

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**  
**A2 GCE**  
**4753/01**  
**MATHEMATICS (MEI)**  
**Methods for Advanced Mathematics (C3)**  
**QUESTION PAPER**

**TUESDAY 18 JUNE 2013: Morning**  
**DURATION: 1 hour 30 minutes**  
**plus your additional time allowance**

**MODIFIED ENLARGED**

**Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.**

**OCR SUPPLIED MATERIALS:**

**Printed Answer Book 4753/01**  
**MEI Examination Formulae and Tables (MF2)**  
**Insert for questions 7(ii) and 8(iv) (inserted)**

**OTHER MATERIALS REQUIRED:**

**Scientific or graphical calculator**

**READ INSTRUCTIONS OVERLEAF**

## **INSTRUCTIONS TO CANDIDATES**

- **Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.**
- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.**
- **IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **Answer ALL the questions.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Final answers should be given to a degree of accuracy appropriate to the context.**

## **INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **NO MARKS** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- Any blank pages are indicated.

## **INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

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## SECTION A (36 marks)

- 1 Fig. 1 below shows the graphs of  $y = |x|$  and  $y = a|x + b|$ , where  $a$  and  $b$  are constants. The intercepts of  $y = a|x + b|$  with the  $x$ - and  $y$ -axes are  $(-1, 0)$  and  $(0, \frac{1}{2})$  respectively.

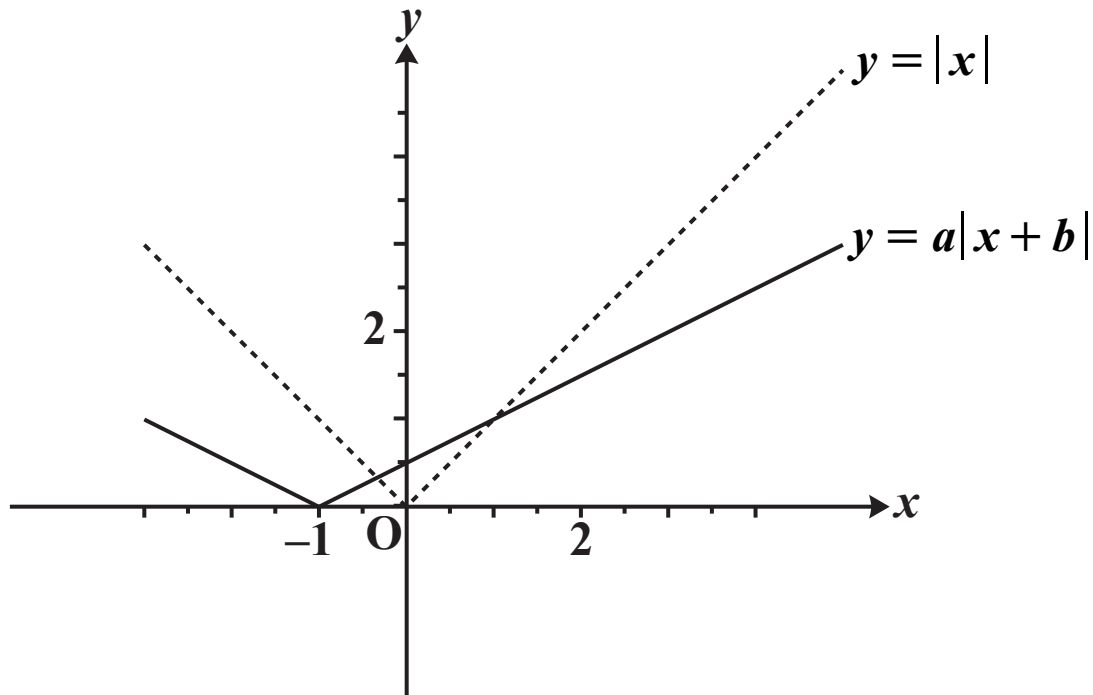


FIG. 1

- (i) Find  $a$  and  $b$ . [2]
- (ii) Find the coordinates of the two points of intersection of the graphs. [4]
- 2 (i) Factorise fully  $n^3 - n$ . [2]
- (ii) Hence prove that, if  $n$  is an integer,  $n^3 - n$  is divisible by 6. [2]

- 3 The function  $f(x)$  is defined by  $f(x) = 1 - 2 \sin x$  for  $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$ . Fig. 3 below shows the curve  $y = f(x)$ .

