OXFORD CAMBRIDGE AND RSA EXAMINATIONS
AS GCE

4755/01

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

MONDAY 13 MAY 2013: Afternoon

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4755/01
MEI Examination Formulae and Tables (MF2)
Insert for question 7(ii) (inserted)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (CW) 1648595153 80053/2

INSTRUCTIONS TO CANDIDATES

- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.**
- **IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **Answer ALL the questions.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Final answers should be given to a degree of accuracy appropriate to the context.**

INFORMATION FOR CANDIDATES

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.**
- **The total number of marks for this paper is 72.**
- **Any blank pages are indicated.**

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

SECTION A (36 marks)

1 Find the values of A, B, C and D in the identity $2x(x^2 - 5) \equiv (x - 2)(Ax^2 + Bx + C) + D$. [5]

2 You are given that $z = \frac{3}{2}$ is a root of the cubic equation $2z^3 + 9z^2 + 2z - 30 = 0$. Find the other two roots. [6]

3 You are given that $\mathbf{N} = \begin{pmatrix} -9 & -2 & -4 \\ 3 & 2 & 2 \\ 5 & 1 & 2 \end{pmatrix}$

and $\mathbf{N}^{-1} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ -\frac{7}{2} & p & -6 \end{pmatrix}$.

(i) Find the value of p. [2]

(ii) Solve the equation $\mathbf{N}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -39 \\ 5 \\ 22 \end{pmatrix}$. [4]

4 The complex number z_1 is $3 - 2\mathrm{j}$ and the complex number z_2 has modulus 5 and argument $\frac{\pi}{4}$.

(i) Express z_2 in the form $a + b\mathrm{j}$, giving a and b in exact form. [2]

(ii) Represent z_1, z_2, $z_1 + z_2$ and $z_1 - z_2$ on a single Argand diagram. [4]

5 You are given that $\dfrac{4}{(4n-3)(4n+1)} \equiv \dfrac{1}{4n-3} - \dfrac{1}{4n+1}$.
Use the method of differences to show that

$$\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+1)} = \frac{n}{4n+1}. \quad [6]$$

6 The cubic equation $x^3 - 5x^2 + 3x - 6 = 0$ has roots α, β and γ. Find a cubic equation with roots $\dfrac{\alpha}{3} + 1$, $\dfrac{\beta}{3} + 1$ and $\dfrac{\gamma}{3} + 1$, simplifying your answer as far as possible. [7]

SECTION B (36 marks)

7 Fig. 7 below shows an incomplete sketch of $y = \dfrac{cx^2}{(bx-1)(x+a)}$ where a, b and c are integers. The asymptotes of the curve are also shown.

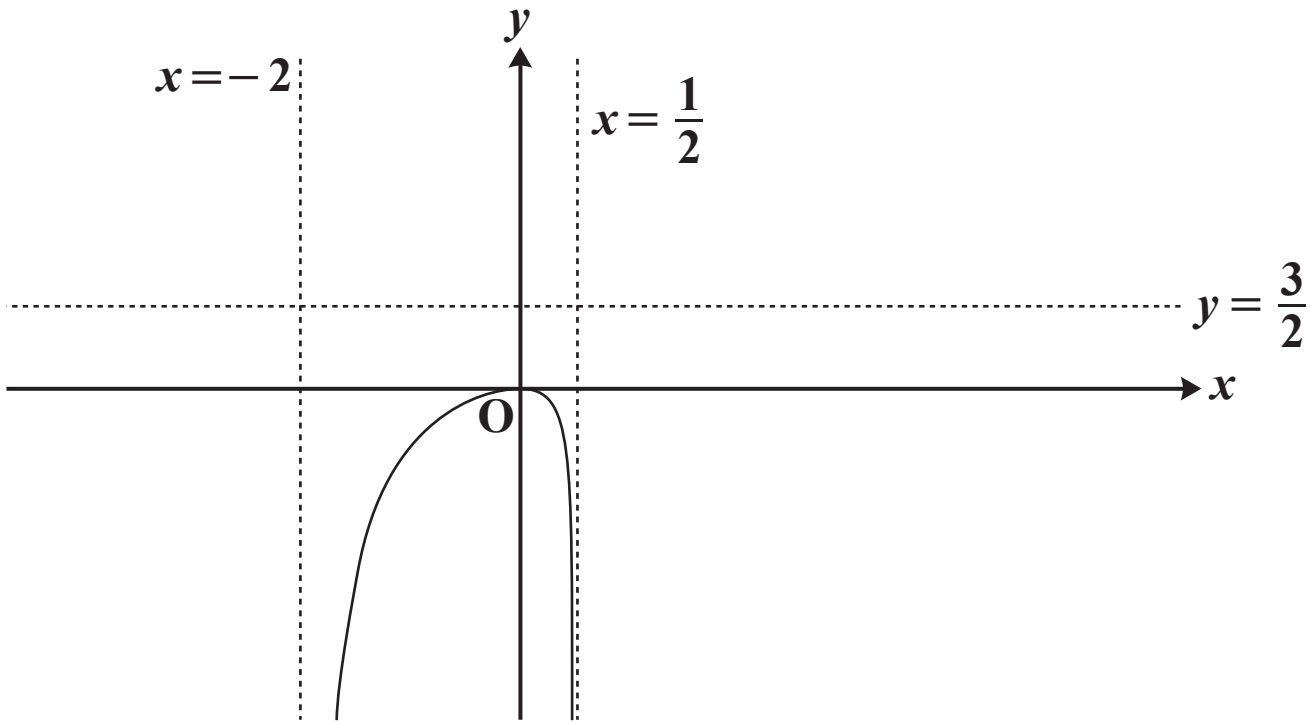

FIG. 7

(i) Determine the values of a, b and c. [4]

Use these values of a, b and c throughout the rest of the question.

(ii) Determine how the curve approaches the horizontal asymptote for large positive values of x, and for large negative values of x, justifying your answer. On the copy of Fig. 7 on the insert, sketch the rest of the curve. [4]

(iii) Find the x coordinates of the points on the curve where $y = 1$. Write down the solution to the inequality $\dfrac{cx^2}{(bx-1)(x+a)} < 1$. [4]

8 (i) Use standard series formulae to show that

$$\sum_{r=1}^{n} [r(r-1)-1] = \frac{1}{3}n(n+2)(n-2). \qquad (*) \text{ [5]}$$

(ii) Prove (*) by mathematical induction. [7]

9 (i) Describe fully the transformation $\mathrm{Q_T}$, represented by the matrix Q, where $\mathrm{Q} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. [2]

The transformation $\mathrm{M_T}$ is represented by the matrix M, where $\mathrm{M} = \begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$.

(ii) $\mathrm{M_T}$ maps all points on the line $y = 2$ onto a single point, P. Find the coordinates of P. [2]

(iii) $\mathrm{M_T}$ maps all points on the plane onto a single line, l. Find the equation of l. [2]

(iv) $\mathrm{M_T}$ maps all points on the line n onto the point $(-6, 6)$. Find the equation of n. [2]

(v) Show that M is singular. Relate this to the transformation it represents. [2]

(vi) $\mathrm{R_T}$ is the composite transformation $\mathrm{M_T}$ followed by $\mathrm{Q_T}$. $\mathrm{R_T}$ maps all points on the plane onto the line q. Find the equation of q. [2]