

Mathematics (MEI)

Advanced GCE

Unit 4769: Statistics 4

Mark Scheme for June 2013

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations

Annotation in scoris	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply — quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 — but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1	(i)	$P(X = x) = \frac{e^{-\theta} \theta^x}{x!}$ $L = \frac{e^{-\theta} \theta^{x_1}}{x_1!} \cdots \frac{e^{-\theta} \theta^{x_n}}{x_n!} = \frac{e^{-n\theta} \theta^{\sum x_i}}{x_1! x_2! \cdots x_n!}$ $\ln L = -n\theta + \sum x_i \ln \theta - \sum \ln x_i!$ $\frac{d \ln L}{d\theta} = -n + \frac{\sum x_i}{\theta}$ $= 0 \text{ for ML Est } \hat{\theta}.$ $\therefore n\hat{\theta} = \sum x_i, \quad \text{i.e. } \hat{\theta} = \bar{x}.$ <p>Confirmation that this is a maximum:</p> $\frac{d^2 \ln L}{d\theta^2} = -\frac{\sum x_i}{\theta^2} < 0.$	M1 A1 M1 A1 M1 A1 M1 A1 A1 M1 A1 [10]	M1 for general product form. A1 (a.e.f.) for answer. M1 is for taking logs (base e). Allow (\pm) constant instead of last term. M1 for differentiating, A1 for answer. Any or all of the four M1 marks down to here can be awarded in part (iv) if not awarded here.
1	(ii)	$P(X = 0) = e^{-\theta}.$ ML Est of $e^{-\theta} = e^{-\hat{\theta}}, \quad \text{i.e. the estimate is } e^{-\bar{x}}.$	M1 M1 A1 [3]	M1 for "invariance property", not necessarily named.
1	(iii)	We have estimate of $P(X = 0) = e^{-5} = 0.0067$, so we might reasonably expect around $1000e^{-5} \approx 6.7$ cases of zero in a sample of size 1000 – finding no such cases seems suspicious.	M1 E1 E1 [3]	Sensible use of $n = 1000$ and $\bar{x} = 5$.

Question		Answer	Marks	Guidance
1	(iv)	<p>X has Poisson distribution "scaled up" so that , therefore ,</p> <p>where $1 = k \sum_{x=1}^{\infty} \frac{e^{\theta} \cdot x}{x!} = k \left\{ \sum_{x=0}^{\infty} \frac{e^{\theta} \cdot x}{x!} - e^{-\theta} \right\} = k \{1 - e^{-\theta}\}$.</p> <p>$\therefore k = \frac{1}{1 - e^{-\theta}}$.</p> <p>$\therefore P(X = x) = \frac{1}{1 - e^{-\theta}} \cdot \frac{e^{-\theta} \theta^x}{x!} = \frac{\theta^x}{(e^{\theta} - 1)x!}$ [for $x = 1, 2, \dots$].</p> <p>$\therefore L = \frac{\theta^{\sum x_i}}{(e^{\theta} - 1)^n x_1! x_2! \dots x_n!}$</p> <p>$\therefore \ln L = \ln \theta - n \ln e - \ln (e^{\theta} - 1) - \sum \ln x_i$</p> <p>$\therefore \frac{d \ln L}{d \theta} = \frac{\sum x_i}{\theta} - \frac{n e^{\theta}}{e^{\theta} - 1}$,</p> <p>and on setting this equal to zero we get that $\hat{\theta}$ satisfies</p> <p>$\frac{\theta e^{\theta}}{e^{\theta} - 1} = \frac{\sum x_i}{n} = \bar{x}$.</p>	M1 M1 M1 A1 A1 A1 A1 A1 A1 [8]	M1 for this idea, however expressed. M1 for sum from 0 to ∞ minus value for $x = 0$. Beware printed answer. Beware printed answer.
2	(i)	<p>(A) $\mu = 0$.</p> <p>(B) $E(X^2) = 8/3$.</p> <p>(C) $\text{Var}(X) = 8/3$.</p>	B1 B1 B1 [3]	
2	(ii)	<p>Mgf of X is $M_X(\theta) = E(e^{\theta X})$</p> $= \left(e^{-2\theta} \cdot \frac{1}{3} \right) + \left(e^0 \cdot \frac{1}{3} \right) + \left(e^{2\theta} \cdot \frac{1}{3} \right)$ $= \frac{1}{3} (1 + e^{2\theta} + e^{-2\theta})$	M1 A1 [2]	Any equivalent form.