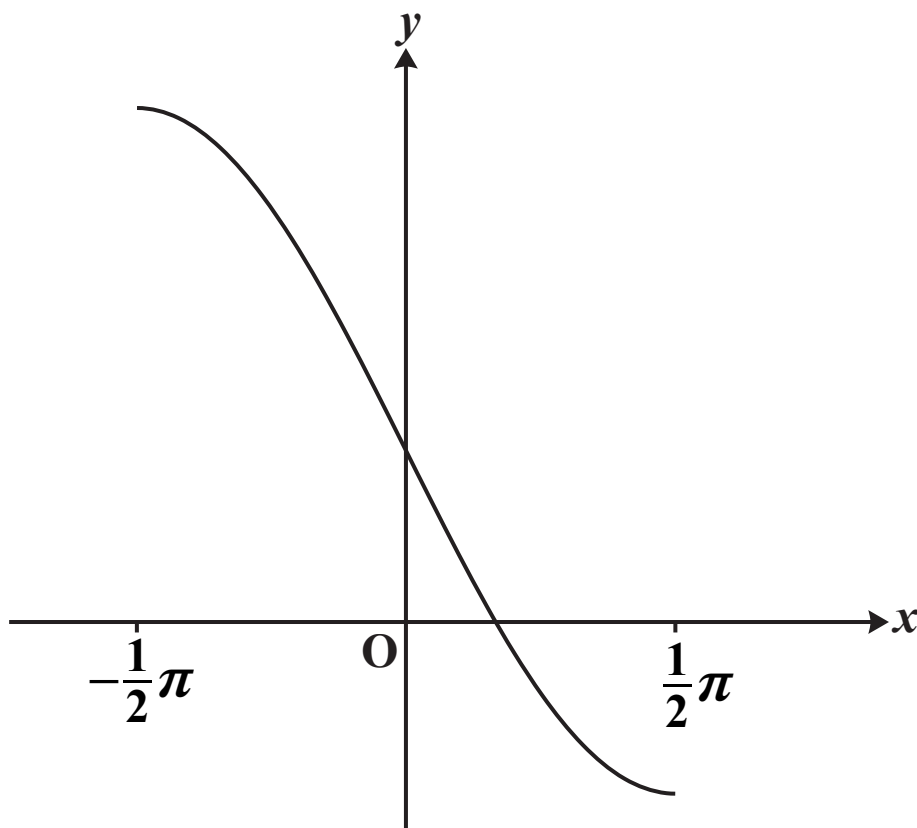


FIG. 3

- (i) Write down the range of the function  $f(x)$ . [2]
- (ii) Find the inverse function  $f^{-1}(x)$ . [3]
- (iii) Find  $f'(0)$ . Hence write down the gradient of  $y = f^{-1}(x)$  at the point  $(1, 0)$ . [3]

- 4 Water flows into a bowl at a constant rate of  $10 \text{ cm}^3 \text{ s}^{-1}$  (see Fig. 4 below).

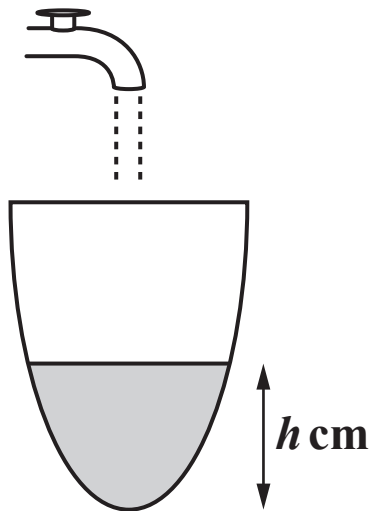


FIG. 4

When the depth of water in the bowl is  $h \text{ cm}$ , the volume of water is  $V \text{ cm}^3$ , where  $V = \pi h^2$ . Find the rate at which the depth is increasing at the instant in time when the depth is  $5 \text{ cm}$ . [5]

- 5 Given that  $y = \ln \left( \sqrt{\frac{2x-1}{2x+1}} \right)$ , show that

$$\frac{dy}{dx} = \frac{1}{2x-1} - \frac{1}{2x+1}. \quad [4]$$

- 6 Using a suitable substitution or otherwise, show that

$$\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{3 + \cos 2x} dx = \frac{1}{2} \ln 2. \quad [5]$$

- 7 (i) Show algebraically that the function  $f(x) = \frac{2x}{1-x^2}$  is odd. [2]

Fig. 7 below shows the curve  $y = f(x)$  for  $0 \leq x \leq 4$ , together with the asymptote  $x = 1$ .

