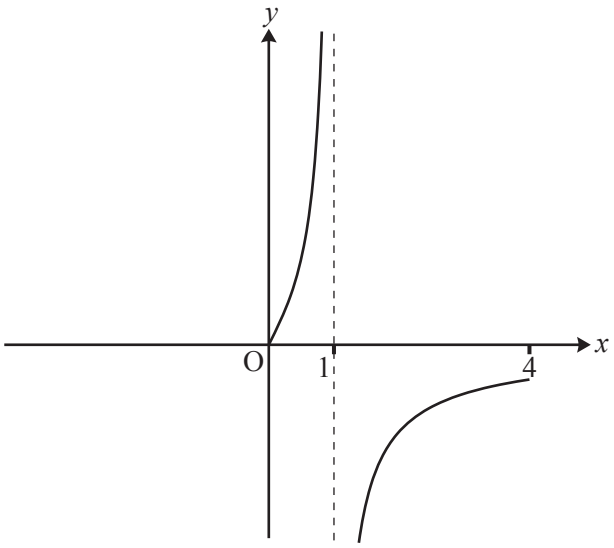

Fig. 7

(ii) Use the copy of Fig. 7 to complete the curve for $-4 \leqslant x \leqslant 4$. **[2]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (1 - x)\mathrm{e}^{2x}$, with its turning point P.

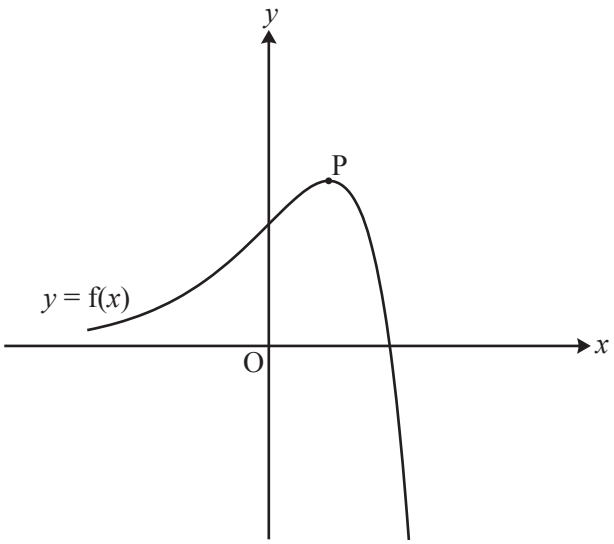

Fig. 8

(i) Write down the coordinates of the intercepts of $y = \mathrm{f}(x)$ with the x- and y-axes. **[2]**

(ii) Find the exact coordinates of the turning point P. **[6]**

(iii) Show that the exact area of the region enclosed by the curve and the x- and y-axes is $\frac{1}{4}(\mathrm{e}^2 - 3)$. **[5]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = 3\mathrm{f}\left(\frac{1}{2}x\right)$.

(iv) Express $\mathrm{g}(x)$ in terms of x.

Sketch the curve $y = \mathrm{g}(x)$ on the copy of Fig. 8, indicating the coordinates of its intercepts with the x- and y-axes and of its turning point. **[4]**

(v) Write down the exact area of the region enclosed by the curve $y = \mathrm{g}(x)$ and the x- and y-axes. **[1]**

9 Fig. 9 shows the curve with equation $y^3 = \dfrac{x^3}{2x-1}$. It has an asymptote $x = a$ and turning point P.

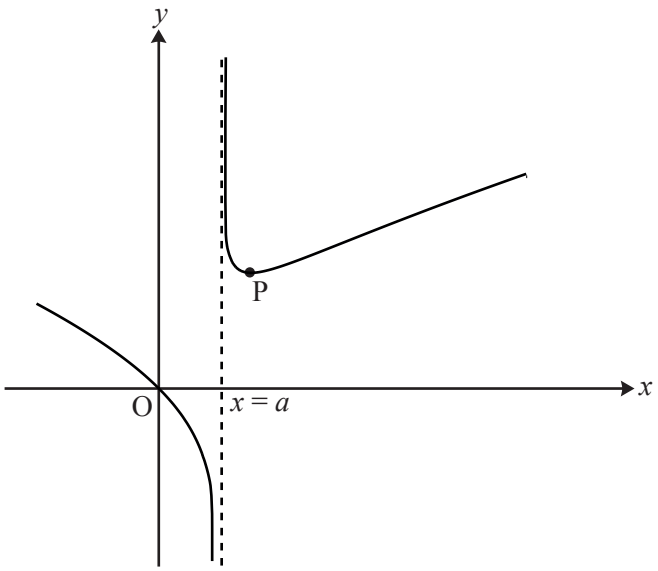

Fig. 9

(i) Write down the value of a. **[1]**

(ii) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4x^3 - 3x^2}{3y^2(2x-1)^2}$.

Hence find the coordinates of the turning point P, giving the y-coordinate to 3 significant figures. **[9]**

(iii) Show that the substitution $u = 2x - 1$ transforms $\displaystyle\int \frac{x}{\sqrt[3]{2x-1}}\mathrm{d}x$ to $\displaystyle\frac{1}{4}\int \left(u^{\frac{2}{3}} + u^{-\frac{1}{3}}\right)\mathrm{d}u$.

Hence find the exact area of the region enclosed by the curve $y^3 = \dfrac{x^3}{2x-1}$, the x-axis and the lines $x = 1$ and $x = 4.5$. **[8]**

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