

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER



Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

1 (a) You are given that $f(x) = \frac{1}{(1-2x)^2}$.

Find $f'(x)$, $f''(x)$ and $f'''(x)$. Hence obtain the Maclaurin series for $f(x)$ as far as the term in x^3 .

By considering the equivalent binomial expansion, give the set of values of x for which the Maclaurin series is valid. [7]

(b) A curve has polar equation $r = a \sin 3\theta$, where a is a positive constant and $0 \leq \theta \leq \frac{1}{3}\pi$.

(i) Sketch the curve. [2]

(ii) Find, in terms of a , the cartesian coordinates of the point on the curve furthest from the origin. [4]

(iii) Find, in terms of a , the area of the region enclosed by the curve. [5]

2 (a) (i) Use de Moivre's theorem to show that

$$\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta. \quad [3]$$

(ii) Given that $\cos 5\theta = 0$ but $\cos \theta \neq 0$, find in surd form the two possible values for $\cos^2 \theta$.

$$\text{Hence show that } \cos 18^\circ = \left(\frac{5 + \sqrt{5}}{8}\right)^{\frac{1}{2}}.$$

Find, in similar form, an expression for $\sin 18^\circ$. [6]

(b) (i) Find the cube roots of the complex number $4(\sqrt{3} + j)$ in the form $r e^{j\theta}$, where $r > 0$ and $0 < \theta < 2\pi$. Illustrate the roots on an Argand diagram. [7]

The points representing the two roots with smallest values of θ are P and Q. The mid-point of PQ is M, and M represents the complex number w .

(ii) Find the argument of w . Write down the smallest positive integer n for which w^n is a real number. [2]

3 You are given the matrix $\mathbf{A} = \begin{pmatrix} k & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix}$.

(i) Show that when $k = 5$ the determinant of \mathbf{A} is zero. Obtain an expression for the inverse of \mathbf{A} when $k \neq 5$. [7]

(ii) Solve the matrix equation

$$\begin{pmatrix} 4 & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ 1 \\ 2 \end{pmatrix},$$

giving your answer in terms of p . [5]

(iii) Find the value of p for which the matrix equation

$$\begin{pmatrix} 5 & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ 1 \\ 2 \end{pmatrix}$$

has a solution. Give the general solution in this case and describe it geometrically. [6]

Section B (18 marks)

4 (i) Prove, using exponential functions, that $\cosh^2 u - \sinh^2 u = 1$. [2]

(ii) Given that $y = \operatorname{arsinh} x$, show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}},$$

and that

$$y = \ln(x + \sqrt{1+x^2}). \quad [9]$$

(iii) Show that

$$\int_0^2 \frac{1}{\sqrt{4+9x^2}} dx = \frac{1}{3} \ln(3 + \sqrt{10}). \quad [4]$$

(iv) Find, in exact logarithmic form,

$$\int_0^1 \frac{1}{\sqrt{1+x^2}} \operatorname{arsinh} x \, dx. \quad [3]$$

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.



Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1GE.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.