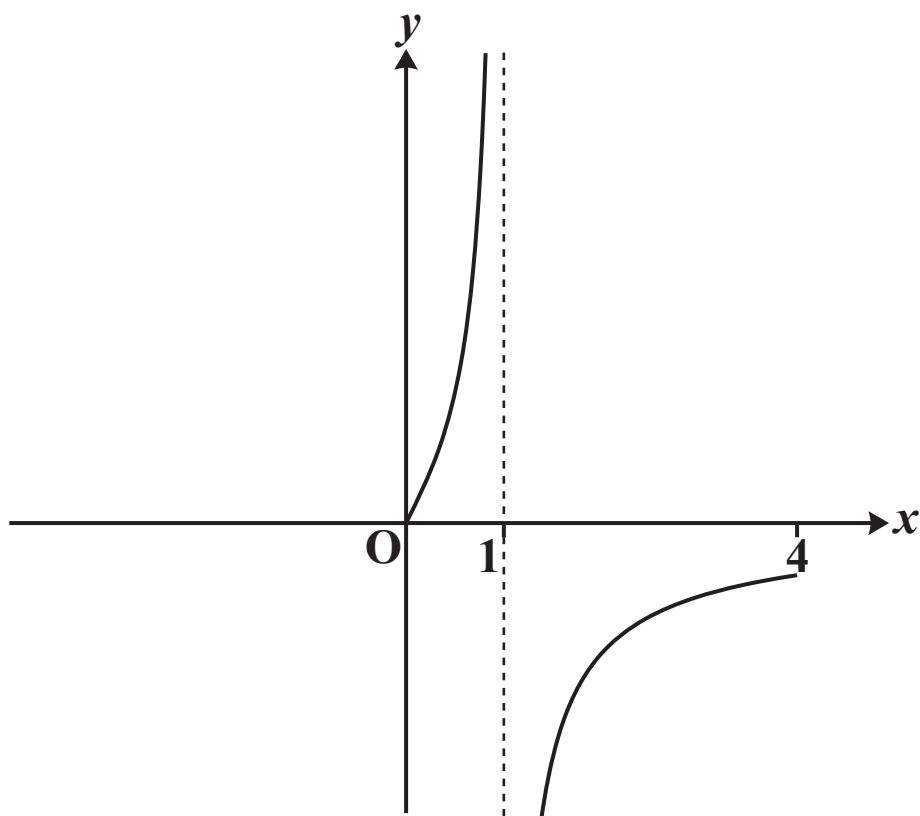


FIG. 7

- (ii) Use the copy of Fig. 7 to complete the curve for  $-4 \leq x \leq 4$ . [You may use the Insert if you wish.] [2]

## SECTION B (36 marks)

- 8 Fig. 8 below shows the curve  $y = f(x)$ , where  $f(x) = (1 - x)e^{2x}$ , with its turning point P.

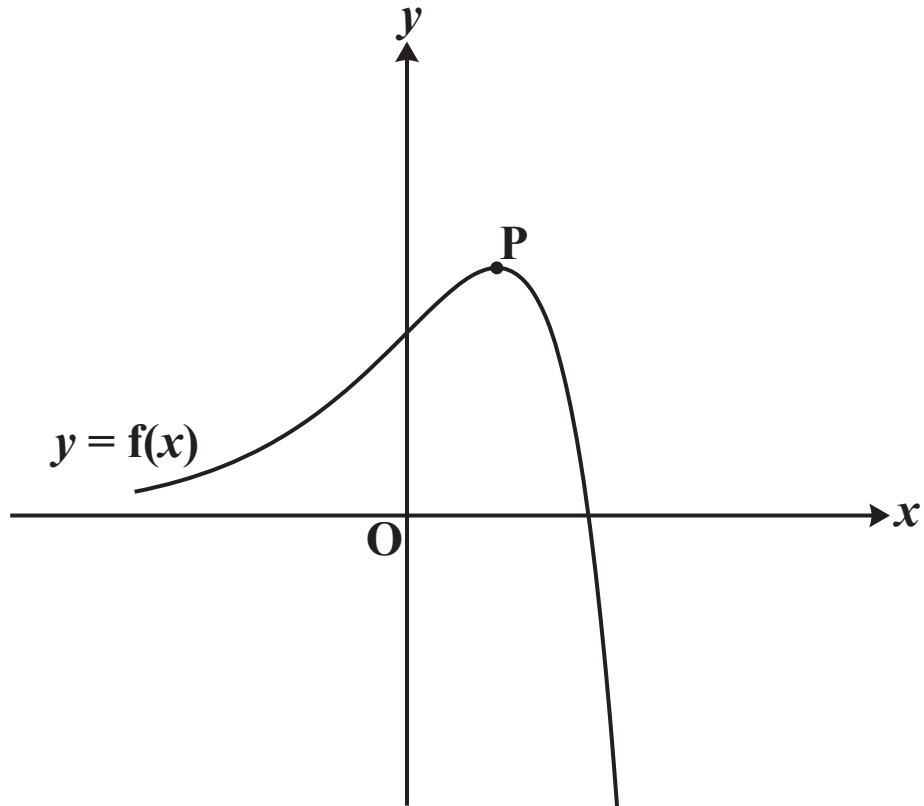


FIG. 8

- (i) Write down the coordinates of the intercepts of  $y = f(x)$  with the  $x$ - and  $y$ -axes. [2]
- (ii) Find the exact coordinates of the turning point P. [6]
- (iii) Show that the exact area of the region enclosed by the curve and the  $x$ - and  $y$ -axes is  $\frac{1}{4}(e^2 - 3)$ . [5]



The function  $g(x)$  is defined by  $g(x) = 3f\left(\frac{1}{2}x\right)$ .