Question		Answer	Marks	Guidance
2	(iii)	<p>General results:</p> <p>[Convolution theorem] Mgf of sum of independent random variables = product of their mgfs.</p> <p>[Linear transformation result] $M_{aX+b}(\theta) = e^{b\theta} M_X(a\theta)$.</p> $M_{\Sigma X}(\theta) = \left\{ \frac{1}{3} (1 + e^{2\theta} + e^{-2\theta}) \right\}^n$ $M_{\bar{X}}(\theta) = \left\{ \frac{1}{3} \left(1 + e^{\frac{2\theta}{n}} + e^{-\frac{2\theta}{n}} \right) \right\}^n$ $Z = \frac{\bar{X} \sqrt{n}}{\sigma} - \frac{n}{\sigma} \frac{\sqrt{n} X \sqrt{3}}{2\sqrt{2}}$ $M_Z(\theta) = \left\{ \frac{1}{3} \left(1 + e^{\frac{2\sqrt{3n}}{n} \theta} + e^{-\frac{2\sqrt{3n}}{n} \theta} \right) \right\}^n$ $= \left\{ \frac{1}{3} \left(1 + e^{\frac{\theta\sqrt{3}}{\sqrt{2n}}} + e^{-\frac{\theta\sqrt{3}}{\sqrt{2n}}} \right) \right\}^n.$	B1 M1 M1 A1 A1 B1 A1 A1 [8]	B1 for explicit mention of "independent". Note M1 mark required for f.t. to A marks below. Allow implicit $b = 0$. Note M1 mark required for f.t. to A marks below. Might be implicit in what follows. Beware printed answer.

Question		Answer	Marks	Guidance
2	(iv)	$ \begin{aligned} M_Z(\theta) &= \left\{ \frac{1}{3}(1 + 1 + \frac{\theta\sqrt{3}}{\sqrt{2n}} + \frac{3\theta^2}{\sqrt{2n} \cdot 2!} + \text{terms in } n^{-3/2}, n^{-2}, \dots) \right. \\ &\quad \left. + 1 - \frac{\theta\sqrt{3}}{\sqrt{2n}} + \frac{3\theta^2}{\sqrt{2n} \cdot 2!} + \text{terms in } n^{-3/2}, n^{-2}, \dots \right\}^n \\ &\quad \text{cancel } \sqrt{\quad} \quad \text{neglect} \\ &\approx \left\{ \frac{1}{3} \left(3 + \frac{3\theta^2}{2n} \right) \right\}^n \\ &= \left(1 + \frac{\theta^2}{2n} \right)^n. \end{aligned} $	M1 A1 A1 M1 M1 A1 A1 [7]	M1 for reasonable attempt to expand exponentials. A1 for this line. A1 for this line. M1 for cancelling first order terms, may be implicit. M1 for neglecting higher order terms in n^{-1} , MUST be explicit. Beware printed answer.
2	(v)	<p>Limit is $e^{\theta^2/2}$. This is mgf of $N(0, 1)$.</p> <p>Mgfs are unique (even in this limiting process). So (approximately) distribution of Z is $N(0, 1)$.</p>	M1 M1 B1 A1 [4]	M1 for limit, M1 for recognising mgf. Bracketed phrase is not needed to earn the mark. Bracketed word is not needed to earn the mark.
3	(i)	<p>Type I error: rejecting null hypothesis when it is true</p> <p>Type II error: accepting null hypothesis when it is false</p> <p>OC: $P(\text{accepting null hypothesis as a function of the parameter under investigation})$</p> <p>Power: $P(\text{rejecting null hypothesis as a function of the parameter under investigation})$</p>	B1 B1 B1 B1 B1 B1 B1 B1 [8]	Allow B1 out of 2 for $P(\dots)$. Allow B1 out of 2 for $P(\dots)$. $P(\text{Type II error} \mid \text{the true value of the parameter})$ scores B1+B1. $P(\text{Type I error} \mid \text{the true value of the parameter})$ scores B1+B1.

