

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4756/01

MATHEMATICS (MEI)

**Further Methods for Advanced
Mathematics (FP2)**

QUESTION PAPER

TUESDAY 18 JUNE 2013: Morning

**DURATION: 1 hour 30 minutes
plus your additional time allowance**

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4756/01

MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

INSTRUCTIONS TO CANDIDATES

- **Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.**
- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.**
- **IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **Answer ALL the questions.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Final answers should be given to a degree of accuracy appropriate to the context.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **NO MARKS** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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SECTION A (54 marks)

1 (a) You are given that $f(x) = \frac{1}{(1-2x)^2}$.

Find $f'(x)$, $f''(x)$ and $f'''(x)$. Hence obtain the Maclaurin series for $f(x)$ as far as the term in x^3 .

By considering the equivalent binomial expansion, give the set of values of x for which the Maclaurin series is valid. [7]

(b) A curve has polar equation $r = a \sin 3\theta$, where a is a positive constant and $0 \leq \theta \leq \frac{1}{3}\pi$.

(i) Sketch the curve. [2]

(ii) Find, in terms of a , the cartesian coordinates of the point on the curve furthest from the origin. [4]

(iii) Find, in terms of a , the area of the region enclosed by the curve. [5]

- 2 (a) (i) Use de Moivre's theorem to show that

$$\cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta. \quad [3]$$

- (ii) Given that $\cos 5\theta = 0$ but $\cos \theta \neq 0$, find in surd form the two possible values for $\cos^2 \theta$.

$$\text{Hence show that } \cos 18^\circ = \left(\frac{5 + \sqrt{5}}{8} \right)^{\frac{1}{2}}.$$

Find, in similar form, an expression for $\sin 18^\circ$. [6]

- (b) (i) Find the cube roots of the complex number $4(\sqrt{3} + j)$ in the form $re^{j\theta}$, where $r > 0$ and $0 < \theta < 2\pi$. Illustrate the roots on an Argand diagram. [7]

The points representing the two roots with smallest values of θ are P and Q. The mid-point of PQ is M, and M represents the complex number w .

- (ii) Find the argument of w . Write down the smallest positive integer n for which w^n is a real number. [2]

3 You are given the matrix $A = \begin{pmatrix} k & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix}$.

**(i) Show that when $k = 5$ the determinant of A is zero.
Obtain an expression for the inverse of A when $k \neq 5$. [7]**

(ii) Solve the matrix equation

$$\begin{pmatrix} 4 & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ 1 \\ 2 \end{pmatrix},$$

giving your answer in terms of p . [5]

(iii) Find the value of p for which the matrix equation

$$\begin{pmatrix} 5 & -7 & 4 \\ 2 & -2 & 3 \\ 1 & -3 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ 1 \\ 2 \end{pmatrix}$$

has a solution. Give the general solution in this case and describe it geometrically. [6]

SECTION B (18 marks)

- 4 (i) Prove, using exponential functions, that $\cosh^2 u - \sinh^2 u = 1$. [2]

- (ii) Given that $y = \operatorname{arsinh} x$, show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}},$$

and that

$$y = \ln(x + \sqrt{1+x^2}). \quad [9]$$

- (iii) Show that

$$\int_0^2 \frac{1}{\sqrt{4+9x^2}} dx = \frac{1}{3} \ln(3 + \sqrt{10}). \quad [4]$$

- (iv) Find, in exact logarithmic form,

$$\int_0^1 \frac{1}{\sqrt{1+x^2}} \operatorname{arsinh} x \, dx. \quad [3]$$

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