(iv) Express  $g(x)$  in terms of  $x$ .

Sketch the curve  $y = g(x)$  on the copy of Fig. 8, indicating the coordinates of its intercepts with the  $x$ - and  $y$ -axes and of its turning point. [You may use the Insert if you wish.] [4]

(v) Write down the exact area of the region enclosed by the curve  $y = g(x)$  and the  $x$ - and  $y$ -axes. [1]

- 9 Fig. 9 below shows the curve with equation  $y^3 = \frac{x^3}{2x-1}$ . It has an asymptote  $x = a$  and turning point P.

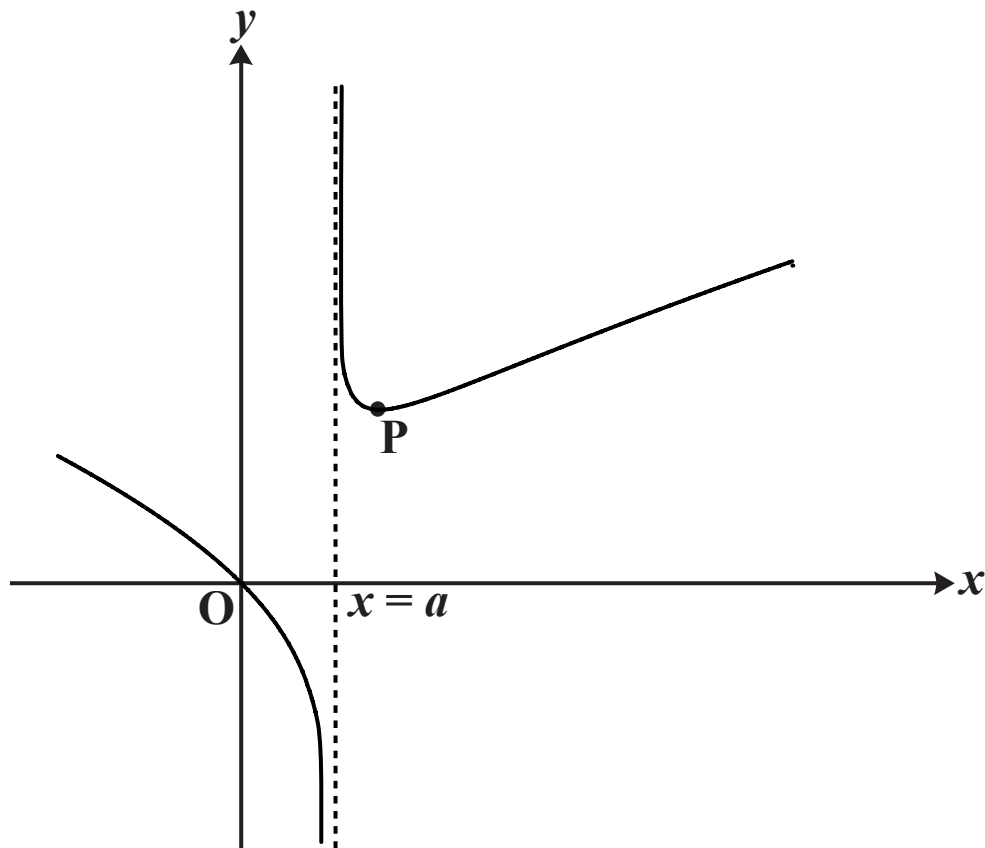


FIG. 9

- (i) Write down the value of  $a$ . [1]
- (ii) Show that  $\frac{dy}{dx} = \frac{4x^3 - 3x^2}{3y^2(2x-1)^2}$ .

Hence find the coordinates of the turning point P, giving the y-coordinate to 3 significant figures. [9]

(iii) Show that the substitution  $u = 2x - 1$  transforms

$$\int \frac{x}{\sqrt[3]{2x-1}} dx \text{ to } \frac{1}{4} \int \left( u^{\frac{2}{3}} + u^{-\frac{1}{3}} \right) du.$$

Hence find the exact area of the region enclosed by the curve  $y^3 = \frac{x^3}{2x-1}$ , the  $x$ -axis and the lines  $x = 1$  and  $x = 4.5$ . [8]

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