OCR
RECOGNISING ACHIEVEMENT

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

1 **(a)** You are given that $f(x) = \dfrac{1}{(1-2x)^2}$.

Find $f'(x)$, $f''(x)$ and $f'''(x)$. Hence obtain the Maclaurin series for $f(x)$ as far as the term in x^3.

By considering the equivalent binomial expansion, give the set of values of x for which the Maclaurin series is valid. **[7]**

(b) A curve has polar equation $r = a\sin 3\theta$, where a is a positive constant and $0 \leqslant \theta \leqslant \frac{1}{3}\pi$.

(i) Sketch the curve. **[2]**

(ii) Find, in terms of a, the cartesian coordinates of the point on the curve furthest from the origin. **[4]**

(iii) Find, in terms of a, the area of the region enclosed by the curve. **[5]**

2 **(a)** **(i)** Use de Moivre's theorem to show that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$ **[3]**

(ii) Given that $\cos 5\theta = 0$ but $\cos\theta \neq 0$, find in surd form the two possible values for $\cos^2\theta$.

Hence show that $\cos 18° = \left(\dfrac{5+\sqrt{5}}{8}\right)^{\frac{1}{2}}$.

Find, in similar form, an expression for $\sin 18°$. **[6]**

(b) **(i)** Find the cube roots of the complex number $4(\sqrt{3} + \mathrm{j})$ in the form $re^{\mathrm{j}\theta}$, where $r > 0$ and $0 < \theta < 2\pi$. Illustrate the roots on an Argand diagram. **[7]**

The points representing the two roots with smallest values of θ are P and Q. The mid-point of PQ is M, and M represents the complex number w.

(ii) Find the argument of w. Write down the smallest positive integer n for which w^n is a real number. **[2]**

3 You are given the matrix $\mathbf{A} = \begin{pmatrix} k & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix}$.

(i) Show that when $k = 5$ the determinant of **A** is zero. Obtain an expression for the inverse of **A** when $k \neq 5$. **[7]**

(ii) Solve the matrix equation

$$\begin{pmatrix} 4 & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ 1 \\ 2 \end{pmatrix},$$

giving your answer in terms of p. **[5]**

(iii) Find the value of p for which the matrix equation

$$\begin{pmatrix} 5 & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ 1 \\ 2 \end{pmatrix}$$

has a solution. Give the general solution in this case and describe it geometrically. **[6]**

Section B (18 marks)

4 **(i)** Prove, using exponential functions, that $\cosh^2 u - \sinh^2 u = 1$. **[2]**

(ii) Given that $y = \operatorname{arsinh} x$, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{1 + x^2}},$$

and that

$$y = \ln\left(x + \sqrt{1 + x^2}\right).$$ **[9]**

(iii) Show that

$$\int_0^2 \frac{1}{\sqrt{4 + 9x^2}} \mathrm{d}x = \tfrac{1}{3}\ln\left(3 + \sqrt{10}\right).$$ **[4]**

(iv) Find, in exact logarithmic form,

$$\int_0^1 \frac{1}{\sqrt{1 + x^2}} \operatorname{arsinh} x \, \mathrm{d}x.$$ **[3]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.