OCR
RECOGNISING ACHIEVEMENT

Friday 17 May 2013 – Morning

AS GCE MATHEMATICS (MEI)

4776/01 Numerical Methods

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4776/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

* 4 7 1 5 9 4 0 6 1 3 *

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 **(i)** Show by sketching two curves on the same axes that the equation

$$x^2 = \cos x,$$

where x is in radians, has exactly one positive root. Give a rough initial estimate of the root. **[3]**

(ii) By re-arranging the equation, find an iterative formula for x_{r+1} in terms of x_r. Use this iterative formula to find the root correct to 2 decimal places. **[5]**

2 This question concerns binomial coefficients of the form $\binom{2n}{n}$, where $\binom{2n}{n} = \frac{(2n)!}{(n!)^2}$.

An approximate formula for $\binom{2n}{n}$ is $\frac{4^n}{\sqrt{n\pi}}$.

(i) Calculate the absolute and relative errors in the approximate formula for $n = 5$ and $n = 10$. Comment briefly on how the absolute errors and relative errors appear to change with n. **[5]**

(ii) It can be shown that the relative errors in part **(i)** are approximately equal to $\frac{1}{kn}$ for some integer k. Use the values calculated in part **(i)** to determine k. **[2]**

3 The function $f(x)$ has the values shown in the table.

x	0.1	0.2	0.3	0.4
$f(x)$	1.641	1.990	1.840	1.192

(i) Show by means of a difference table that $f(x)$ can be closely approximated by a quadratic function. **[3]**

(ii) Use Newton's forward difference interpolation formula to obtain an estimate of $f(0.15)$. **[4]**

4 **(i)** Show, graphically or otherwise, that the equation

$$2^x + 3^x = 4 \qquad (*)$$

has exactly one root.

Show that the root lies in the interval [0.7, 0.8]. **[4]**

(ii) Use the method of false position to find the root of (*) correct to 2 decimal places. **[4]**

5 The values of the function $g(x)$ in the table are correct to 4 decimal places.

x	–0.2	–0.15	–0.1	–0.05	0	0.05	0.1	0.15	0.2
$g(x)$	1.1292	1.1540	1.1766	1.1974	1.2163	1.2335	1.2489	1.2625	1.2745

(i) Use the central difference formula with suitable values of h to obtain a sequence of three estimates of $g'(0)$. **[4]**

(ii) Hence give a value for $g'(0)$ to an appropriate degree of accuracy, explaining your reasoning. **[2]**

Section B (36 marks)

6 In this question, $I = \int_0^{0.5} \sqrt{1 + \tan x}\, dx$, where x is in radians. Estimates of I should be given correct to 6 decimal places.

(i) Obtain the trapezium rule and mid-point rule estimates of I with $h = 0.5$.

Use these two values to obtain a Simpson's rule estimate of I. **[3]**

(ii) Find, as efficiently as possible, two further trapezium rule estimates, two further mid-point rule estimates, and two further Simpson's rule estimates.

Give the value of I to the accuracy that is justified. **[7]**

(iii) Find the differences and the ratio of differences for the trapezium rule estimates and also for the mid-point rule estimates.

What do the ratios of differences indicate?

State, with a reason, whether either of the mid-point and trapezium rules gives more accurate estimates than the other. **[8]**

[Question 7 is printed overleaf]

7 The series $S_n = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}}$ is summed, for various values of n, using a spreadsheet. The spreadsheet gives the answers $\mathrm{S}_{100} = 18.5896$ and $\mathrm{S}_{200} = 26.8593$. For the purposes of this question, these values may be regarded as exact.

(i) The same calculations are now carried out with each term in the series rounded to 4 decimal places. The answers obtained are 18.5897 and 26.8589 respectively.

Explain how it arises that one sum is too large and the other is too small. **[2]**

(ii) Now suppose that the same calculations were carried out with each term in the series chopped to 4 decimal places. Estimate the answers that would be obtained, explaining your reasoning. **[4]**

(iii) Show, by using the mid-point rule on the integral $\int_{k-0.5}^{k+0.5} \frac{1}{\sqrt{x}} \mathrm{d}x$, that

$$\frac{1}{\sqrt{k}} \approx 2\left(\sqrt{k+0.5} - \sqrt{k-0.5}\right).$$ **[4]**

(iv) It follows from the result in part **(iii)** that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} \approx 2\left(\sqrt{n+0.5} - \sqrt{0.5}\right).$$

Use this result to find approximations for S_{100} and S_{200}. Find the errors in these approximations. What do you notice about the values of these errors? **[5]**

(v) Making a suitable assumption about the error, obtain as accurate an estimate of S_{1000} as you can. **[3]**