OXFORD CAMBRIDGE AND RSA EXAMINATIONS
AS GCE

4776/01

MATHEMATICS (MEI)

Numerical Methods

QUESTION PAPER

FRIDAY 17 MAY 2013: Morning
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4776/01
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (KN) 2464471161 80447/1

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.
- IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer ALL the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- Any blank pages are indicated.

INSTRUCTIONS TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

SECTION A (36 marks)

1 (i) Show by sketching two curves on the same axes that the equation

$$x^2 = \cos x,$$

where x is in radians, has exactly one positive root. Give a rough initial estimate of the root. [3]

(ii) By re-arranging the equation, find an iterative formula for x_{r+1} in terms of x_r. Use this iterative formula to find the root correct to 2 decimal places. [5]

2 This question concerns binomial coefficients of the form $\binom{2n}{n}$, where $\binom{2n}{n} = \dfrac{(2n)!}{(n!)^2}$.

An approximate formula for $\binom{2n}{n}$ is $\dfrac{4^n}{\sqrt{n\pi}}$.

(i) Calculate the absolute and relative errors in the approximate formula for $n = 5$ and $n = 10$. Comment briefly on how the absolute errors and relative errors appear to change with n. [5]

(ii) It can be shown that the relative errors in part (i) are approximately equal to $\dfrac{1}{kn}$ for some integer k. Use the values calculated in part (i) to determine k. [2]

3 **The function f(x) has the values shown in the following table.**

x	0.1	0.2	0.3	0.4
f(x)	1.641	1.990	1.840	1.192

(i) Show by means of a difference table that f(x) can be closely approximated by a quadratic function. [3]

(ii) Use Newton's forward difference interpolation formula to obtain an estimate of f(0.15). [4]

4 **(i) Show, graphically or otherwise, that the equation**

$$2^x + 3^x = 4 \qquad\qquad (*)$$

has exactly one root.

Show that the root lies in the interval [0.7, 0.8]. [4]

(ii) Use the method of false position to find the root of (*) correct to 2 decimal places. [4]

5 **The values of the function g(x) in the following table are correct to 4 decimal places.**

x	g(x)
–0.2	1.1292
–0.15	1.1540
–0.1	1.1766
–0.05	1.1974
0	1.2163
0.05	1.2335
0.1	1.2489
0.15	1.2625
0.2	1.2745

(i) Use the central difference formula with suitable values of h to obtain a sequence of three estimates of g′(0). [4]

(ii) Hence give a value for g′(0) to an appropriate degree of accuracy, explaining your reasoning. [2]

SECTION B (36 marks)

6 In this question, $I = \int_0^{0.5} \sqrt{1 + \tan x}\, dx$, where x is in radians.

Estimates of I should be given correct to 6 decimal places.

(i) Obtain the trapezium rule and mid-point rule estimates of I with $h = 0.5$.

Use these two values to obtain a Simpson's rule estimate of I. [3]

(ii) Find, as efficiently as possible, two further trapezium rule estimates, two further mid-point rule estimates, and two further Simpson's rule estimates.

Give the value of I to the accuracy that is justified. [7]

(iii) Find the differences and the ratio of differences for the trapezium rule estimates and also for the mid-point rule estimates.

What do the ratios of differences indicate?

State, with a reason, whether either of the mid-point and trapezium rules gives more accurate estimates than the other. [8]

Question 7 begins on page 8.

7 **The series $S_n = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}}$ is summed, for various values of n, using a spreadsheet. The spreadsheet gives the answers $S_{100} = 18.5896$ and $S_{200} = 26.8593$. For the purposes of this question, these values may be regarded as exact.**

(i) The same calculations are now carried out with each term in the series rounded to 4 decimal places. The answers obtained are 18.5897 and 26.8589 respectively.

Explain how it arises that one sum is too large and the other is too small. [2]

(ii) Now suppose that the same calculations were carried out with each term in the series chopped to 4 decimal places. Estimate the answers that would be obtained, explaining your reasoning. [4]

(iii) Show, by using the mid-point rule on the integral

$$\int_{k-0.5}^{k+0.5} \frac{1}{\sqrt{x}}\,\mathrm{d}x, \text{ that}$$

$$\frac{1}{\sqrt{k}} \approx 2\left(\sqrt{k+0.5} - \sqrt{k-0.5}\right). \quad [4]$$

(iv) It follows from the result in part (iii) that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} \approx 2\left(\sqrt{n+0.5} - \sqrt{0.5}\right).$$

Use this result to find approximations for S_{100} and S_{200}. Find the errors in these approximations. What do you notice about the values of these errors? [5]

(v) Making a suitable assumption about the error, obtain as accurate an estimate of S_{1000} as you can. [3]

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