OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4763/01

MATHEMATICS (MEI)

Mechanics 3

QUESTION PAPER

THURSDAY 6 JUNE 2013: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4763/01
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (CW/SW) 2799835478 81721

INSTRUCTIONS TO CANDIDATES

- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.**
- **IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **Answer ALL questions.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Final answers should be given to a degree of accuracy appropriate to the context.**
- **The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 (a) A particle P of mass 1.5 kg is connected to a fixed point by a light inextensible string of length 3.2 m. The particle P is moving as a conical pendulum in a horizontal circle at a constant angular speed of 2.5 rad s^{-1}.

(i) Find the tension in the string. [4]

(ii) Find the angle that the string makes with the vertical. [2]

(b) A particle Q of mass m moves on a smooth horizontal surface, and is connected to a fixed point on the surface by a light elastic string of natural length d and stiffness k. With the string at its natural length, Q is set in motion with initial speed u perpendicular to the string. In the subsequent motion, the maximum length of the string is $2d$, and the string first returns to its natural length after time t.

You are given that $u = \sqrt{\dfrac{4kd^2}{3m}}$ and $t = Ak^{\alpha}d^{\beta}m^{\gamma}$, where A is a dimensionless constant.

(i) Show that the dimensions of k are MT^{-2}. [1]

(ii) Show that the equation $u = \sqrt{\dfrac{4kd^2}{3m}}$ is dimensionally consistent. [2]

(iii) Find α, β and γ. [4]

You are now given that Q has mass 5 kg, and the string has natural length 0.7 m and stiffness 60 N m^{-1}.

(iv) Find the initial speed u, and use conservation of energy to find the speed of Q at the instant when the length of the string is double its natural length. [5]

2 A particle P of mass 0.25 kg is connected to a fixed point O by a light inextensible string of length a metres, and is moving in a vertical circle with centre O and radius a metres. When P is vertically below O, its speed is $8.4\,\mathrm{m\,s^{-1}}$. When OP makes an angle θ with the downward vertical, and the string is still taut, P has speed $v\,\mathrm{m\,s^{-1}}$ and the tension in the string is T N, as shown in Fig. 2 below.

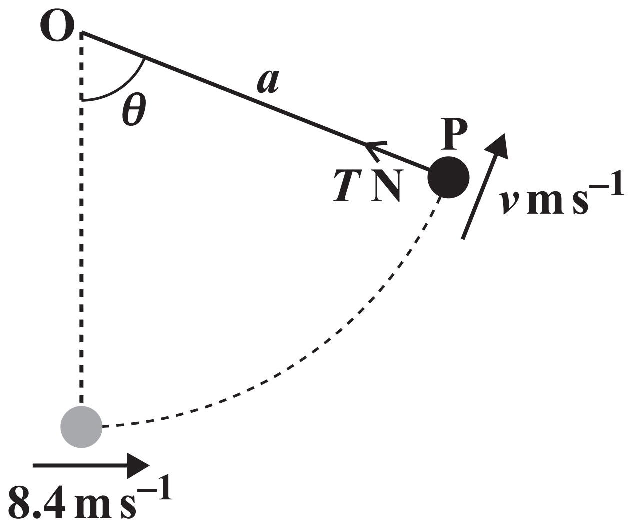

FIG. 2

(i) Find an expression for v^2 in terms of a and θ, and show that

$$T = \frac{17.64}{a} + 7.35\cos\theta - 4.9.$$ [7]

(ii) Given that $a = 0.9$, show that P moves in a complete circle. Find the maximum and minimum magnitudes of the tension in the string. [4]

(iii) Find the largest value of a for which P moves in a complete circle. [3]

(iv) Given that $a = 1.6$, find the speed of P at the instant when the string first becomes slack. [4]

3 **A light spring, with modulus of elasticity 686 N, has one end attached to a fixed point A. The other end is attached to a particle P of mass 18 kg which hangs in equilibrium when it is 2.2 m vertically below A.**

(i) Find the natural length of the spring AP. [2]

Another light spring has natural length 2.5 m and modulus of elasticity 145 N. One end of this spring is now attached to the particle P, and the other end is attached to a fixed point B which is 2.5 m vertically below P (so leaving the equilibrium position of P unchanged). While in its equilibrium position, P is set in motion with initial velocity 3.4 m s^{-1} vertically downwards, as shown in Fig. 3 below. It now executes simple harmonic motion along part of the vertical line AB.

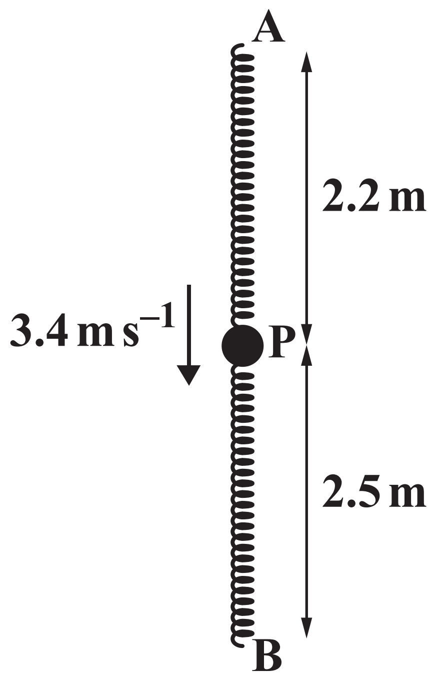

FIG. 3

At time t seconds after it is set in motion, P is x metres below its equilibrium position.

