

**Tuesday 18 June 2013 – Morning**

**A2 GCE MATHEMATICS (MEI)**

**4756/01** Further Methods for Advanced Mathematics (FP2)

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

## Section A (54 marks)

- 1 (a) You are given that  $f(x) = \frac{1}{(1-2x)^2}$ .

Find  $f'(x)$ ,  $f''(x)$  and  $f'''(x)$ . Hence obtain the Maclaurin series for  $f(x)$  as far as the term in  $x^3$ .

By considering the equivalent binomial expansion, give the set of values of  $x$  for which the Maclaurin series is valid. [7]

- (b) A curve has polar equation  $r = a \sin 3\theta$ , where  $a$  is a positive constant and  $0 \leq \theta \leq \frac{1}{3}\pi$ .

(i) Sketch the curve. [2]

(ii) Find, in terms of  $a$ , the cartesian coordinates of the point on the curve furthest from the origin. [4]

(iii) Find, in terms of  $a$ , the area of the region enclosed by the curve. [5]

- 2 (a) (i) Use de Moivre's theorem to show that

$$\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta. \quad [3]$$

(ii) Given that  $\cos 5\theta = 0$  but  $\cos \theta \neq 0$ , find in surd form the two possible values for  $\cos^2 \theta$ .

$$\text{Hence show that } \cos 18^\circ = \left( \frac{5 + \sqrt{5}}{8} \right)^{\frac{1}{2}}.$$

Find, in similar form, an expression for  $\sin 18^\circ$ . [6]

- (b) (i) Find the cube roots of the complex number  $4(\sqrt{3} + j)$  in the form  $re^{j\theta}$ , where  $r > 0$  and  $0 < \theta < 2\pi$ . Illustrate the roots on an Argand diagram. [7]

The points representing the two roots with smallest values of  $\theta$  are P and Q. The mid-point of PQ is M, and M represents the complex number  $w$ .

(ii) Find the argument of  $w$ . Write down the smallest positive integer  $n$  for which  $w^n$  is a real number. [2]

3 You are given the matrix  $\mathbf{A} = \begin{pmatrix} k & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix}$ .

(i) Show that when  $k = 5$  the determinant of  $\mathbf{A}$  is zero. Obtain an expression for the inverse of  $\mathbf{A}$  when  $k \neq 5$ . [7]

(ii) Solve the matrix equation

$$\begin{pmatrix} 4 & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ 1 \\ 2 \end{pmatrix},$$

giving your answer in terms of  $p$ .

[5]

(iii) Find the value of  $p$  for which the matrix equation

$$\begin{pmatrix} 5 & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ 1 \\ 2 \end{pmatrix}$$

has a solution. Give the general solution in this case and describe it geometrically.

[6]

### Section B (18 marks)

4 (i) Prove, using exponential functions, that  $\cosh^2 u - \sinh^2 u = 1$ . [2]

(ii) Given that  $y = \operatorname{arsinh} x$ , show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}},$$

and that

$$y = \ln(x + \sqrt{1+x^2}). \quad [9]$$

(iii) Show that

$$\int_0^2 \frac{1}{\sqrt{4+9x^2}} dx = \frac{1}{3} \ln(3 + \sqrt{10}). \quad [4]$$

(iv) Find, in exact logarithmic form,

$$\int_0^1 \frac{1}{\sqrt{1+x^2}} \operatorname{arsinh} x \, dx. \quad [3]$$

**THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.**



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