Question		Answer	Marks	Guidance
3	(ii)	Correct "spike" shape, or point shown. Location of spike or point at θ_0 correct and correctly labelled. Height of spike correct and correctly labelled as 1.	G1 G1 G1 [3]	
3	(iii)	$X \sim N(\mu, 9)$. $n = 25$. $H_0: \mu = 0$. $H_1: \mu \neq 0$. Accept H_0 if $-1 < \bar{x} < 1$. $P(\text{Type I error}) = P\left(\left \bar{X}\right > 1 \mid \bar{X} \sim N\left(0, \frac{9}{25}\right)\right)$ $= P\left(\left N(0, 1)\right > \frac{1-0}{3/5}\right) = 2 \times 0.0478 = 0.0956.$ $P(\text{Type II error when } \mu = 0.5)$ $= P\left(-1 < \bar{X} < 1 \mid \bar{X} \sim N\left(0.5, \frac{9}{25}\right)\right)$ $= P\left(\frac{-1.5}{3/5} < N(0, 1) < \frac{0.5}{3/5}\right) = P(-2.5 < N(0, 1) < 0.8333)$ $= 0.7976 - 0.0062 = 0.7914.$ This is high. We are trying to detect only a small departure from H_0 , the "error" (σ^2) being comparatively large.	M1 M1 A1 M1 M1 M1 A1 E1 E1 [9]	M1 for use of $ \bar{X} > 1$, M1 for distribution of \bar{X} . Either or both marks might be implicit in what follows. Accept a.w.r.t. 0.096. As for the two M1 marks for the Type I error. Standardising with correct end-points.
3	(iv)	Correct shape – must be symmetrical, and reasonable approximation to Normal pdf. "Centre" at 0 and clearly labelled. Height at 0 distinctly less than 1 by about 10% (ft candidate's $P(\text{Type I error})$). Some indication that height at 0.5 is about 0.8 (ft candidate's $P(\text{Type II error})$).	G1 G1 G1 G1 [4]	

Question		Answer	Marks	Guidance
4	(i)	<p>Randomisation is mainly to guard against possible sources of bias, which may be due to subjective allocation of treatments to units or unsuspected.</p> <p>Replication enables an estimate of experimental error to be made.</p>	B1 B1 B1 B1 B1 B1 [6]	<p>Award up to B3 marks. These include B1 for idea of "sources of bias" and another B1 for either idea of "unsuspected" or "subjective".</p> <p>Award up to B3 marks. These include B1 for <i>some</i> concept of replication addressing experimental error and another B1 for the explicit idea that it enables this error to be estimated. The third B1 should be awarded for the general quality of the response, especially the lack of extraneous material in it</p> <p>Alternative suggestions offered by candidates may be rewarded <i>provided they are statistically sound</i>. Suggestions that are limited to particular designs (eg for the advantages of randomisation within a randomised blocks design) may be rewarded if statistically sound, but for at most 2 of the B3 marks in each set.</p>
4	(ii)	<p>μ is the population mean for the entire experiment.</p> <p>α_i is the population amount by which the mean for the ith treatment differs from μ.</p> <p>$e_{ij} \sim \text{ind N}(0, \sigma^2)$</p>	B1 B1 B1 B1 B1 B1 B1 [7]	<p>B1 for explicit mention of "population", B1 for idea of mean for whole experiment.</p> <p>B1 for explicit mention of "population", B1 for idea of difference of means.</p> <p>For "ind N"; allow "uncorrelated".</p> <p>For mean 0.</p> <p>For variance σ^2 [i.e. that the variance is constant].</p>

Question		Answer					Marks	Guidance																
4	(iii)	<p>Null hypothesis: $\alpha_1 = \alpha_2 = \dots = \alpha_k (= 0)$ Alternative hypothesis: not all α_i are equal Note that alternative hypothesis is NOT that all the α_i are different – B0 for alternative hypothesis if this is stated.</p> <table border="1"> <thead> <tr> <th>Source of variation</th> <th>Sum of squares</th> <th>d.f.</th> <th>Mean squares</th> <th>MS ratio</th> </tr> </thead> <tbody> <tr> <td>Between fertilisers</td> <td>219.2</td> <td>4</td> <td>54.8</td> <td>2.7</td> </tr> <tr> <td>Residual</td> <td>304.5</td> <td>15</td> <td>20.3</td> <td></td> </tr> <tr> <td>Total</td> <td>523.7</td> <td>19</td> <td></td> <td></td> </tr> </tbody> </table> <p>Refer 2.7 to $F_{4,15}$. Upper 5% point is 3.06. Not significant. Seems mean effects of fertilisers are all the same.</p>	Source of variation	Sum of squares	d.f.	Mean squares	MS ratio	Between fertilisers	219.2	4	54.8	2.7	Residual	304.5	15	20.3		Total	523.7	19			B1 B1 B1 B1 M1 M1 A1 M1 A1 A1 E1 [11]	<p>No need for definition of k as the number of treatments, and accept simply $\alpha_1 = \alpha_2 = \dots$ with no explicit upper end to the sequence. Accept hypotheses stated verbally provided it is clear that <i>population</i> parameters (means) are being referred to. B1 for <i>each</i> d.f. (4 and 15).</p> <p>For <i>method</i> of mean squares.</p> <p>For <i>method</i> of mean square ratio.</p> <p>A1 c.a.o. for 2.7 (2.6695). f.t. from here provided all M marks earned.</p> <p>No f.t. if wrong but allow M1 for F with candidate's df if both positive and totalling 19. cao. No f.t. if wrong (or if not quoted).</p> <p>Verbal conclusion in context, and not "too assertive".</p>
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