(ii) Show that the tension in the spring AP is $(176.4 + 392x)$N, and write down an expression for the thrust in the spring BP. [3]

(iii) Show that $\frac{d^2x}{dt^2} = -25x$. [3]

(iv) Find the period and the amplitude of the motion. [3]

(v) Find the magnitude and direction of the velocity of P when $t = 2.4$. [3]

(vi) Find the total distance travelled by P during the first 2.4 seconds of its motion. [4]

4 (a) A uniform solid of revolution S is formed by rotating the region enclosed between the x-axis and the curve $y = x\sqrt{4-x}$ for $0 \leqslant x \leqslant 4$ through 2π radians about the x-axis, as shown in Fig. 4.1 below. O is the origin and the end A of the solid is at the point (4, 0).

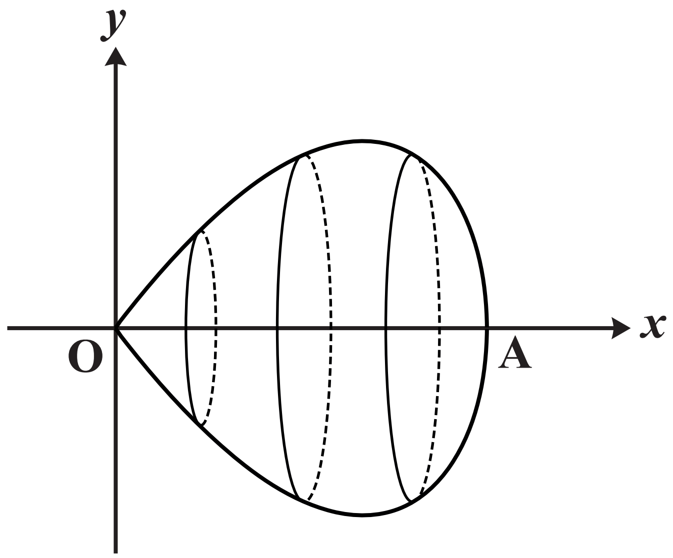

FIG. 4.1

(i) Find the x-coordinate of the centre of mass of the solid S. [6]

The solid S has weight W, and it is freely hinged to a fixed point at O. A horizontal force, of magnitude W acting in the vertical plane containing OA, is applied at the point A, as shown in Fig. 4.2 below. S is in equilibrium.

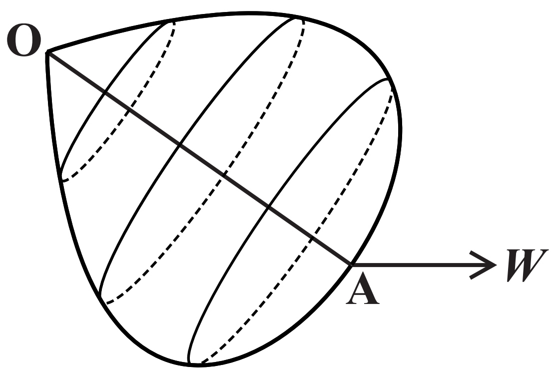

FIG. 4.2

(ii) Find the angle that OA makes with the vertical. [3]

(b) Fig. 4.3 below shows the region bounded by the x-axis, the y-axis, the line $y = 8$ and the curve $y = (x - 2)^3$ for $0 \leqslant y \leqslant 8$.

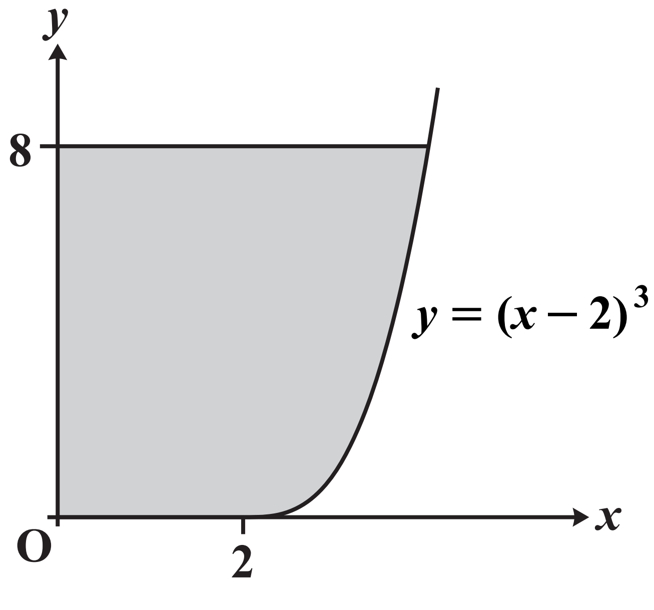

FIG. 4.3

Find the coordinates of the centre of mass of a uniform lamina occupying this region. [9]

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