

Thursday 6 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01 Mechanics 3

QUESTION PAPER



Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $gm s^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 (a) A particle P of mass 1.5 kg is connected to a fixed point by a light inextensible string of length 3.2 m. The particle P is moving as a conical pendulum in a horizontal circle at a constant angular speed of 2.5 rad s^{-1} .

(i) Find the tension in the string. [4]

(ii) Find the angle that the string makes with the vertical. [2]

(b) A particle Q of mass m moves on a smooth horizontal surface, and is connected to a fixed point on the surface by a light elastic string of natural length d and stiffness k . With the string at its natural length, Q is set in motion with initial speed u perpendicular to the string. In the subsequent motion, the maximum length of the string is $2d$, and the string first returns to its natural length after time t .

You are given that $u = \sqrt{\frac{4kd^2}{3m}}$ and $t = Ak^\alpha d^\beta m^\gamma$, where A is a dimensionless constant.

(i) Show that the dimensions of k are MT^{-2} . [1]

(ii) Show that the equation $u = \sqrt{\frac{4kd^2}{3m}}$ is dimensionally consistent. [2]

(iii) Find α , β and γ . [4]

You are now given that Q has mass 5 kg, and the string has natural length 0.7 m and stiffness 60 N m^{-1} .

(iv) Find the initial speed u , and use conservation of energy to find the speed of Q at the instant when the length of the string is double its natural length. [5]

2 A particle P of mass 0.25 kg is connected to a fixed point O by a light inextensible string of length a metres, and is moving in a vertical circle with centre O and radius a metres. When P is vertically below O, its speed is 8.4 m s^{-1} . When OP makes an angle θ with the downward vertical, and the string is still taut, P has speed $v \text{ m s}^{-1}$ and the tension in the string is $T \text{ N}$, as shown in Fig. 2.

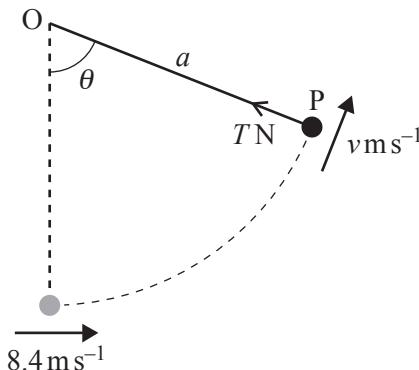


Fig. 2

(i) Find an expression for v^2 in terms of a and θ , and show that

$$T = \frac{17.64}{a} + 7.35 \cos \theta - 4.9. \quad [7]$$

(ii) Given that $a = 0.9$, show that P moves in a complete circle. Find the maximum and minimum magnitudes of the tension in the string. [4]

(iii) Find the largest value of a for which P moves in a complete circle. [3]

(iv) Given that $a = 1.6$, find the speed of P at the instant when the string first becomes slack. [4]

3 A light spring, with modulus of elasticity 686 N, has one end attached to a fixed point A. The other end is attached to a particle P of mass 18 kg which hangs in equilibrium when it is 2.2 m vertically below A.

(i) Find the natural length of the spring AP. [2]

Another light spring has natural length 2.5 m and modulus of elasticity 145 N. One end of this spring is now attached to the particle P, and the other end is attached to a fixed point B which is 2.5 m vertically below P (so leaving the equilibrium position of P unchanged). While in its equilibrium position, P is set in motion with initial velocity 3.4 m s^{-1} vertically downwards, as shown in Fig. 3. It now executes simple harmonic motion along part of the vertical line AB.

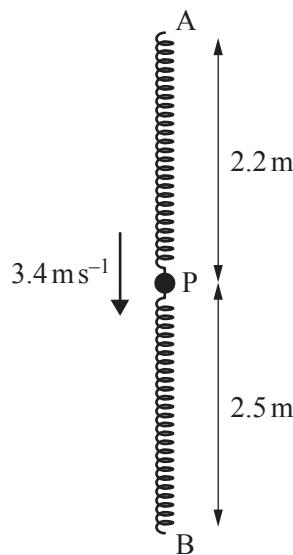


Fig. 3

At time t seconds after it is set in motion, P is x metres below its equilibrium position.

(ii) Show that the tension in the spring AP is $(176.4 + 392x)$ N, and write down an expression for the thrust in the spring BP. [3]

(iii) Show that $\frac{d^2x}{dt^2} = -25x$. [3]

(iv) Find the period and the amplitude of the motion. [3]

(v) Find the magnitude and direction of the velocity of P when $t = 2.4$. [3]

(vi) Find the total distance travelled by P during the first 2.4 seconds of its motion. [4]

4 (a) A uniform solid of revolution S is formed by rotating the region enclosed between the x -axis and the curve $y = x\sqrt{4-x}$ for $0 \leq x \leq 4$ through 2π radians about the x -axis, as shown in Fig. 4.1. O is the origin and the end A of the solid is at the point $(4, 0)$.

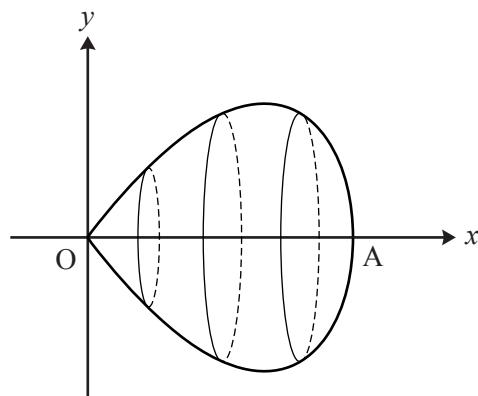


Fig. 4.1

(i) Find the x -coordinate of the centre of mass of the solid S .

[6]

The solid S has weight W , and it is freely hinged to a fixed point at O. A horizontal force, of magnitude W acting in the vertical plane containing OA, is applied at the point A, as shown in Fig. 4.2. S is in equilibrium.

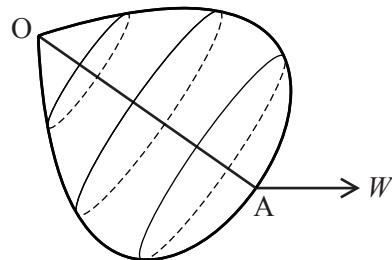


Fig. 4.2

(ii) Find the angle that OA makes with the vertical.

[3]

[Question 4(b) is printed overleaf]

(b) Fig. 4.3 shows the region bounded by the x -axis, the y -axis, the line $y = 8$ and the curve $y = (x - 2)^3$ for $0 \leq y \leq 8$.

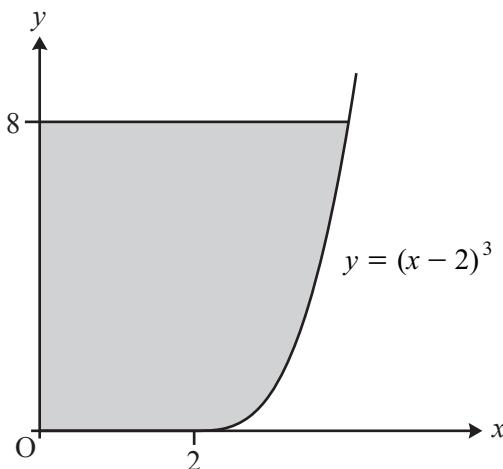


Fig. 4.3

Find the coordinates of the centre of mass of a uniform lamina occupying this region.

[9]

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