Monday 24 June 2013 – Afternoon

A2 GCE MATHEMATICS (MEI)

4773/01 Decision Mathematics Computation

Candidates answer on the Answer Booklet.

OCR supplied materials:
- 12 page Answer Booklet (OCR12) (sent with general stationery)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator
- Computer with appropriate software and printing facilities

Duration: 2 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Answer Booklet. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Additional sheets, including computer print-outs, should be fastened securely to the Answer Booklet.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- In each of the questions you are required to write spreadsheet or other routines to carry out various processes.
- For each question you attempt, you should submit print-outs showing the routine you have written and the output it generates.
- You are not expected to print out and submit everything your routine produces, but you are required to submit sufficient evidence to convince the examiner that a correct procedure has been used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

COMPUTING RESOURCES

- Candidates will require access to a computer with a spreadsheet program, a linear programming package and suitable printing facilities throughout the examination.

1 The bread man calls early at a remote mountain village on every third day, including weekends. Ioanna always buys either one or two loaves, randomly and each with probability 0.5.

The following random variable is a good model of Ioanna's daily bread requirements.

Daily requirements (loaves)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
Probability	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

(i) Build a spreadsheet simulation of this system and run it for 100 days, starting with a day on which Ioanna starts with half a loaf in stock and on which the bread man calls. **[7]**

(ii) Define two measures of system performance. Add these measures to your simulation. **[2]**

(iii) Repeat your simulation a number of times and report on the behaviour of your two measures. **[4]**

Ioanna's friend advises her that it would be better if she took account of her stock level when purchasing bread. She suggests that Ioanna should purchase one loaf if her stock is 0.75 loaves or more, and two loaves otherwise.

(iv) Investigate and report on how this system performs, comparing it with the original system. **[5]**

2 (This question is concerned only with working days, and ignores weekends.)

Ioanna likes to keep €500 in her current account. Her bank is a small agricultural bank in a rural region, which does not offer internet banking. Ioanna has a separate savings account, and she either transfers money from this to her current account, or vice-versa, to keep the current account balance at €500. However, her instructions to move money take 3 days to be put into effect.

At the close of banking on day 1, Ioanna's current account has a balance of €450, and she issues an instruction to move €50 in from her savings account.

At the end of day 2, the current account contains €520, and she instructs that €20 be moved to her savings account.

At the end of day 3, the current account contains €410, and she instructs that €90 be moved in to it from her savings account.

She continues to operate this strategy in subsequent days.

During day 4, the €50 from her day 1 instruction arrives in the current account. Assume that this is the only change to the current account balance during day 4.

(i) Assuming that there are no changes to the current account in subsequent days other than those following Ioanna's instructions, give a recurrence relation for e_n, the amount in the current account on day n, in terms of e_{n-1} and e_{n-3} ($n \geqslant 4$). **[1]**

(ii) Construct a spreadsheet to show how Ioanna's current account balance varies over a period of 25 working days. **[2]**

Ioanna is promised an improved service in which her instructions are put into effect after 2 working days.

(iii) Give a recurrence relation for e_n, the amount in the current account on day n, in terms of e_{n-1} and e_{n-2} ($n \geqslant 3$) under this new service. **[1]**

(iv) Construct a spreadsheet to show how Ioanna's current account balance would vary under this new service over a period of 25 working days, starting with €450 in the account on day 1, and €520 on day 2. During day 3, €110 leaves the current account and €50 arrives following Ioanna's day 1 instruction. Subsequently the only changes are due to Ioanna's instructions. **[2]**

Ioanna is still unhappy with the fluctuations in the level of her current account, and complains to her bank manager. He cannot improve the bank's service level any further, but he advises her to make her daily adjustments equal to a proportion of the difference between €500 and the amount in her current account, ie so that the daily change is €p(500 – balance), where $0 \leqslant p \leqslant 1$.

(v) Give the new recurrence relation for e_n when Ioanna implements this advice. **[1]**

(vi) Solve the recurrence relation when $p = \frac{2}{9}$, $e_1 = 450$ and $e_2 = 520$. **[9]**

(vii) Construct a spreadsheet to check your answer to part **(vi)**. **[2]**

3 The manager of an athletics club has 8 runners to allocate to positions 1, 2, 3 and 4 in two sprint relay teams. The table shows historical information giving the past mean times (in seconds) of the athletes when running in each of the four positions. She wants to minimise the expected total running time.

athlete \ position	1	2	3	4
A	11.12	11.34	11.74	11.63
B	12.01	12.23	11.89	12.17
C	11.24	11.09	11.56	11.65
D	13.34	12.95	12.67	13.01
E	12.54	12.37	12.21	12.45
F	11.87	11.74	11.35	11.21
G	11.52	11.42	11.37	11.74
H	12.08	12.43	12.32	12.57

The manager sets up the problem as an allocation problem. There are 8 athletes to be allocated to 8 positions. Numbers 1, 2, 3 and 4 represent the positions in one team, and numbers 5, 6, 7 and 8 represent the corresponding positions in the other team.

(i) Set up an LP to solve this allocation problem. Solve it and interpret your solution. **[7]**

(ii) Athlete C, one of the fastest, complains that this method of team selection will not maximise his chances of winning a medal. Why might he argue thus? **[1]**

(iii) Set up LPs to choose the best team out of the 8 athletes, and their best positions, and the best positions for the athletes in the second team. Solve your LPs and interpret the solutions. **[10]**

4 Each of the customers at a restaurant orders a main meal. In addition, some have a starter, some have a dessert, and some have both starter and dessert.

The individual dishes vary in price, but the management is encouraging custom by making two offers: any starter and main for £15; any main and dessert for £10.

In addition the management is encouraging custom by not restricting the offers to orders placed by individuals. So, for instance, if two people share a meal in which one has a starter, a main and a dessert, and the other has just a main, then they could choose to pay £25, ie £15 for a starter and a main, and £10 for a main and a dessert.

A party of 8 diners orders the following dishes.

5 starters priced at £8.50, £7.65, £4.32, £5.67 and £5.67
8 mains priced at £12.42, £9.85, £13.36, £21.25, £12.42, £17.85, £13.63 and £13.63
4 desserts priced at £6.85, £5.32, £3.42 and £10.18

The following LP computes the minimum price payable by the party.

```
min  8.50s1+7.65s2+4.32s3+5.67s4+5.67s5
     +12.42m1+9.85m2+13.36m3+21.25m4+12.42m5+17.85m6+13.63m7+13.63m8
     +6.85d1+5.32d2+3.42d3+10.18d4
     +15sm+10md
st   m1+m2+m3+m4+m5+m6+m7+m8+sm+md=8
     s1+s2+s3+s4+s5+sm=5
     d1+d2+d3+d4+md=4
end
int 17
```

(i) Run the LP, and interpret the output. **[3]**

(ii) Explain what the variables represent, and the meaning of "int 17". **[6]**

(iii) The optimal solution involves the party paying separately for the cheapest starter. Explain why this is so, when the cheapest starter costs more than the cheapest dessert. **[1]**

To encourage even more custom the restaurant's management considers introducing a special price of £17.50 for any 3-course meal, starter, main and dessert, with the same rules as before, ie which dishes individual party members eat is ignored in the pricing.

(iv) Modify the LP to find the minimum price now payable by the party of 8. **[5]**

(v) Run your modified LP and show that the new offer is of no worth to the party. **[1]**

(vi) Verify that, to be of any worth at all to the party, the 3-course meal would have to be priced at £15.14. Interpret the corresponding solution. **[2]**

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